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Compositions of Transformations in Triangle Geometry

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Abstract. In [1], Darij Grinberg announced the following relation: The cyclocevian conjugate of a point is the isotomic conjugate of the anticomplement of the isogonal conjugate of the complement of the isotomic conjugate of the point. We note the following computer-generated result: There is no other non-trivial relation between compositions of these 5 transformations, if we consider relations between compositions having together no more than 6 transformations.

In [1] (see also Kimberling [3], Glossary, article *Cyclocevian Conjugate*), Darij Grinberg stated the following theorem:

Given a point P. Then Cyclocevian conjugate of P = Isotomic conjugate of the anticomplement of the isogonal conjugate of the complement of the isotomic conjugate of P.

Below by transformation we mean one of the following 5 transformations (with relation to a given triangle): Complement, Anticomplement, Isogonal Conjugate, Isotomic Conjugate, Cyclocevian Conjugate. By composition of one transformation we mean one of these transformations. Trivial relation is one of the following 5 relations:

Complement of the Anticomplement of P = P.
Anticompliment of the Complement of P = P.
Isogonal Conjugate of the Isogonal Conjugate of P = P.
Isotomic Conjugate of the Isotomic Conjugate of P = P.
Cyclocevian Conjugate of the Cyclocevian Conjugate of P = P.

If we order to a computer to give us all non-trivial relations between compositions of transformations, except for trivial relations, such that the number of all transformations in any composition is no more than 3, we obtain the following 6 relations (not included in the first edition of [2]):

Isogonal Conjugate of Complement of Isotomic Conjugate of Point P = Complement of Isotomic Conjugate of Cyclocevian Conjugate of Point P.

Isotomic Conjugate of Anticomplement of Isogonal Conjugate of Point P = Cyclocevian Conjugate of Isotomic Conjugate of Anticomplement of Point P.

Anticomplement of Isogonal Conjugate of Complement of Point P = Isotomic Conjugate of Cyclocevian Conjugate of Isotomic Conjugate of Point P.

Complement of Isotomic Conjugate of Cyclocevian Conjugate of Point P = Isogonal

Conjugate of Complement of Isotomic Conjugate of Point P.

Cyclocevian Conjugate of Isotomic Conjugate of Anticomplement of Point P = Isotomic Conjugate of Anticomplement of Isogonal Conjugate of Point P.

Isotomic Conjugate of Cyclocevian Conjugate of Isotomic Conjugate of Point P = Anticomplement of Isogonal Conjugate of Complement of Point P.

It is easy to see that any of the above 6 relations can be rewritten to the Grinberg's relation. We can make the following conclusion: If we consider relations between compositions having together no more than 6 transformations, we will not obtain new relations. Indeed, any relation of the form

composition of two transformations = composition of 4 transformations
or of the form
one transformation = composition of 5 transformations

rewrites to relation of the form

composition of 3 transformations = composition of 3 transformations,

so that any relation between compositions having together 6 transformations rewrites to Grinberg's relation.

In addition, since in the general case there is no relations between compositions having together 2, 3, 4 and 5 transformations, we can conclude that in the general case, a composition of one or two or three transformations of P does not coincide with any composition of one or two transformations of P. E.g., in the general case, all points - compositions having no more than 2 transformations of a given point, are different. Clearly, there are 25 non-trivial compositions of no more than two transformations, e.g. Complement of P, Complement of the Complement of P, Isogonal Conjugate of Complement of P, etc.

Clearly, in special cases the points-images can coincide, e.g. Complement of the Orthocenter = Isogonal Conjugate of the Orthocenter, and so on.

In [1], Grinberg introduced the following composition of transformations:

Isotomic Complement of P = Complement of the Isotomic Conjugate of P.

Additional compositions of transformations are defined in [2]. The above remarks give us the basis to define additional compositions of transformations.

I would like to note the usefulness of compositions of transformations in Triangle Geometry. By using compositions of transformations, we are able to find useful relations between points, triangles, circles. In many cases these relations give us convenient ways we to construct new points by using straightedge and compass.

References

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