

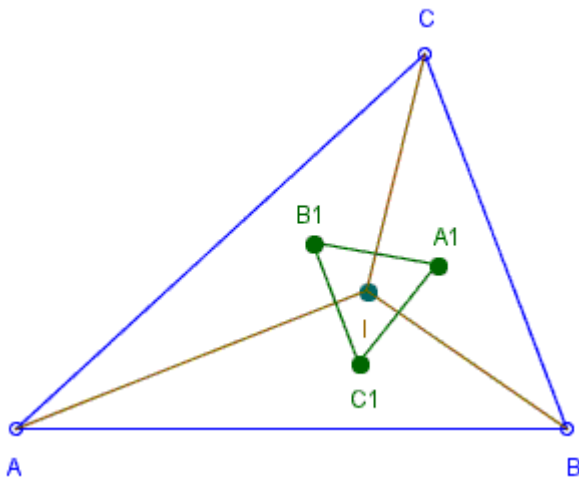
Triangulation Triangles

Deko Dekov

Abstract. By using the computer program "Machine for Questions and Answers", we study perspectors of basic triangles and triangles of triangle centers of triangulation triangles.

Given a triangle ABC and a triangle center of kind 1, labeled by P . Construct triangle centers A_1, B_1, C_1 of kind 2 (possibly different from the kind 1) of triangles BCP, CAP, ABP , respectively. We call triangle $A_1B_1C_1$ the *Triangle of the Triangle Centers of kind 2 of the Triangulation triangles of the Triangle Center of kind 1*.

See the Figure:



I - Triangle Center of kind 1;

A_1, B_1, C_1 - Triangle Centers of kind 2 of triangles BCP, CAP, ABP , respectively;

$A_1B_1C_1$ - Triangle of the Triangle Centers of kind 2 of the Triangulation triangles of the Triangle Center of kind 1;

In this Figure:

I - Incenter of triangle ABC ;

A_1, B_1, C_1 - Incenters of triangles BCP, CAP, ABP , respectively;

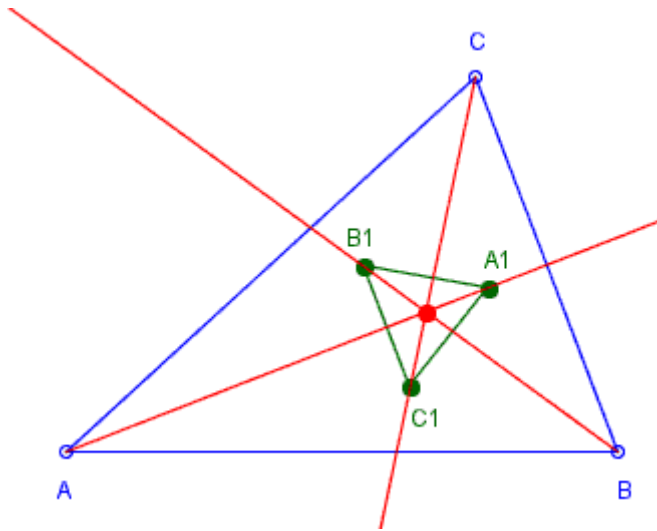
$A_1B_1C_1$ - Triangle of the Incenters of the Triangulation triangles of the Incenter;

De Villiers (see [1 - 4]) proved that

Triangle ABC and the Triangle of the Incenters of the Triangulation Triangles of the

Incenter are perspective.

See the Figure:



$A_1B_1C_1$ - Triangle of the Incenters of the Triangulation triangles of the Incenter;
Triangles ABC and $A_1B_1C_1$ are perspective.

Darij Grinberg gave a description of the perspector and named it the *First de Villiers Point*. The Triangle of the Incenters of the Triangulation Triangles of the Incenter is known as the *BCI Triangle*; we use the term *de Villiers Triangle*.

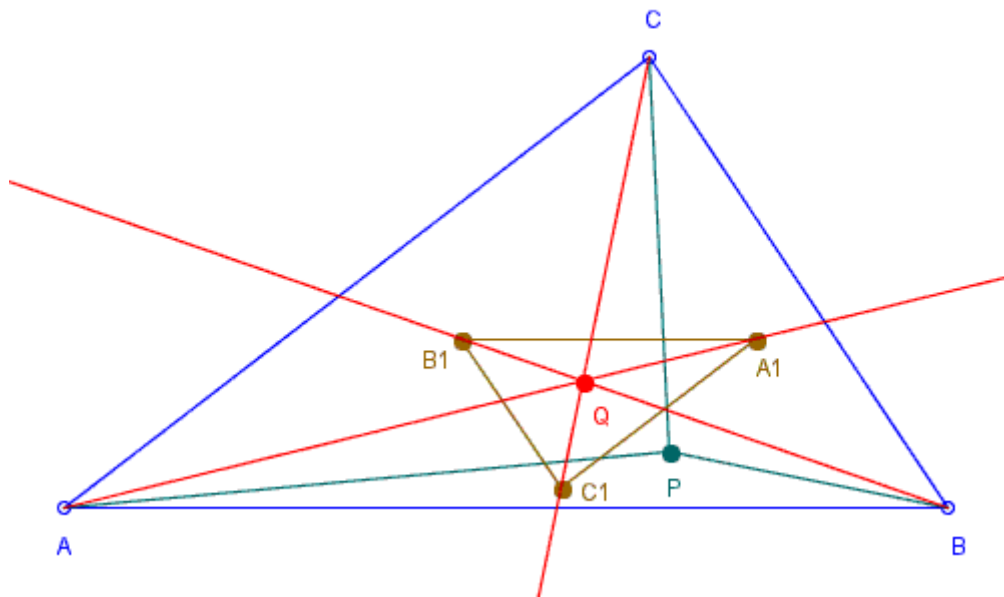
A few similar results are known (the reader is invited to submit a note/paper with additional references), e.g.

Triangle ABC and the Triangle of the Centroids of the Triangulation triangles of the Circumcenter are homothetic with homothetic center the Complement of the Nine-Point Center.

Triangle ABC and the Triangle of the Circumcenters of the Triangulation triangles of the Circumcenter are perspective with perspector the Kosnita Point.

Triangle ABC and the Triangle of the Incenters of the Triangulation triangles of the Center of the Inner Soddy Circle are perspective with perspector the Inner Eppstein Point.

Given a triangle ABC and a triangle center P. Construct the centroids A_1 , B_1 and C_1 of triangles BCP, CAP and ABP, respectively. Triangle ABC is perspective with triangle $A_1B_1C_1$. See the figure:



Examples

The Machine for Questions and Answers produces examples. A few examples are given below.

Triangle ABC is perspective with the de Villiers Triangle.

Triangle ABC is perspective with the Triangle of the Circumcenters of the Triangulation Triangles of the Incenter.

Triangle ABC is perspective with the Triangle of the Outer Fermat Points of the Triangulation Triangles of the Incenter.

Triangle ABC is perspective with the Triangle of the Inner Fermat Points of the Triangulation Triangles of the Incenter.

Triangle ABC is perspective with the Triangle of the First Isodynamic Points of the Triangulation Triangles of the Incenter.

Triangle ABC is perspective with the Triangle of the Second Isodynamic Points of the Triangulation Triangles of the Incenter.

Triangle ABC is perspective with the Triangle of the Clawson Points of the Triangulation Triangles of the Incenter.

Triangle ABC is perspective with the Triangle of the Circumcenters of the Triangulation Triangles of the Circumcenter.

Triangle ABC is homothetic to the Triangle of the First Feuerbach Points of the Triangulation Triangles of the Circumcenter.

Triangle ABC is homothetic to the Triangle of the Kiepert Centers of the Triangulation Triangles of the Circumcenter.

Triangle ABC is homothetic to the Triangle of the Circumcenters of the Triangulation Triangles of the Orthocenter.

Triangle ABC is perspective with the Triangle of the Symmedian Points of the Triangulation Triangles of the Orthocenter.

Triangle ABC is homothetic to the Triangle of the de Longchamps Points of the Triangulation Triangles of the Orthocenter.

Triangle ABC is perspective with the Triangle of the Schiffler Points of the Triangulation Triangles of the Orthocenter.

Triangle ABC is perspective with the Triangle of the Exeter Points of the Triangulation Triangles of the Orthocenter.

Triangle ABC is perspective with the Triangle of the Far-Out Points of the Triangulation Triangles of the Orthocenter.

Triangle ABC is perspective with the Triangle of the Gibert Points of the Triangulation Triangles of the Orthocenter.

Triangle ABC is perspective with the Triangle of the Prasolov Points of the Triangulation Triangles of the Orthocenter.

Triangle ABC is homothetic to the Triangle of the Centers of the Orthocentroidal Circles of the Triangulation Triangles of the Orthocenter.

Triangle ABC is homothetic to the Triangle of the Centers of the Taylor Circles of the Triangulation Triangles of the Orthocenter.

Triangle ABC is homothetic to the Triangle of the Skordev Points of the Triangulation Triangles of the Orthocenter.

Triangle ABC is perspective with the Triangle of the Malfatti-Moses Points of the Triangulation Triangles of the Orthocenter.

Triangle ABC is perspective with the Triangle of the Nine-Point Centers of the Triangulation Triangles of the Nine-Point Center.

Triangle ABC is perspective with the Triangle of the Symmedian Points of the Triangulation Triangles of the Symmedian Point.

Triangle ABC is perspective with the Triangle of the Incenters of the Triangulation Triangles of the Outer Fermat Point.

Triangle ABC is perspective with the Triangle of the Circumcenters of the Triangulation

Triangles of the Outer Fermat Point.

Triangle ABC is perspective with the Triangle of the Inner Fermat Points of the Triangulation Triangles of the Outer Fermat Point.

Triangle ABC is perspective with the Triangle of the First Isodynamic Points of the Triangulation Triangles of the Outer Fermat Point.

Triangle ABC is perspective with the Triangle of the Second Isodynamic Points of the Triangulation Triangles of the Outer Fermat Point.

Triangle ABC is perspective with the Triangle of the Circumcenters of the Triangulation Triangles of the Inner Fermat Point.

Triangle ABC is perspective with the Triangle of the Incenters of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Circumcenters of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Symmedian Points of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Outer Fermat Points of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Inner Fermat Points of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the First Isodynamic Points of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Second Power Points of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Third Power Points of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Brocard Midpoints of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Centers of the Brocard Circles of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Schoute Centers of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Inner Kenmotsu Points of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Outer Kenmotu Points of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Centers of the Taylor Circles of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Fourth Power Points of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Centers of the Apollonius Circles of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Radical Centers of the Lucas Circles of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Danneels-Apollonius Prespectors of the Triangulation Triangles of the First Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Incenters of the Triangulation Triangles of the Second Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Circumcenters of the Triangulation Triangles of the Second Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Symmedian Points of the Triangulation Triangles of the Second Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Second Power Points of the Triangulation Triangles of the Second Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Third Power Points of the Triangulation Triangles of the Second Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Brocard Midpoints of the Triangulation Triangles of the Second Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Centers of the Brocard Circles of the Triangulation Triangles of the Second Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Schoute Centers of the Triangulation Triangles of the Second Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Inner Kenmotu Points of the Triangulation Triangles of the Second Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Outer Kenmotu Points of the Triangulation Triangles of the Second Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Centers of the Taylor Circles of the

Triangulation Triangles of the Second Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Fourth Power Points of the Triangulation Triangles of the Second Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Centers of the Apollonius Circles of the Triangulation Triangles of the Second Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Radical Centers of the Lucas Circles of the Triangulation Triangles of the Second Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Danneels-Apollonius Prespectors of the Triangulation Triangles of the Second Isodynamic Point.

Triangle ABC is perspective with the Triangle of the Outer Napoleon Points of the Triangulation Triangles of the Outer Napoleon Point.

Triangle ABC is perspective with the Triangle of the Inner Napoleon Points of the Triangulation Triangles of the Outer Napoleon Point.

Triangle ABC is perspective with the Triangle of the de Longchamps Points of the Triangulation Triangles of the de Longchamps Point.

Triangle ABC is perspective with the Triangle of the Second Power Points of the Triangulation Triangles of the Second Power Point.

Triangle ABC is perspective with the Triangle of the Third Power Points of the Triangulation Triangles of the Third Power Point.

Triangle ABC is perspective with the Triangle of the Incenters of the Triangulation Triangles of the Center of the Outer Soddy Circle.

Triangle ABC is perspective with the Triangle of the Gergonne Points of the Triangulation Triangles of the Center of the Outer Soddy Circle.

Triangle ABC is perspective with the Triangle of the Nagel Points of the Triangulation Triangles of the Center of the Outer Soddy Circle.

Triangle ABC is perspective with the Triangle of the de Longchamps Points of the Triangulation Triangles of the Center of the Outer Soddy Circle.

Triangle ABC is perspective with the Triangle of the Bevan Points of the Triangulation Triangles of the Center of the Outer Soddy Circle.

Triangle ABC is perspective with the Triangle of the Inner Eppstein Points of the Triangulation Triangles of the Center of the Outer Soddy Circle.

Triangle ABC is perspective with the Triangle of the Incenters of the Triangulation Triangles of the Center of the Inner Soddy Circle.

Triangle ABC is perspective with the Triangle of the Gergonne Points of the Triangulation Triangles of the Center of the Inner Soddy Circle.

Triangle ABC is perspective with the Triangle of the Nagel Points of the Triangulation Triangles of the Center of the Inner Soddy Circle.

Triangle ABC is perspective with the Triangle of the de Longchamps Points of the Triangulation Triangles of the Center of the Inner Soddy Circle.

Triangle ABC is perspective with the Triangle of the Bevan Points of the Triangulation Triangles of the Center of the Inner Soddy Circle.

Triangle ABC is perspective with the Triangle of the Centers of the Inner Soddy Circles of the Triangulation Triangles of the Center of the Inner Soddy Circle.

Triangle ABC is perspective with the Triangle of the Inner Eppstein Points of the Triangulation Triangles of the Center of the Inner Soddy Circle.

Triangle ABC is perspective with the Triangle of the Outer Kenmotu Points of the Triangulation Triangles of the Inner Kenmotu Point.

Triangle ABC is perspective with the Triangle of the Centers of the Orthocentroidal Circles of the Triangulation Triangles of the Center of the Orthocentroidal Circle.

Triangle ABC is perspective with the Triangle of the Circumcenters of the Triangulation Triangles of the Evans Perspector.

Triangle ABC is perspective with the Triangle of the Outer Vecten Points of the Triangulation Triangles of the Outer Vecten Point.

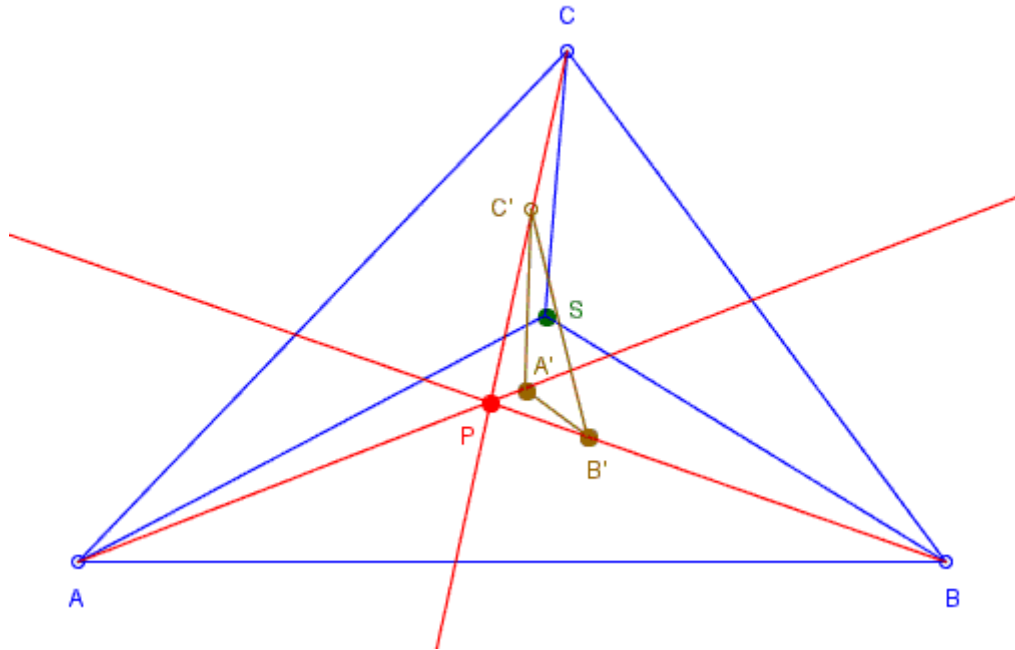
Triangle ABC is perspective with the Triangle of the Inner Vecten Points of the Triangulation Triangles of the Outer Vecten Point.

Triangle ABC is perspective with the Triangle of the Fourth Power Points of the Triangulation Triangles of the Fourth Power Point.

Triangle ABC is perspective with the Triangle of the Malfatti-Moses Points of the Triangulation Triangles of the Malfatti-Moses Point.

Triangle ABC is perspective with the Triangle of the Skordev Points of the Triangulation Triangles of the Skordev Point.

We illustrate the last example of the above list. Given a triangle ABC. Construct the Skordev point S of triangle ABC. Then construct Skordev points A', B', C' of triangles BCS, CAS and ABS, respectively. Triangle ABC is perspective with triangle A'B'C'. See the figure (the perspector is labelled P):



The Machine for Questions and Answers produces descriptions of the perspectors. E.g., we obtain the following extended results:

Triangle ABC and the Triangle of the Centroids of the Triangulation Triangles of the Incenter are homothetic with homothetic center the Complement of the Spieker Center.

Triangle ABC and the Triangle of the Circumcenters of the Triangulation Triangles of the Incenter are perspective with perspector the Incenter.

Triangle ABC and the Triangle of the Centroids of the Triangulation Triangles of the Centroid are homothetic with homothetic center the Centroid.

Triangle ABC and the Triangle of the Centroids of the Triangulation Triangles of the Circumcenter are homothetic with homothetic center the Complement of the Nine-Point Center.

Triangle ABC and the Triangle of the Circumcenters of the Triangulation Triangles of the Circumcenter are perspective with perspector the Kosnita Point.

Triangle ABC and the Triangle of the First Feuerbach Points of the Triangulation Triangles of the Circumcenter are homothetic with homothetic center the Centroid.

Triangle ABC and the Triangle of the Kiepert Centers of the Triangulation Triangles of the Circumcenter are homothetic with homothetic center the Centroid.

Triangle ABC and the Triangle of the Centroids of the Triangulation Triangles of the Orthocenter are homothetic with homothetic center the Nine-Point Center.

Triangle ABC and the Triangle of the Circumcenters of the Triangulation Triangles of the

Orthocenter are homothetic with homothetic center the Nine-Point Center.

Triangle ABC and the Triangle of the Symmedian Points of the Triangulation Triangles of the Orthocenter are perspective with perspector the Symmedian Point of the Orthic Triangle.

Triangle ABC and the Triangle of the de Longchamps Points of the Triangulation Triangles of the Orthocenter are homothetic with homothetic center the Nine-Point Center.

Triangle ABC and the Triangle of the Schiffler Points of the Triangulation Triangles of the Orthocenter are perspective with perspector the Nine-Point Center.

Triangle ABC and the Triangle of the Exeter Points of the Triangulation Triangles of the Orthocenter are perspective with perspector the Nine-Point Center.

Triangle ABC and the Triangle of the Far-Out Points of the Triangulation Triangles of the Orthocenter are perspective with perspector the Nine-Point Center.

Triangle ABC and the Triangle of the Gibert Points of the Triangulation Triangles of the Orthocenter are perspective with perspector the Nine-Point Center.

Triangle ABC and the Triangle of the Prasolov Points of the Triangulation Triangles of the Orthocenter are perspective with perspector the Orthocenter of the Orthic Triangle.

Triangle ABC and the Triangle of the Centers of the Orthocentroidal Circles of the Triangulation Triangles of the Orthocenter are homothetic with homothetic center the Nine-Point Center.

Triangle ABC and the Triangle of the Centers of the Taylor Circles of the Triangulation Triangles of the Orthocenter are homothetic with homothetic center the Centroid of the Orthic Triangle.

Triangle ABC and the Triangle of the Skordev Points of the Triangulation Triangles of the Orthocenter are homothetic with homothetic center the Nine-Point Center.

Triangle ABC and the Triangle of the Centroids of the Triangulation Triangles of the Gergonne Point are homothetic with homothetic center the Complement of the Mittenpunkt.

Triangle ABC and the Triangle of the Centroids of the Triangulation Triangles of the Nagel Point are homothetic with homothetic center the Spieker Center.

Triangle ABC and the Triangle of the Incenters of the Triangulation Triangles of the Outer Fermat Point are perspective with perspector the Outer Fermat Point.

Triangle ABC and the Triangle of the First Isodynamic Points of the Triangulation Triangles of the Outer Fermat Point are perspective with perspector the Outer Fermat Point.

Triangle ABC and the Triangle of the Circumcenters of the Triangulation Triangles of the Inner Fermat Point are perspective with perspector the Inner Napoleon Point.

Triangle ABC and the Triangle of the Outer Napoleon Points of the Triangulation Triangles

of the Outer Napoleon Point are perspective with perspector the Outer Napoleon Point.

Triangle ABC and the Triangle of the Centroids of the Triangulation Triangles of the de Longchamps Point are homothetic with homothetic center the Circumcenter.

Triangle ABC and the Triangle of the de Longchamps Points of the Triangulation Triangles of the de Longchamps Point are perspective with perspector the Orthocenter.

Triangle ABC and the Triangle of the Centroids of the Triangulation Triangles of the Bevan Point are homothetic with homothetic center the Midpoint of the Circumcenter and the Spieker Center.

Triangle ABC and the Triangle of the Incenters of the Triangulation Triangles of the Center of the Outer Soddy Circle are perspective with perspector the Outer Eppstein Point.

Triangle ABC and the Triangle of the Gergonne Points of the Triangulation Triangles of the Center of the Outer Soddy Circle are perspective with perspector the Outer Eppstein Point.

Triangle ABC and the Triangle of the de Longchamps Points of the Triangulation Triangles of the Center of the Outer Soddy Circle are perspective with perspector the Outer Eppstein Point.

Triangle ABC and the Triangle of the Inner Eppstein Points of the Triangulation Triangles of the Center of the Outer Soddy Circle are perspective with perspector the Outer Eppstein Point.

Triangle ABC and the Triangle of the Incenters of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Inner Eppstein Point.

Triangle ABC and the Triangle of the Gergonne Points of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Inner Eppstein Point.

Triangle ABC and the Triangle of the de Longchamps Points of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Inner Eppstein Point.

Triangle ABC and the Triangle of the Centers of the Inner Soddy Circles of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Inner Eppstein Point.

Triangle ABC and the Triangle of the Inner Eppstein Points of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Inner Eppstein Point.

Triangle ABC and the Triangle of the Centroids of the Triangulation Triangles of the Equal Parallelians Point are homothetic with homothetic center the Grinberg Point.

Triangle ABC and the Triangle of the Centroids of the Triangulation Triangles of the Center of the Fuhrmann Circle are homothetic with homothetic center the Midpoint of the Nine-

Point Center and the Spieker Center.

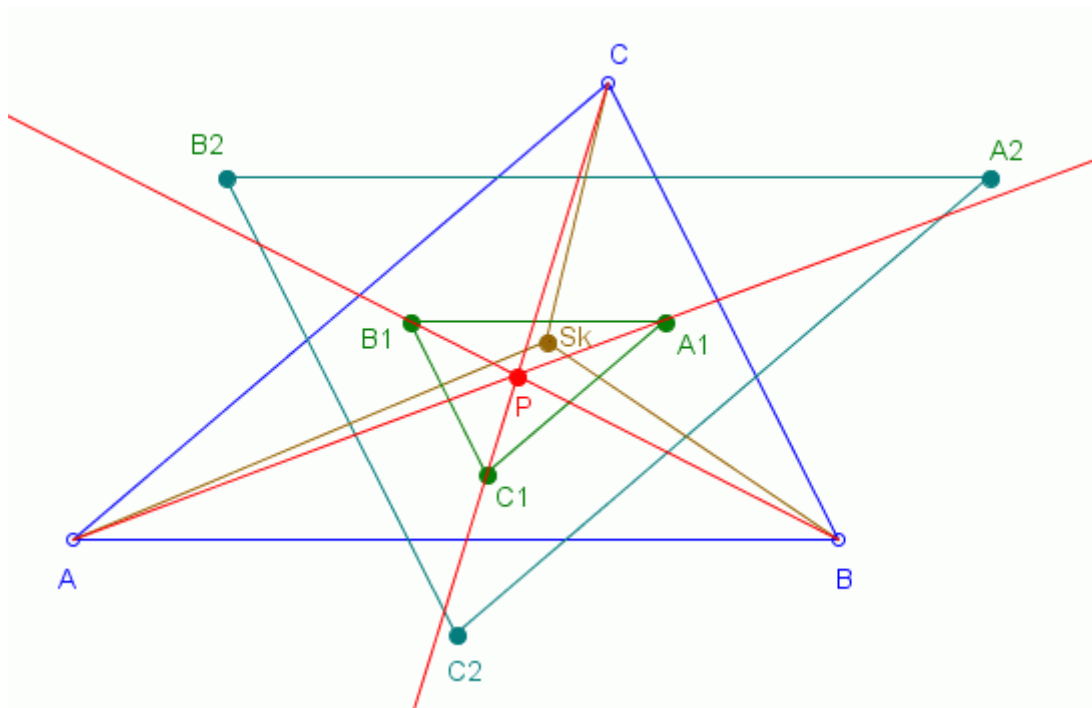
Triangle ABC and the Triangle of the Centroids of the Triangulation Triangles of the Center of the Orthocentroidal Circle are homothetic with homothetic center the Midpoint of the Centroid and the Nine-Point Center.

Triangle ABC and the Triangle of the Outer Vecten Points of the Triangulation Triangles of the Outer Vecten Point are perspective with perspector the Outer Vecten Point.

If we need to obtain a description of a perspector, we could use the Machine for Questions and Answers to find a list of descriptions.. E.g., if we want to obtain a description of the perspector of Triangle ABC and the Triangle of the Centroids of the Triangulation triangles of the Skordev Point, we obtain a list containing the following the result:

Triangle ABC and the Triangle of the Centroids of the Triangulation triangles of the Skordev Point are homothetic with homothetic center the Skordev Point of the Johnson Triangle.

See the figure:



- Sk - Skordev Point of triangle ABC;
- $A_1B_1C_1$ - Triangle of the Centroids of the Triangulation triangles of the Skordev Point;
- $A_2B_2C_2$ - Johnson Triangle;
- P - Skordev Point of the Johnson Triangle = Perspector of triangles ABC and $A_1B_1C_1$.

Perspectors of Special Triangles

The Machine for Questions and Answers produces examples of perspectors of basic triangles and Triangulation triangles. A few examples are given below.

The Incentral Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Incenter are perspective with perspector the Incenter.

The Incentral Triangle and the Triangle of the Incenters of the Triangulation Triangles of the First Isodynamic Point are perspective with perspector the First Isodynamic Point.

The Incentral Triangle and the Triangle of the Incenters of the Triangulation Triangles of the Second Isodynamic Point are perspective with perspector the Second Isodynamic Point.

The Medial Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Incenter are perspective with perspector the Mittenpunkt.

The Medial Triangle and the Triangle of the Nine-Point Centers of the Triangulation Triangles of the Incenter are perspective with perspector the Spieker Center.

The Medial Triangle and the Triangle of the Symmedian Points of the Triangulation Triangles of the Centroid are perspective with perspector the Symmedian Point.

The Medial Triangle and the Triangle of the Malfatti-Moses Points of the Triangulation Triangles of the Centroid are perspective with perspector the Malfatti-Moses Point.

The Orthic Triangle and the Triangle of the Prasolov Points of the Triangulation Triangles of the Incenter are perspective with perspector the Incenter.

The Orthic Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Circumcenter are homothetic with homothetic center the Gibert Point.

The Orthic Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Circumcenter are perspective with perspector the Symmedian Point.

The Orthic Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Orthocenter are perspective with perspector the Orthocenter of the Tangential Triangle.

The Orthic Triangle and the Triangle of the Symmedian Points of the Triangulation Triangles of the Orthocenter are perspective with perspector the Homothetic Center of the Orthic Triangle and the Tangential Triangle.

The Orthic Triangle and the Triangle of the de Longchamps Points of the Triangulation Triangles of the de Longchamps Point are perspective with perspector the Orthocenter.

The Orthic Triangle and the Triangle of the Symmedian Points of the Triangulation Triangles of the Far-Out Point are perspective with perspector the Homothetic Center of the Orthic Triangle and the Tangential Triangle.

The Orthic Triangle and the Triangle of the Inner Vecten Points of the Triangulation Triangles of the Inner Kenmotu Point are perspective with perspector the Inner Kenmotu Point.

The Symmedial Triangle and the Triangle of the Symmedian Points of the Triangulation

Triangles of the First Isodynamic Point are perspective with perspector the First Isodynamic Point.

The Symmedial Triangle and the Triangle of the First Isodynamic Points of the Triangulation Triangles of the First Isodynamic Point are perspective with perspector the Second Isodynamic Point.

The Symmedial Triangle and the Triangle of the Symmedian Points of the Triangulation Triangles of the Second Isodynamic Point are perspective with perspector the Second Isodynamic Point.

The Intouch Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Incenter are homothetic with homothetic center the External Center of Similitude of the Incircle and the Circumcircle.

The Intouch Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Incenter are perspective with perspector the Incenter.

The Intouch Triangle and the Triangle of the Gibert Points of the Triangulation Triangles of the Incenter are perspective with perspector the Circumcenter.

The Intouch Triangle and the Triangle of the de Longchamps Points of the Triangulation Triangles of the Bevan Point are perspective with perspector the Incenter.

The Intouch Triangle and the Triangle of the Incenters of the Triangulation Triangles of the Center of the Outer Soddy Circle are perspective with perspector the Incenter.

The Intouch Triangle and the Triangle of the Gergonne Points of the Triangulation Triangles of the Center of the Outer Soddy Circle are perspective with perspector the Center of the Outer Soddy Circle.

The Intouch Triangle and the Triangle of the Incenters of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Incenter.

The Intouch Triangle and the Triangle of the Gergonne Points of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Center of the Inner Soddy Circle.

The Extouch Triangle and the Triangle of the de Longchamps Points of the Triangulation Triangles of the Incenter are perspective with perspector the Bevan Point.

The Extouch Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Bevan Point are perspective with perspector the Bevan Point.

The Extouch Triangle and the Triangle of the Nagel Points of the Triangulation Triangles of the Center of the Outer Soddy Circle are perspective with perspector the Center of the Outer Soddy Circle.

The Extouch Triangle and the Triangle of the Bevan Points of the Triangulation Triangles of

the Center of the Outer Soddy Circle are perspective with perspector the Bevan Point.

The Extouch Triangle and the Triangle of the Nagel Points of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Center of the Inner Soddy Circle.

The Extouch Triangle and the Triangle of the Bevan Points of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Bevan Point.

The Excentral Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Incenter are homothetic with homothetic center the Incenter.

The Excentral Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Incenter are perspective with perspector the Mittenpunkt.

The Excentral Triangle and the Triangle of the de Longchamps Points of the Triangulation Triangles of the Incenter are perspective with perspector the Bevan Point.

The Excentral Triangle and the Triangle of the Bevan Points of the Triangulation Triangles of the Incenter are perspective with perspector the Incenter of the Excentral Triangle.

The Excentral Triangle and the Triangle of the First Feuerbach Points of the Triangulation Triangles of the Circumcenter are perspective with perspector the Mittenpunkt.

The Excentral Triangle and the Triangle of the Kiepert Centers of the Triangulation Triangles of the Circumcenter are perspective with perspector the Mittenpunkt.

The Excentral Triangle and the Triangle of the Centroids of the Triangulation Triangles of the Mittenpunkt are perspective with perspector the Mittenpunkt.

The Excentral Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Bevan Point are perspective with perspector the Bevan Point.

The Excentral Triangle and the Triangle of the Bevan Points of the Triangulation Triangles of the Center of the Outer Soddy Circle are perspective with perspector the Bevan Point.

The Excentral Triangle and the Triangle of the Bevan Points of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Bevan Point.

The Anticomplementary Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Incenter are perspective with perspector the Nagel Point.

The Anticomplementary Triangle and the Triangle of the Centroids of the Triangulation Triangles of the Centroid are homothetic with homothetic center the Centroid.

The Anticomplementary Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Circumcenter are perspective with perspector the Symmedian Point of the Anticomplementary Triangle.

The Anticomplementary Triangle and the Triangle of the First Feuerbach Points of the

Triangulation Triangles of the Circumcenter are homothetic with homothetic center the Centroid.

The Anticomplementary Triangle and the Triangle of the Kiepert Centers of the Triangulation Triangles of the Circumcenter are homothetic with homothetic center the Centroid.

The Anticomplementary Triangle and the Triangle of the Centroids of the Triangulation Triangles of the Orthocenter are homothetic with homothetic center the Skordev Point.

The Anticomplementary Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Orthocenter are homothetic with homothetic center the Orthocenter.

The Anticomplementary Triangle and the Triangle of the de Longchamps Points of the Triangulation Triangles of the Orthocenter are homothetic with homothetic center the de Longchamps Point.

The Anticomplementary Triangle and the Triangle of the Centroids of the Triangulation Triangles of the Spieker Center are homothetic with homothetic center the Internal Center of Similitude of the Bevan Circle and the Nine-Point Circle.

The Anticomplementary Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the de Longchamps Point are perspective with perspector the de Longchamps Point.

The Tangential Triangle and the Triangle of the Symmedian Points of the Triangulation Triangles of the Orthocenter are perspective with perspector the Homothetic Center of the Orthic Triangle and the Tangential Triangle.

The Tangential Triangle and the Triangle of the Prasolov Points of the Triangulation Triangles of the Orthocenter are perspective with perspector the Orthocenter of the Tangential Triangle.

The Tangential Triangle and the Triangle of the First Isodynamic Points of the Triangulation Triangles of the First Isodynamic Point are perspective with perspector the First Isodynamic Point.

The Tangential Triangle and the Triangle of the Symmedian Points of the Triangulation Triangles of the Far-Out Point are perspective with perspector the Homothetic Center of the Orthic Triangle and the Tangential Triangle.

The Circum-Medial Triangle and the Triangle of the Centroids of the Triangulation Triangles of the Centroid are perspective with perspector the Centroid.

The Circum-Medial Triangle and the Triangle of the First Feuerbach Points of the Triangulation Triangles of the Circumcenter are perspective with perspector the Centroid.

The Circum-Medial Triangle and the Triangle of the Kiepert Centers of the Triangulation

Triangles of the Circumcenter are perspective with perspector the Centroid.

The Circum-Orthic Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Circumcenter are homothetic with homothetic center the Inverse of the Orthocenter in the Circumcircle.

The Circum-Orthic Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Circumcenter are perspective with perspector the Kosnita Point.

The Circum-Orthic Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Orthocenter are perspective with perspector the Kiepert-Parry Point.

The Circum-Orthic Triangle and the Triangle of the de Longchamps Points of the Triangulation Triangles of the de Longchamps Point are perspective with perspector the Orthocenter.

The Euler Triangle and the Triangle of the First Feuerbach Points of the Triangulation Triangles of the Circumcenter are homothetic with homothetic center the Nine-Point Center.

The Euler Triangle and the Triangle of the Kiepert Centers of the Triangulation Triangles of the Circumcenter are homothetic with homothetic center the Nine-Point Center.

The Euler Triangle and the Triangle of the Centroids of the Triangulation Triangles of the Orthocenter are homothetic with homothetic center the Skordev Point.

The Euler Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Orthocenter are homothetic with homothetic center the Center of the Orthocentroidal Circle.

The Euler Triangle and the Triangle of the Centers of the Orthocentroidal Circles of the Triangulation Triangles of the Orthocenter are homothetic with homothetic center the Circumcenter.

The Euler Triangle and the Triangle of the Skordev Points of the Triangulation Triangles of the Orthocenter are homothetic with homothetic center the Centroid.

The Euler Triangle and the Triangle of the Centroids of the Triangulation Triangles of the Nine-Point Center are homothetic with homothetic center the Nine-Point Center.

The Euler Triangle and the Triangle of the de Longchamps Points of the Triangulation Triangles of the de Longchamps Point are perspective with perspector the Orthocenter.

The Euler Triangle and the Triangle of the Centroids of the Triangulation Triangles of the Bevan Point are homothetic with homothetic center the Internal Center of Similitude of the Bevan Circle and the Nine-Point Circle.

The Intangents Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Incenter are perspective with perspector the Incenter.

The Intangents Triangle and the Triangle of the Circumcenters of the Triangulation

Triangles of the Circumcenter are homothetic with homothetic center the Moses Point.

The Intangents Triangle and the Triangle of the de Longchamps Points of the Triangulation Triangles of the Bevan Point are perspective with perspector the Incenter.

The Intangents Triangle and the Triangle of the Incenters of the Triangulation Triangles of the Center of the Outer Soddy Circle are perspective with perspector the Incenter.

The Intangents Triangle and the Triangle of the Incenters of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Incenter.

The Extangents Triangle and the Triangle of the de Longchamps Points of the Triangulation Triangles of the Incenter are perspective with perspector the Bevan Point.

The Extangents Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Bevan Point are perspective with perspector the Bevan Point.

The Extangents Triangle and the Triangle of the Bevan Points of the Triangulation Triangles of the Center of the Outer Soddy Circle are perspective with perspector the Bevan Point.

The Extangents Triangle and the Triangle of the Bevan Points of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Bevan Point.

The Mixtilinear Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Incenter are perspective with perspector the Incenter.

The Fuhrmann Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Incenter are perspective with perspector the Nagel Point.

The Fuhrmann Triangle and the Triangle of the Nine-Point Centers of the Triangulation Triangles of the Incenter are homothetic with homothetic center the Incenter.

The Mid-Arc Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Incenter are perspective with perspector the Incenter.

The Reflection Triangle and the Triangle of the First Isodynamic Points of the Triangulation Triangles of the Outer Fermat Point are perspective with perspector the First Isodynamic Point.

The Reflection Triangle and the Triangle of the Internal Centers of Similitude of the Incircles and the Circumcircles of the Triangulation Triangles of the Outer Fermat Point are perspective with perspector the First Isodynamic Point.

The Reflection Triangle and the Triangle of the Miquel Points of the Incenters of the Triangulation Triangles of the Outer Fermat Point are perspective with perspector the First Isodynamic Point.

The Reflection Triangle and the Triangle of the Inner Fermat Points of the Triangulation Triangles of the First Isodynamic Point are perspective with perspector the First Isodynamic Point.

Point.

The Reflection Triangle and the Triangle of the de Longchamps Points of the Triangulation Triangles of the de Longchamps Point are perspective with perspector the Orthocenter.

The Reflection Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Kosnita Point are perspective with perspector the Symmedian Point.

The Fourth Brocard Triangle and the Triangle of the Centroids of the Triangulation Triangles of the Centroid are perspective with perspector the Centroid.

The Fourth Brocard Triangle and the Triangle of the First Feuerbach Points of the Triangulation Triangles of the Circumcenter are perspective with perspector the Centroid.

The Fourth Brocard Triangle and the Triangle of the Kiepert Centers of the Triangulation Triangles of the Circumcenter are perspective with perspector the Centroid.

The Fourth Brocard Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Outer Fermat Point are perspective with perspector the First Isodynamic Point.

The Fourth Brocard Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Inner Fermat Point are perspective with perspector the Second Isodynamic Point.

The Fourth Brocard Triangle and the Triangle of the Outer Napoleon Points of the Triangulation Triangles of the First Isodynamic Point are perspective with perspector the First Isodynamic Point.

The Yff Central Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Incenter are homothetic with homothetic center the Dimovski Point.

The Malfatti Central Triangle and the de Villiers Triangle are perspective with perspector the Radical Center of the Malfatti Circles.

The Malfatti Central Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Incenter are perspective with perspector the Incenter.

The Malfatti Squares Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Malfatti-Moses Point are perspective with perspector the Malfatti-Moses Point.

The Lucas Central Triangle and the Triangle of the Outer Kenmotu Points of the Triangulation Triangles of the First Isodynamic Point are perspective with perspector the Symmedian Point.

The Lucas Central Triangle and the Triangle of the Centroids of the Triangulation Triangles of the de Longchamps Point are perspective with perspector the Circumcenter.

The Hexyl Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of

the Incenter are perspective with perspector the Incenter.

The Hexyl Triangle and the Triangle of the de Longchamps Points of the Triangulation Triangles of the Bevan Point are perspective with perspector the Incenter.

The Hexyl Triangle and the Triangle of the Incenters of the Triangulation Triangles of the Center of the Outer Soddy Circle are perspective with perspector the Incenter.

The Hexyl Triangle and the Triangle of the Incenters of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Incenter.

The Inner Johnson-Yff Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Incenter are perspective with perspector the Incenter.

The Inner Johnson-Yff Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Orthocenter are homothetic with homothetic center the Second Feuerbach Point.

The Inner Johnson-Yff Triangle and the Triangle of the Centers of the Orthocentroidal Circles of the Triangulation Triangles of the Orthocenter are homothetic with homothetic center the First Feuerbach Point.

The Inner Johnson-Yff Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Center of the Inner Johnson-Yff Circle are perspective with perspector the Center of the Inner Johnson-Yff Circle.

The Outer Johnson-Yff Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Incenter are perspective with perspector the Incenter.

The Outer Johnson-Yff Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Orthocenter are homothetic with homothetic center the First Feuerbach Point.

The Outer Johnson-Yff Triangle and the Triangle of the Centers of the Orthocentroidal Circles of the Triangulation Triangles of the Orthocenter are homothetic with homothetic center the Second Feuerbach Point.

The Outer Johnson-Yff Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Center of the Outer Johnson-Yff Circle are perspective with perspector the Center of the Outer Johnson-Yff Circle.

The Machine for Questions and Answers produces also theorems about an arbitrary triangle center. In the theorems below "Triangle Center" could be replaced by any triangle center, e.g., by Incenter, Centroid, Circumcenter, Orthocenter, etc.

For any Triangle Center, the Medial Triangle and the Triangle of the Triangle Centers of the Triangulation Triangles of the Circumcenter are perspective with perspector the Circumcenter.

For any Triangle Center, the Medial Triangle and the Triangle of the Centroids of the

Triangulation Triangles of the Triangle Center are homothetic with homothetic center the Triangle Center.

For any Triangle Center, the Medial Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Triangle Center are homothetic with homothetic center the Circumcenter.

For any Triangle Center, the Tangential Triangle and the Triangle of the Triangle Centers of the Triangulation Triangles of the Circumcenter are perspective with perspector the Circumcenter.

For any Triangle Center, the Tangential Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Triangle Center are perspective with perspector the Circumcenter.

For any Triangle Center, the Fuhrmann Triangle and the Triangle of the Triangle Centers of the Triangulation Triangles of the Circumcenter are perspective with perspector the Circumcenter.

For any Triangle Center, the Fuhrmann Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Triangle Center are perspective with perspector the Circumcenter.

For any Triangle Center, the First Brocard Triangle and the Triangle of the Triangle Centers of the Triangulation Triangles of the Circumcenter are perspective with perspector the Circumcenter.

For any Triangle Center, the First Brocard Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Triangle Center are perspective with perspector the Circumcenter.

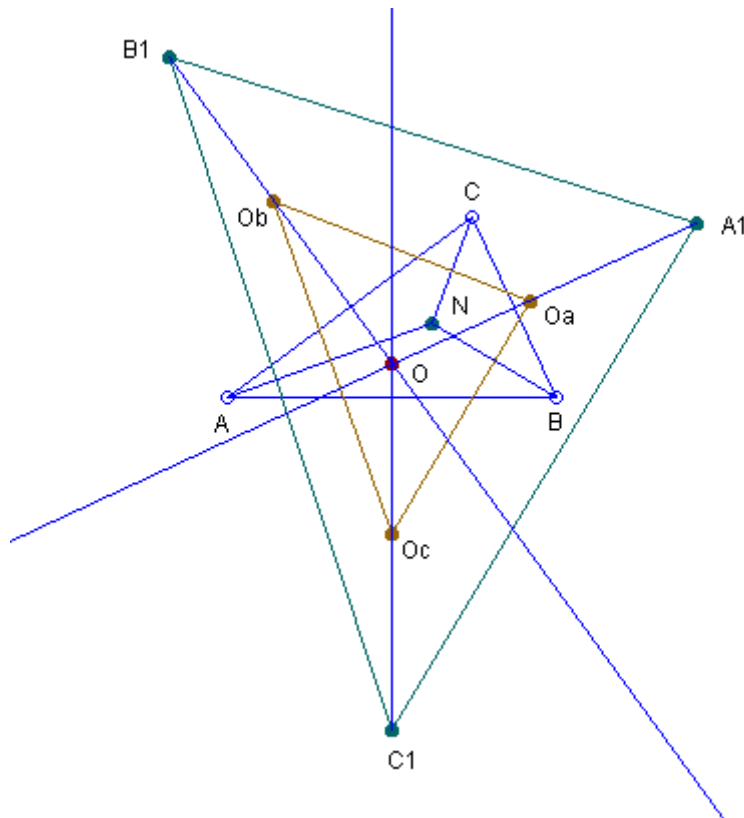
For any Triangle Center, the Neuberg Triangle and the Triangle of the Triangle Centers of the Triangulation Triangles of the Circumcenter are perspective with perspector the Circumcenter.

For any Triangle Center, the Neuberg Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Triangle Center are perspective with perspector the Circumcenter.

For any Triangle Center, the Reflected Neuberg Triangle and the Triangle of the Triangle Centers of the Triangulation Triangles of the Circumcenter are perspective with perspector the Circumcenter.

For any Triangle Center, the Reflected Neuberg Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Triangle Center are perspective with perspector the Circumcenter.

We illustrate the last theorem. In this illustration "Triangle Center" is replaced by "Nine-Point Center". See the Figure:



$A_1B_1C_1$ - Reflected Neuberg Triangle;
 N - Nine-Point Center;
 O - Circumcenter;
 $O_aO_bO_c$ - Triangle of the Circumcenters of the Triangulation Triangles of the Nine-Point Center;
 Triangles $A_1B_1C_1$ and $O_aO_bO_c$ are perspective at the Circumcenter.

Invitation

The reader is invited to submit a note/paper containing synthetic proofs of results from the above lists.

Definitions

We use the definitions in accordance with [1 - 4].

New definitions:

- Malfatti-Moses Point = Centroid of the Malfatti Squares Triangle.
- Gibert Point = Perspector of Triangle ABC and the Orthic Triangle of the Orthic Triangle.
- Moses Point = Perspector of Triangle ABC and the Orthic Triangle of the Incentral Triangle.
- Grinberg Point = Perspector of Triangle ABC and the Medial Triangle of the Incentral Triangle.
- de Villiers Triangle = BCI Triangle.
- Product = Barycentric Product.

The Level

The Machine for Questions and Answers is used to produce results in this paper. Currently the Machine has 6 levels of depths - 0,1,2,3,4,5. We use for this paper the level 0, that is, the Machine produces only elementary results. If we need deeper investigation, we have to use a level bigger than 0. Since the Machine for Questions and Answers produces too many results, it is suitable we to use bigger levels upon request, that is, for specific questions.

Thanks

The figures in this note are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann' s program is available for download in the Web: [Rene Grothmann' s C.a.R.](#) It is free and open source. The reader may verify easily the statements of this paper by using C.a.R. Many thanks to Rene Grothmann for his wonderful program.

References

1. Quim Castellsaguer, The Triangles Web, <http://www.xtec.es/~qcastell/ttw/ttweng/portada.html>
2. D. Dekov, Computer-Generated Encyclopedia of Euclidean Geometry, First Edition, 2006, <http://www.dekovsoft.com/>.
3. C. Kimberling, Encyclopedia of Triangle Centers, available at <http://faculty.evansville.edu/ck6/encyclopedia/>, see triangle center X1127.
4. Eric W. Weisstein, "First de Villiers Point." From MathWorld - A Wolfram Web Resource. <http://mathworld.wolfram.com/FirstdeVilliersPoint.html>.

Publication Date: 3 November 2007

Dr.Deko Dekov, ddekov@dekovsoft.com.