Construction of the Malfatti Squares Triangle

Deko Dekov

Abstract. By using the computer program "Machine for Questions and Answers", we find six different ways to construct the Malfatti Squares Triangle.

The Malfatti Squares Triangle is studied by Floor van Lamoen and Paul Yiu (under the name Malfatti triangle) [4]. Floor van Lamoen and Paul Yiu gave a simple construction of the Malfatti Squares Triangle.

We use the Machine for Questions and Answers to find six additional ways how to construct the Malfatti Squares Triangle.

The Malfatti-Moses Point

We use the Malfatti-Moses Point, that is, the Centroid of the Malfatti Squares Triangle. Construction of the Malfatti-Moses Point is given by Peter J.C. Moses ([6], Proposition 2). We give below two additional constructions of the Malfatti-Moses Point.

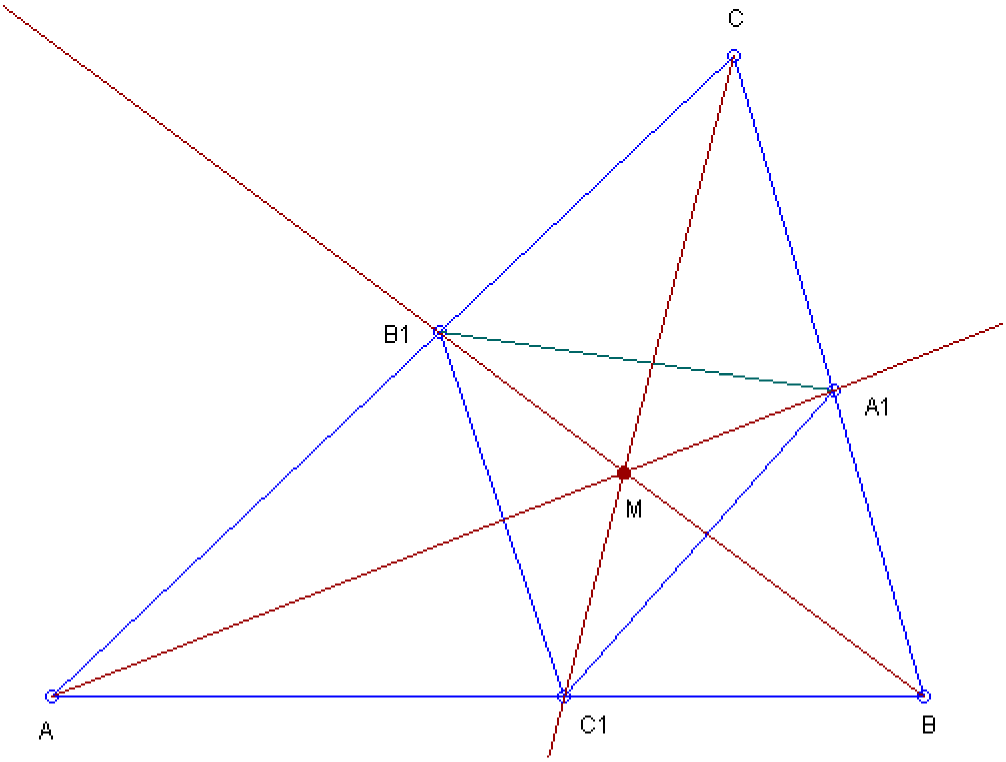
Given a point, the Machine for Questions and Answers produces theorems related to properties of the point. The Machine for Questions and Answers produces theorems related to properties of the Malfatti-Moses Point. We select the following three theorems:

Malfatti-Moses Point = Perspector of Triangle ABC and the Stevanovic Triangle of the Outer Vecten Points of the Triangulation triangles of the Inner Kenmotu Point.

The Malfatti-Moses Point lies on the Line through the Centroid and the Symmedian Point.

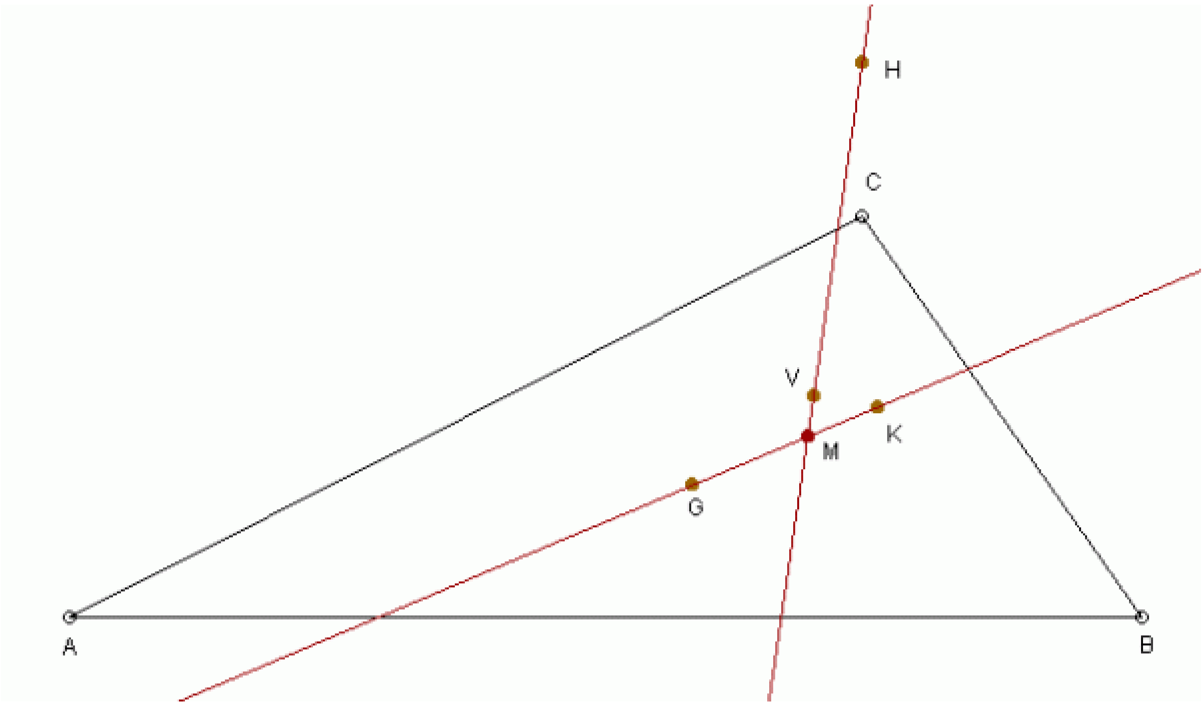
The Malfatti-Moses Point lies on the Line through the Orthocenter and the Outer Vecten Point.

We use the first theorem to construct the Malfatti-Moses Point as perspector of Triangle ABC and the Stevanovic Triangle of the Outer Vecten Points of the Triangulation triangles of the Inner Kenmotu Point. See the Figure:



$A_1B_1C_1$ - Stevanovic Triangle of the Outer Vecten Points of the Triangulation triangles of the Inner Kenmotu Point;
M - the Malfatti-Moses Point = Perspector of Triangle ABC and the Stevanovic Triangle of the Outer Vecten Points of the Triangulation triangles of the Inner Kenmotu Point.

We use the second and the third theorems for an addition construction of the Malfatti-Moses Point, as the point of intersection of two lines: the Line through the Centroid and the Symmedian Point, and the Line through the Orthocenter and the Outer Vecten Point. See the Figure:



G - Centroid;
K - Symmedian Point;
H - Orthocenter;
V - Outer Vecten Point;
M - Malfatti-Moses Point = the point of intersection of the Line through the Centroid and the Symmedian Point, and the Line through the Orthocenter and the Outer Vecten Point.

The Malfatti Squares Triangle

We can construct a triangle, if we can construct

- A triangle perspective to the triangle, and the perspector,
- and, a second triangle perspective to the triangle, and the second perspector.

We use the Machine for Questions and Answers to specify a few theorems given in the paper [3]. We obtain the following theorems (the list below could be extended by the reader by adding additional similar theorems):

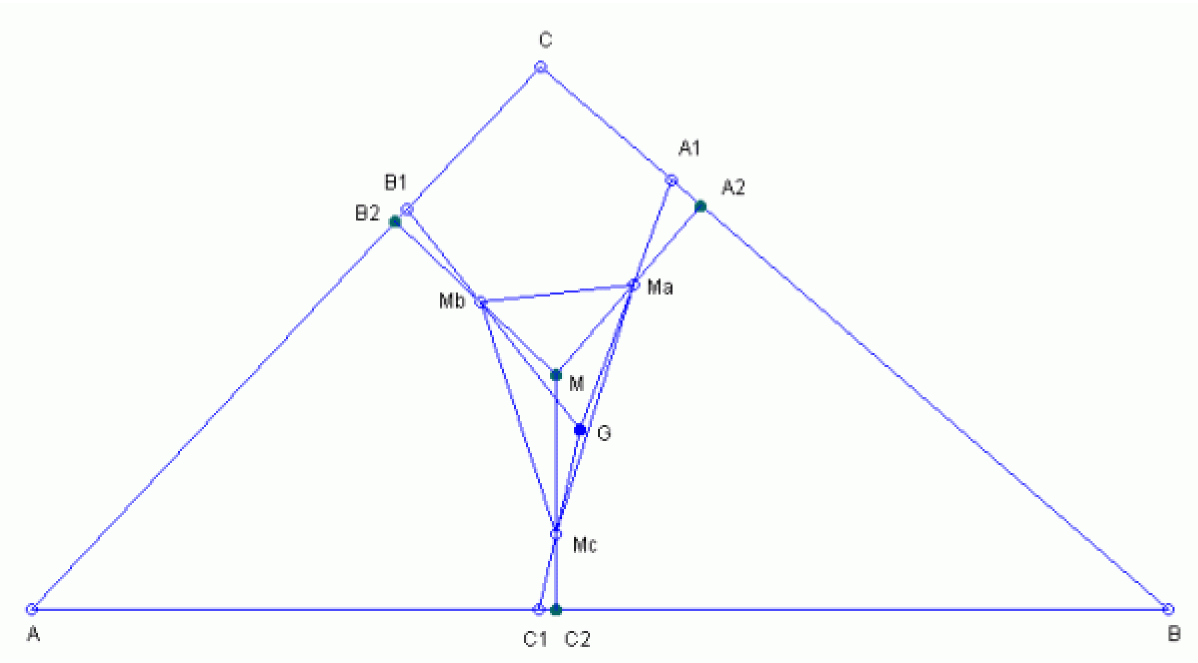
1. The Malfatti Squares Triangle and the Pedal Triangle of the Symmedian Point are homothetic with homothetic center the Centroid.
2. The Malfatti Squares Triangle and the Pedal Triangle of the Malfatti-Moses Point are perspective with perspector the Malfatti-Moses Point.
3. The Malfatti Squares Triangle and the Hatzipolakis Triangle of the Orthocenter are homothetic with homothetic center the Inner Kenmotu Point.
4. The Malfatti Squares Triangle and the Hatzipolakis Triangle of the Centroid are homothetic with homothetic center the Centroid of the Pedal Triangle of the Inner Kenmotu Point.

The first two of the above theorems, and Solution 1 below, are given by Floor van Lamoen and Paul Yiu [4].

We use the above theorems to obtain six ways how to construct the Malfatti Squares Triangle: The above four perspectives give six ways (Clearly, if the reader extends the list of the perspectives, he will obtain additional ways.)

Solution 1

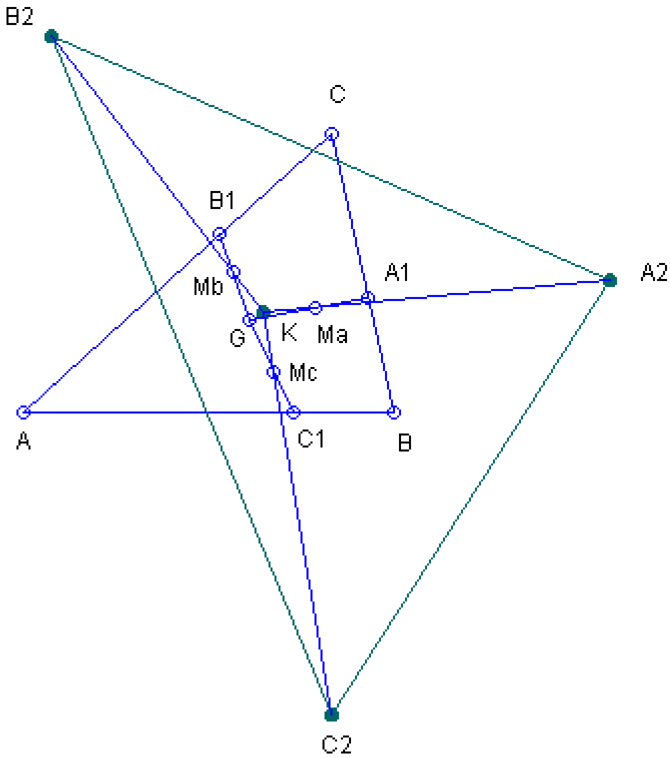
We use Theorems 1 and 2. See the Figure:



G - Centroid;
$A_1B_1C_1$ - Pedal Triangle of the Symmedian Point;
M - Malfatti-Moses Point;
$A_2B_2C_2$ - Pedal Triangle of the Malfatti-Moses Point;
M_a - the intersection of lines GA_1 and MA_2;
M_b - the intersection of lines GB_1 and MB_2;
M_c - the intersection of lines GC_1 and MC_2;
$M_aM_bM_c$ - the Malffati Squares Triangle.

Solution 2

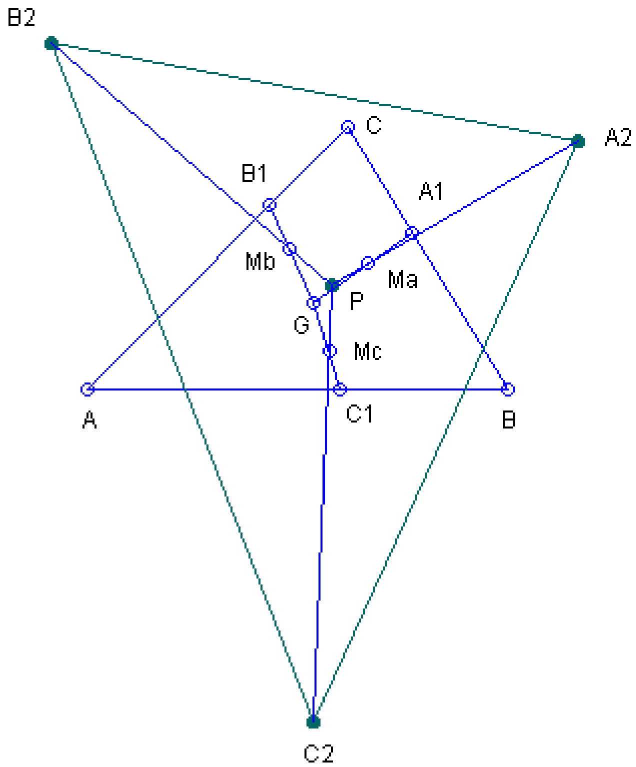
We use Theorems 1 and 3. See the Figure:



G - Centroid;
$A_1B_1C_1$ - Pedal Triangle of the Symmedian Point;
K - Inner Kenmotu Point;
$A_2B_2C_2$ - Hatzipolakis Triangle of the Orthocenter;
M_a - the intersection of lines GA_1 and KA_2;
M_b - the intersection of lines GB_1 and KB_2;
M_c - the intersection of lines GC_1 and KC_2;
$M_aM_bM_c$ - the Malffati Squares Triangle.

Solution 3

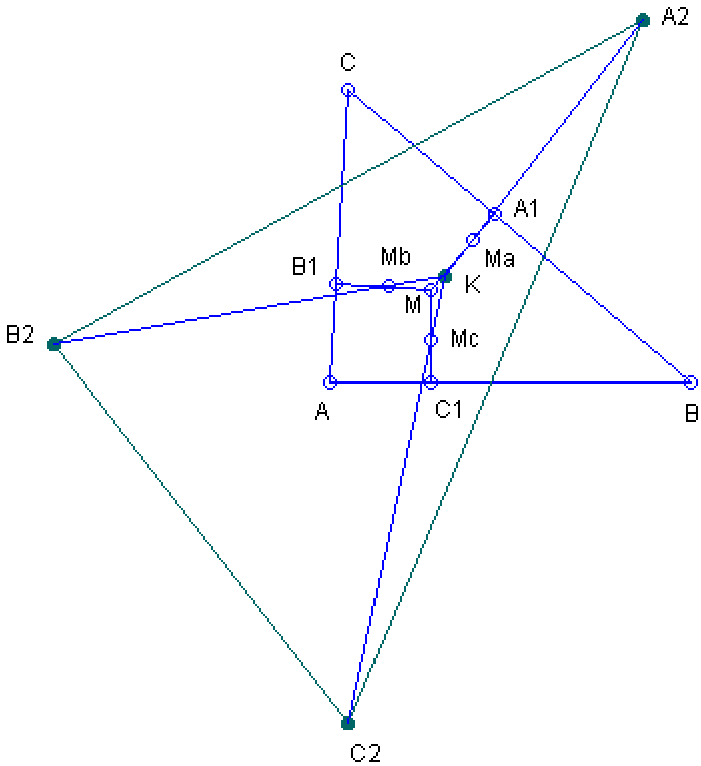
We use Theorems 1 and 4. See the Figure:



G - Centroid;
$A_1B_1C_1$ - Pedal Triangle of the Symmedian Point;
P - Centroid of the Pedal Triangle of the Inner Kenmotu Point;
$A_2B_2C_2$ - Hatzipolakis Triangle of the Centroid;
M_a - the intersection of lines GA_1 and PA_2;
M_b - the intersection of lines GB_1 and PB_2;
M_c - the intersection of lines GC_1 and PC_2;
$M_aM_bM_c$ - the Malffati Squares Triangle.

Solution 4

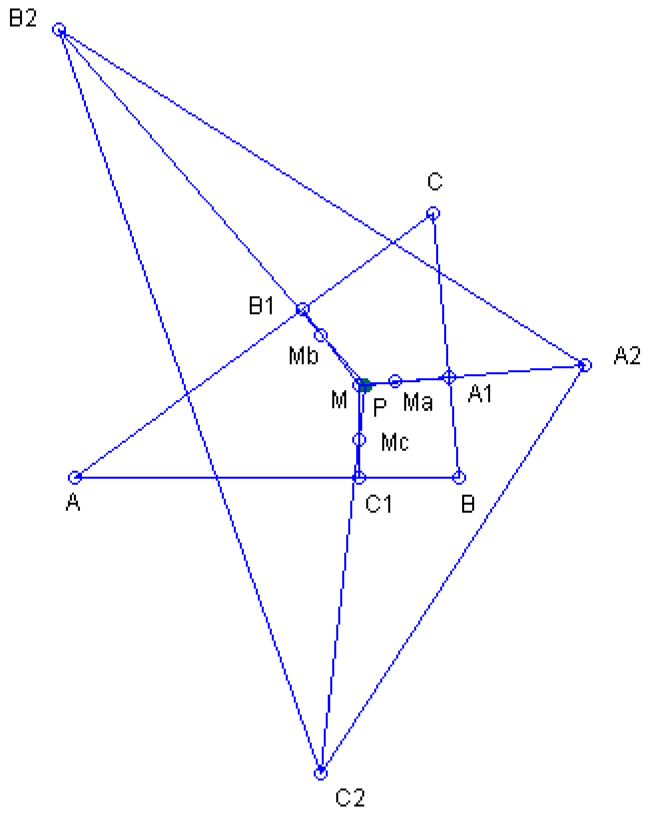
We use Theorems 2 and 3. See the Figure:



M - Malfatti-Moses Point;
$A_1B_1C_1$ - Pedal Triangle of the Malfatti-Moses Point;
K - Inner Kenmotu Point;
$A_2B_2C_2$ - Hatzipolakis Triangle of the Orthocenter;
M_a - the intersection of lines MA_1 and KA_2;
M_b - the intersection of lines MB_1 and KB_2;
M_c - the intersection of lines MC_1 and KC_2;
$M_aM_bM_c$ - the Malffati Squares Triangle.

Solution 5

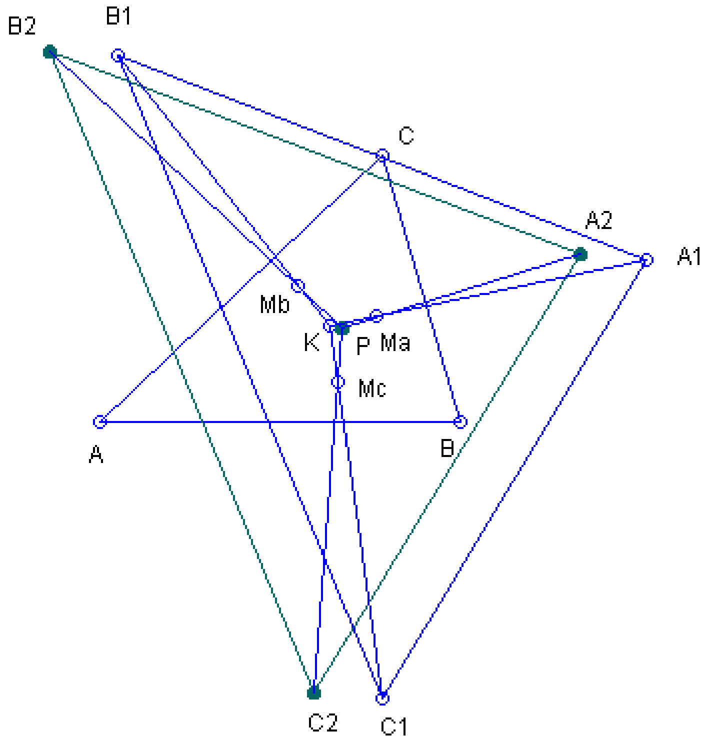
We use Theorems 2 and 4. See the Figure:



M - Malfatti-Moses Point;
$A_1B_1C_1$ - Pedal Triangle of the Malfatti-Moses Point;
P - Centroid of the Pedal Triangle of the Inner Kenmotu Point;
$A_2B_2C_2$ - Hatzipolakis Triangle of the Centroid;
M_a - the intersection of lines MA_1 and PA_2;
M_b - the intersection of lines MB_1 and PB_2;
M_c - the intersection of lines MC_1 and PC_2;
$M_aM_bM_c$ - the Malffati Squares Triangle.

Solution 6

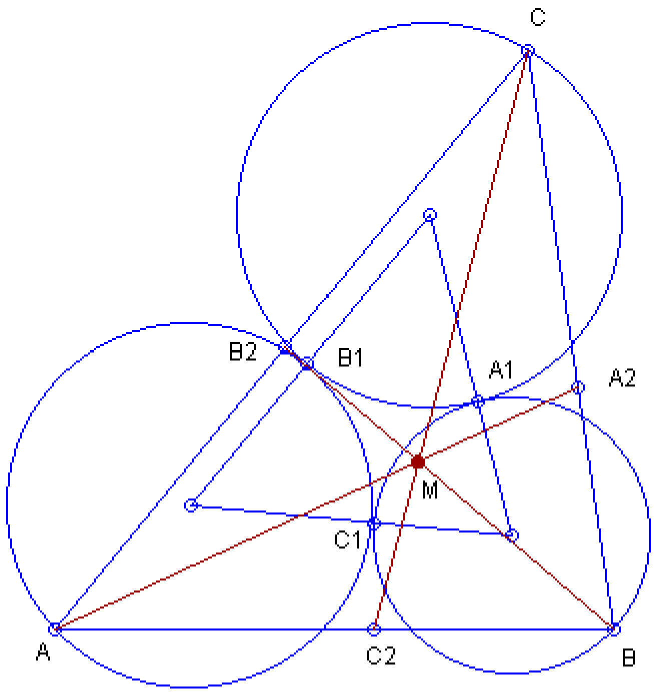
We use Theorems 3 and 4. See the Figure:



K - Inner Kenmotu Point;
$A_1B_1C_1$ - Hatzipolakis Triangle of the Orthocenter;
P - Centroid of the Pedal Triangle of the Inner Kenmotu Point;
$A_2B_2C_2$ - Hatzipolakis Triangle of the Centroid;
M_a - the intersection of lines KA_1 and PA_2;
M_b - the intersection of lines KB_1 and PB_2;
M_c - the intersection of lines KC_1 and PC_2;
$M_aM_bM_c$ - the Malffati Squares Triangle.

The Moses construction of the Malfatti-Moses Point

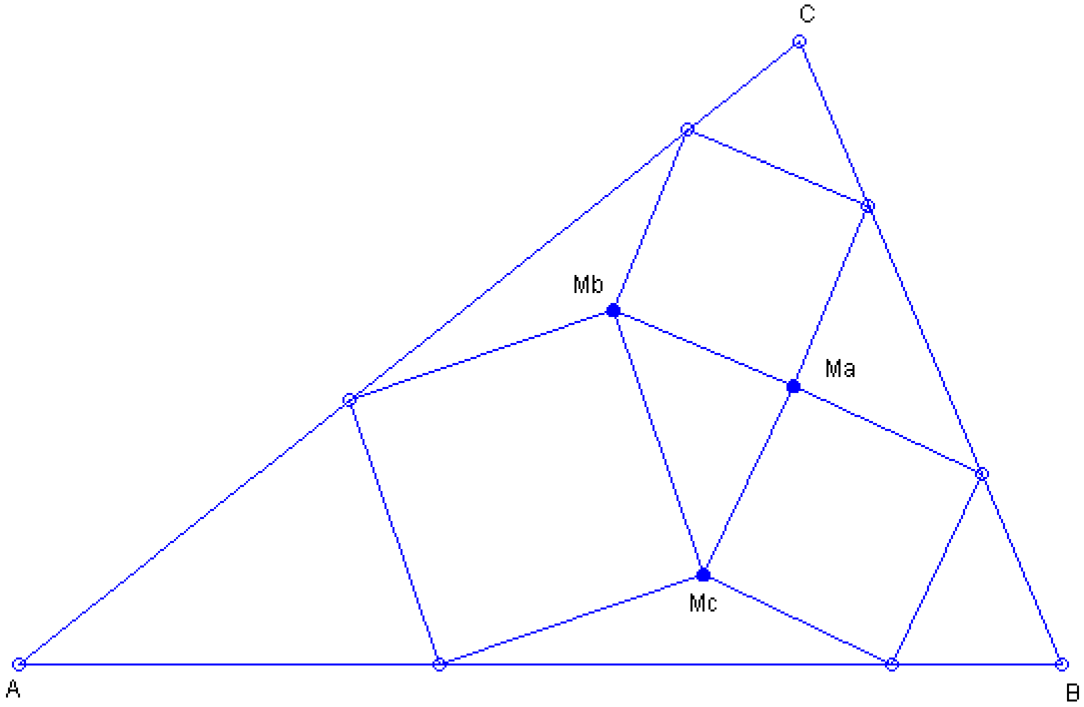
We recall the Peter J.C. Moses construction of the Malfatti-Moses Point given in [6], Proposition 2. See the Figure:



$A_1B_1C_1$ - Contact Lucas Triangle = Intouch Triangle of the Lucas Central Triangle;
A_2 - the foot of the perpendicular from A_1 to BC;
B_2 - the foot of the perpendicular from B_1 to CA;
C_2 - the foot of the perpendicular from C_1 to AB;
M - Malfatti-Moses Point = intersection of lines AA_2, BB_2, and CC_2.

From Malfatti Squares Triangle to Malfatti Squares

If we have constructed the Malfatti Squares Triangle, we could easily construct the Malfatti Squares. See the Figure:



$M_aM_bM_c$ - the Malffati Squares Triangle.

Invitation

The reader is invited to submit a note/paper containing

- synthetic proofs of theorems from this paper,
- or, applications of theorems from this paper,
- or, additional references related to this paper.

Definitions

We use the definitions in accordance with [1 - 8] and papers published in this journal.

The Level

The Machine for Questions and Answers is used to produce results in this paper. Currently the Machine has 6 levels of depths - 0,1,2,3,4,5. We use for this paper the level 0, that is, the Machine produces only elementary results. If we need deeper investigation, we have to use a level bigger than 0. Since the Machine for Questions and Answers produces too many results, it is suitable we to use bigger levels upon request, that is, for specific questions.

Thanks

The figures in this note are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for

download in the Web: Rene Grothmann's C.a.R.. It is free and open source. The reader may verify easily the statements of this paper by using C.a.R. Many thanks to Rene Grothmann for his wonderful program.

References

1. Quim Castellsaguer, The Triangles Web, http://www.xtec.es/~qcastell/ttw/ttweng/portada.html
2. D. Dekov, Computer-Generated Encyclopedia of Euclidean Geometry, First Edition, 2006, http://www.dekovsoft.com/
3. D. Dekov, Malfatti Squares Triangle, Journal of Computer-Generated Euclidean Geometry, volume 2, 2007, http://www.dekovsoft.com/j/
4. Floor van Lamoen and Paul Yiu, Construction of Malfatti Squares, preprint, 2003.
5. C. Kimberling, Encyclopedia of Triangle Centers, http://faculty.evansville.edu/ck6/encyclopedia/
6. Peter J.C. Moses, Circles and Triangle Centers Associated with the Lucas Circles, Forum Geometricorum, volume 5, 2005, 97-106.
7. Eric W. Weisstein, MathWorld - A Wolfram Web Resource. http://mathworld.wolfram.com/
8. Paul Yiu, Introduction to the Geometry of the Triangle, 2001, http://www.math.fau.edu/yiu/geometry.html

Publication Date: 14 November 2007

Dr.Deko Dekov, ddekov@dekovsoft.com.