

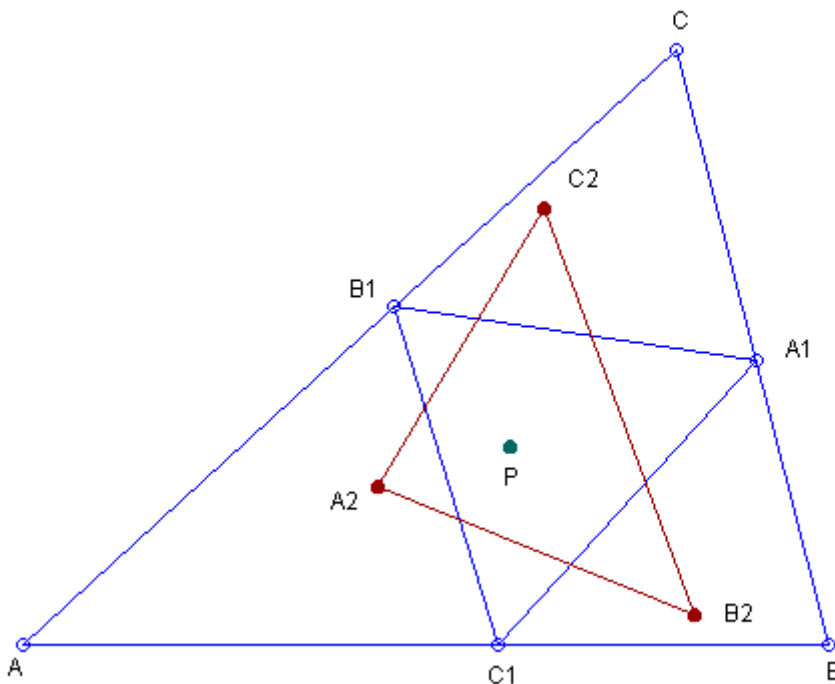
Triangles of Reflections. Part 1.

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Abstract. By using the computer program "Machine for Questions and Answers", we study perspectors of basic triangles and triangles of reflections of a triangle center in the sides of the cevian triangle of a triangle center.

Given a triangle ABC and a Triangle Center of kind 1, labeled by P . Let $A_1B_1C_1$ be the cevian triangle of a Triangle Center of kind 2 (possibly different from kind 1). Construct the reflection A_2 of P in the sideline B_1C_1 . Construct the reflection B_2 of P in the sideline C_1A_1 . Construct the reflection C_2 of P in the sideline A_1B_1 . We call triangle $A_2B_2C_2$ the *Triangle of the reflections of the Triangle Center of kind 1 in the sides of the Cevian Triangle of the Triangle Center of kind 2*.

See the Figure:



- P - Triangle Center of kind 1;
- $A_1B_1C_1$ - cevian triangle of Triangle Center of kind 2;
- A_2 - reflection of P in the sideline B_1C_1 ;
- B_2 - reflection of P in the sideline C_1A_1 ;

C_2 - reflection of P in the sideline A_1B_1 ;
 $A_2B_2C_2$ - Triangle of the reflections of the Triangle Center of kind 1 in the sides of the Cevian Triangle of the Triangle Center of kind 2.
 In this Figure:
 P - Centroid;
 $A_1B_1C_1$ - cevian triangle of the Incenter = Incentral Triangle;
 A_2 - reflection of P in the sideline B_1C_1 ;
 B_2 - reflection of P in the sideline C_1A_1 ;
 C_2 - reflection of P in the sideline A_1B_1 ;
 $A_2B_2C_2$ - Triangle of the reflections of the Centroid in the sides of the Incentral Triangle.

In 2003 Jean-Pierre Ehrmann [3] proved that

For any Triangle Center, Triangle ABC and the Triangle of the reflections of the Triangle Center in the sides of the cevian triangle of the Triangle Center are perspective.

Jean-Pierre Ehrmann [3] gave a description of the perspector in the above theorem, and a few examples. The reader is invited to submit a note/paper with additional references.

Examples

The Machine for Questions and Answers produces examples of perspectives between triangles and Triangles of reflections. A few examples are given below.

For any Triangle Center, Triangle ABC and the Triangle of the reflections of the Triangle Center in the sides of the cevian triangle of the Triangle Center are perspective.

For any Triangle Center, the Tangential Triangle and the Triangle of the reflections of the Triangle Center in the sides of the Medial Triangle are perspective.

Triangle ABC and the Triangle of the reflections of the Incenter in the sides of the Incentral Triangle are perspective with perspector the Moses Point.

Triangle ABC and the Triangle of the reflections of the Centroid in the sides of the Medial Triangle are perspective with perspector the Symmedian Point of the Anticomplementary Triangle.

Triangle ABC and the Triangle of the reflections of the Circumcenter in the sides of the Medial Triangle are perspective with perspector the Prasolov Point.

Triangle ABC and the Triangle of the reflections of the Orthocenter in the sides of the Medial Triangle are perspective with perspector the Orthocenter.

Triangle ABC and the Triangle of the reflections of the Nine-Point Center in the sides of the Medial Triangle are homothetic with homothetic center the Circumcenter.

Triangle ABC and the Triangle of the reflections of the Circumcenter in the sides of the Orthic Triangle are perspective with perspector the Circumcenter.

Triangle ABC and the Triangle of the reflections of the Orthocenter in the sides of the

Orthic Triangle are homothetic with homothetic center the Gibert Point.

Triangle ABC and the Triangle of the reflections of the Symmedian Point in the sides of the Symmedian Triangle are perspective with perspector the Perspector of Triangle ABC and the Orthic Triangle of the Symmedian Triangle.

Triangle ABC and the Triangle of the reflections of the Incenter in the sides of the Intouch Triangle are perspective with perspector the Incenter.

Triangle ABC and the Triangle of the reflections of the Gergonne Point in the sides of the Intouch Triangle are perspective with perspector the Isogonal Conjugate of the Mittenpunkt.

Triangle ABC and the Triangle of the reflections of the Nagel Point in the sides of the Extouch Triangle are perspective with perspector the Perspector of Triangle ABC and the Orthic Triangle of the Extouch Triangle.

The Incentral Triangle and the Triangle of the reflections of the Incenter in the sides of the Intouch Triangle are perspective with perspector the Incenter.

The Medial Triangle and the Triangle of the reflections of the Circumcenter in the sides of the Medial Triangle are perspective with perspector the Circumcenter.

The Medial Triangle and the Triangle of the reflections of the Nine-Point Center in the sides of the Medial Triangle are homothetic with homothetic center the Complement of the Nine-Point Center.

The Medial Triangle and the Triangle of the reflections of the Outer Fermat Point in the sides of the Medial Triangle are perspective with perspector the First Isodynamic Point.

The Medial Triangle and the Triangle of the reflections of the Inner Fermat Point in the sides of the Medial Triangle are perspective with perspector the Second Isodynamic Point.

The Medial Triangle and the Triangle of the reflections of the Nine-Point Center in the sides of the Orthic Triangle are perspective with perspector the Nine-Point Center.

The Medial Triangle and the Triangle of the reflections of the Spieker Center in the sides of the Intouch Triangle are perspective with perspector the Spieker Center.

The Orthic Triangle and the Triangle of the reflections of the Circumcenter in the sides of the Medial Triangle are homothetic with homothetic center the Nine-Point Center.

The Orthic Triangle and the Triangle of the reflections of the Orthocenter in the sides of the Medial Triangle are perspective with perspector the Orthocenter.

The Orthic Triangle and the Triangle of the reflections of the Prasolov Point in the sides of the Medial Triangle are perspective with perspector the Circumcenter.

The Orthic Triangle and the Triangle of the reflections of the Center of the Taylor Circle in the sides of the Cevian Triangle of the Circumcenter are perspective with perspector the

Center of the Taylor Circle.

The Intouch Triangle and the Triangle of the reflections of the Incenter in the sides of the Medial Triangle are perspective with perspector the Incenter.

The Intouch Triangle and the Triangle of the reflections of the Spieker Center in the sides of the Medial Triangle are homothetic with homothetic center the Complement of the Spieker Center.

The Extouch Triangle and the Triangle of the reflections of the Bevan Point in the sides of the Medial Triangle are perspective with perspector the Bevan Point.

The Excentral Triangle and the Triangle of the reflections of the Circumcenter in the sides of the Incentral Triangle are perspective with perspector the Circumcenter.

The Excentral Triangle and the Triangle of the reflections of the Bevan Point in the sides of the Medial Triangle are perspective with perspector the Bevan Point.

The Excentral Triangle and the Triangle of the reflections of the Incenter in the sides of the Intouch Triangle are homothetic with homothetic center the Incenter.

The Anticomplementary Triangle and the Triangle of the reflections of the de Longchamps Point in the sides of the Medial Triangle are perspective with perspector the de Longchamps Point.

The Anticomplementary Triangle and the Triangle of the reflections of the Orthocenter in the sides of the Orthic Triangle are homothetic with homothetic center the Orthocenter.

The Anticomplementary Triangle and the Triangle of the reflections of the Nagel Point in the sides of the Intouch Triangle are perspective with perspector the Nagel Point.

The Tangential Triangle and the Triangle of the reflections of the Circumcenter in the sides of the Medial Triangle are homothetic with homothetic center the Circumcenter.

The Circum-Incentral Triangle and the Triangle of the reflections of the Circumcenter in the sides of the Medial Triangle are perspective with perspector the Circumcenter.

The Circum-Incentral Triangle and the Triangle of the reflections of the Incenter in the sides of the Intouch Triangle are homothetic with homothetic center the Incenter.

The Circum-Orthic Triangle and the Triangle of the reflections of the Circumcenter in the sides of the Medial Triangle are homothetic with homothetic center the Centroid.

The Circum-Orthic Triangle and the Triangle of the reflections of the Orthocenter in the sides of the Medial Triangle are perspective with perspector the Orthocenter.

The Half-Altitude Triangle and the Triangle of the reflections of the Orthocenter in the sides of the Medial Triangle are homothetic with homothetic center the Orthocenter.

The Half-Altitude Triangle and the Triangle of the reflections of the Center of the Taylor

Circles on the sides of the Orthic Triangle are perspective with perspector the Center of the Taylor Circle.

The Euler Triangle and the Triangle of the reflections of the Orthocenter in the sides of the Medial Triangle are perspective with perspector the Orthocenter.

The Euler Triangle and the Triangle of the reflections of the Nine-Point Center in the sides of the Orthic Triangle are perspective with perspector the Nine-Point Center.

The Euler Triangle and the Triangle of the reflections of the Incenter in the sides of the Intouch Triangle are perspective with perspector the First Feuerbach Point.

The Euler Triangle and the Triangle of the reflections of the Circumcenter in the sides of the Intouch Triangle are perspective with perspector the First Feuerbach Point.

The Euler Triangle and the Triangle of the reflections of the Moses Point in the sides of the Intouch Triangle are perspective with perspector the First Feuerbach Point.

The Euler Triangle and the Triangle of the reflections of the Bevan Point in the sides of the Intouch Triangle are perspective with perspector the First Feuerbach Point.

The Euler Triangle and the Triangle of the reflections of the Internal Center of Similitude of the Incircle and the Circumcircle in the sides of the Intouch Triangle are perspective with perspector the First Feuerbach Point.

The Euler Triangle and the Triangle of the reflections of the External Center of Similitude of the Incircle and the Circumcircle in the sides of the Intouch Triangle are perspective with perspector the First Feuerbach Point.

The Euler Triangle and the Triangle of the reflections of the Weill Point in the sides of the Intouch Triangle are perspective with perspector the First Feuerbach Point.

The Euler Triangle and the Triangle of the reflections of the Evans Perspector in the sides of the Intouch Triangle are perspective with perspector the First Feuerbach Point.

The Intangents Triangle and the Triangle of the reflections of the Incenter in the sides of the Medial Triangle are perspective with perspector the Incenter.

The Extangents Triangle and the Triangle of the reflections of the Bevan Point in the sides of the Medial Triangle are perspective with perspector the Bevan Point.

The Mixtilinear Triangle and the Triangle of the reflections of the Incenter in the sides of the Intouch Triangle are perspective with perspector the Incenter.

The Fuhrmann Triangle and the Triangle of the reflections of the Circumcenter in the sides of the Medial Triangle are perspective with perspector the Circumcenter.

The Fuhrmann Triangle and the Triangle of the reflections of the Nagel Point in the sides of the Intouch Triangle are perspective with perspector the Nagel Point.

The Mid-Arc Triangle and the Triangle of the reflections of the Incenter in the sides of the Intouch Triangle are perspective with perspector the Incenter.

The Reflection Triangle and the Triangle of the reflections of the Moses Point in the sides of the Incidental Triangle are perspective with perspector the Incenter.

The Reflection Triangle and the Triangle of the reflections of the Orthocenter in the sides of the Medial Triangle are perspective with perspector the Orthocenter.

The First Brocard Triangle and the Triangle of the reflections of the Circumcenter in the sides of the Medial Triangle are perspective with perspector the Circumcenter.

The Malfatti Central Triangle and the Triangle of the reflections of the Incenter in the sides of the Intouch Triangle are perspective with perspector the Incenter.

The Malfatti Squares Triangle and the Triangle of the reflections of the Malfatti-Moses Point in the sides of the Medial Triangle are perspective with perspector the Malfatti-Moses Point.

The Lucas Central Triangle and the Triangle of the reflections of the Nine-Point Center in the sides of the Medial Triangle are perspective with perspector the Circumcenter.

The Lucas Central Triangle and the Triangle of the reflections of the Circumcenter in the sides of the Orthic Triangle are perspective with perspector the Circumcenter.

The Neuberg Triangle and the Triangle of the reflections of the Circumcenter in the sides of the Medial Triangle are perspective with perspector the Circumcenter.

The Reflected Neuberg Triangle and the Triangle of the reflections of the Circumcenter in the sides of the Medial Triangle are perspective with perspector the Circumcenter.

The Hexyl Triangle and the Triangle of the reflections of the Circumcenter in the sides of the Incidental Triangle are perspective with perspector the Circumcenter.

The Hexyl Triangle and the Triangle of the reflections of the Incenter in the sides of the Medial Triangle are perspective with perspector the Incenter.

The Hexyl Triangle and the Triangle of the reflections of the Incenter in the sides of the Intouch Triangle are homothetic with homothetic center the Isogonal Conjugate of the Mittenpunkt.

The Hexyl Triangle and the Triangle of the reflections of the Bevan Point in the sides of the Intouch Triangle are perspective with perspector the Bevan Point.

The Johnson Triangle and the Triangle of the reflections of the Circumcenter in the sides of the Medial Triangle are perspective with perspector the Circumcenter.

The Johnson Triangle and the Triangle of the reflections of the Nine-Point Center in the sides of the Medial Triangle are homothetic with homothetic center the Centroid.

The Johnson Triangle and the Triangle of the reflections of the Orthocenter in the sides of the Orthic Triangle are homothetic with homothetic center the Orthocenter.

The Johnson Triangle and the Triangle of the reflections of the Center of the Fuhrmann Circle in the sides of the Intouch Triangle are perspective with perspector the Fuhrmann Center.

The Inner Johnson-Yff Triangle and the Triangle of the reflections of the Nine-Point Center in the sides of the Medial Triangle are homothetic with homothetic center the External Center of Similitude of the Incircle and the Circumcircle.

The Inner Johnson-Yff Triangle and the Triangle of the reflections of the Center of the Inner Johnson-Yff Circle in the sides of the Medial Triangle are perspective with perspector the Center of the Inner Johnson-Yff Circle.

The Inner Johnson-Yff Triangle and the Triangle of the reflections of the Internal Center of Similitude of the Incircle and the Circumcircle in the sides of the Orthic Triangle are perspective with perspector the Internal Center of Similitude of the Incircle and the Circumcircle.

The Inner Johnson-Yff Triangle and the Triangle of the reflections of the Incenter in the sides of the Intouch Triangle are perspective with perspector the Incenter.

The Outer Johnson-Yff Triangle and the Triangle of the reflections of the Nine-Point Center in the sides of the Medial Triangle are homothetic with homothetic center the Internal Center of Similitude of the Incircle and the Circumcircle.

The Outer Johnson-Yff Triangle and the Triangle of the reflections of the Center of the Outer Johnson-Yff Circle in the sides of the Medial Triangle are perspective with perspector the Center of the Outer Johnson-Yff Circle.

The Outer Johnson-Yff Triangle and the Triangle of the reflections of the External Center of Similitude of the Incircle and the Circumcircle in the sides of the Orthic Triangle are perspective with perspector the External Center of Similitude of the Incircle and the Circumcircle.

The Outer Johnson-Yff Triangle and the Triangle of the reflections of the Incenter in the sides of the Intouch Triangle are perspective with perspector the Incenter.

Note

The above list contains examples in which the perspectors are between the most basic points. The Machine for Questions and Answers could identify the other perspectors upon request.

Invitation

The reader is invited to submit a note/paper containing

- synthetic proofs of theorems from this paper,

- or, applications of theorems from this paper,
- or, additional references related to this paper.

Definitions

We use the definitions in accordance with [1 - 6] and papers published in this journal.

The Level

The Machine for Questions and Answers is used to produce results in this paper. Currently the Machine has 6 levels of depths - 0,1,2,3,4,5. We use for this paper the level 0, that is, the Machine produces only elementary results. If we need deeper investigation, we have to use a level bigger than 0. Since the Machine for Questions and Answers produces too many results, it is suitable we to use bigger levels upon request, that is, for specific questions.

Thanks

The figures in this note are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download in the Web: [Rene Grothmann's C.a.R.](#). It is free and open source. The reader may verify easily the statements of this paper by using C.a.R. Many thanks to Rene Grothmann for his wonderful program.

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