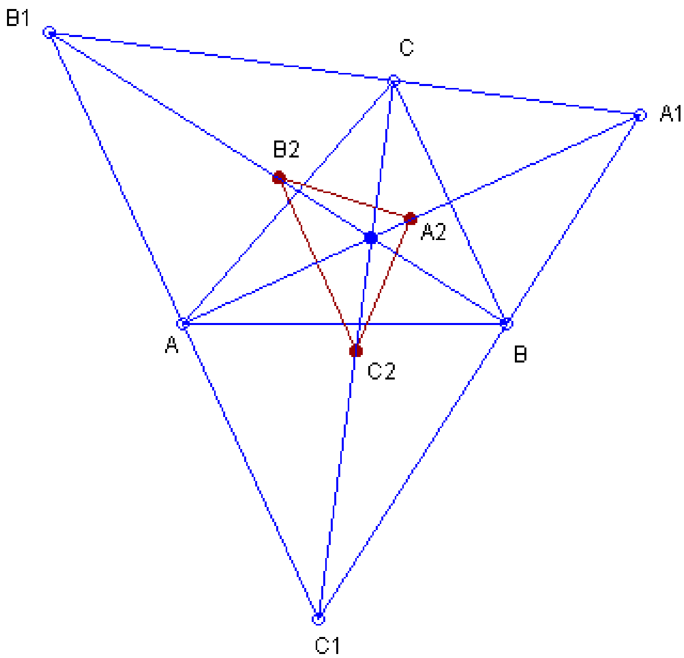
Half-Anticevian Triangles

Deko Dekov

Abstract. We define Half-Cevian triangles and use the computer program "Machine for Questions and Answers" to study perspectives between Half-Anticevian triangles and other triangles.

Given a triangle ABC and a Triangle Center, labeled by P. Construct the Anticevian Triangle $A_1B_1C_1$ of P. Construct the midpoints A_2, B_2, and C_2 of segments AA_1, BB_1, and CC_1, respectively. We call triangle $A_2B_2C_2$ the *Half-Anticevian Triangle of the Triangle Center P.*

See the Figure:



P - Triangle Center;
$A_1B_1C_1$ - anticevians triangle of P;
A_2, B_2, and C_2 midpoints of segments AA_1, BB_1, and CC_1, respectively;
$A_2B_2C_2$ - Half-Anticevian Triangle of the Triangle Center.
In this Figure:
P - Incenter;
$A_1B_1C_1$ - anticevians triangle of I = Excentral Triangle;

A_2, B_2, and C_2 are the midpoints of segments AA_1, BB_1, and CC_1, respectively;
$A_2B_2C_2$ is the Half-Anticevian Triangle of the Incenter.

The Half-Anticevian Triangle of the Centroid is the Medial Triangle.

Examples

The Machine for Questions and Answers produces theorems on perspectives between Half-Anticevian triangles and other triangles. A few examples are given below.

The Incentral Triangle and the Half-Anticevian Triangle of the Spieker Center are perspective.

The Orthic Triangle and the Half-Anticevian Triangle of the Circumcenter are perspective.

The Symmedial Triangle and the Half-Anticevian Triangle of the Symmedian Point are perspective.

The Intouch Triangle and the Half-Anticevian Triangle of the Mittenpunkt are perspective.

The Extouch Triangle and the Half-Anticevian Triangle of the Incenter are perspective.

The Circum-Incentral Triangle and the Half-Anticevian Triangle of the Incenter are perspective.

The Circum-Orthic Triangle and the Half-Anticevian Triangle of the Orthocenter are perspective.

The Half-Altitude Triangle and the Half-Anticevian Triangle of the Circumcenter are perspective.

The Half-Altitude Triangle and the Half-Anticevian Triangle of the Orthocenter are perspective.

The Half-Altitude Triangle and the Half-Anticevian Triangle of the Nine-Point Center are perspective.

The Half-Altitude Triangle and the Half-Anticevian Triangle of the de Longchamps Point are perspective.

The Half-Altitude Triangle and the Half-Anticevian Triangle of the Schiffler Point are perspective.

The Half-Altitude Triangle and the Half-Anticevian Triangle of the Exeter Point are perspective.

The Half-Altitude Triangle and the Half-Anticevian Triangle of the Far-Out Point are perspective.

The Half-Altitude Triangle and the Half-Anticevian Triangle of the Gibert Point are perspective.

The Half-Altitude Triangle and the Half-Anticevian Triangle of the Center of the Orthocentroidal Circle are perspective.

The Half-Altitude Triangle and the Half-Anticevian Triangle of the Skordev Point are perspective.

The Euler Triangle and the Half-Anticevian Triangle of the Orthocenter are perspective.

The Feuerbach Triangle and the Half-Anticevian Triangle of the Incenter are perspective.

The Feuerbach Triangle and the Half-Anticevian Triangle of the Spieker Center are perspective.

The Feuerbach Triangle and the Half-Anticevian Triangle of the Second Feuerbach Point are perspective.

The Mixtilinear Triangle and the Half-Anticevian Triangle of the Incenter are perspective.

The Mid-Arc Triangle and the Half-Anticevian Triangle of the Incenter are perspective.

The Reflection Triangle and the Half-Anticevian Triangle of the Circumcenter are perspective.

The Reflection Triangle and the Half-Anticevian Triangle of the Orthocenter are perspective.

The Second Brocard Triangle and the Half-Anticevian Triangle of the Symmedian Point are perspective.

The Third Brocard Triangle and the Half-Anticevian Triangle of the Third Power Point are perspective.

The de Villiers Triangle and the Half-Anticevian Triangle of the First de Villiers Point are perspective.

The Malfatti Central Triangle and the Half-Anticevian Triangle of the Incenter are perspective.

The Lucas Central Triangle and the Half-Anticevian Triangle of the Circumcenter are perspective.

The Inner Lucas Triangle and the Half-Anticevian Triangle of the Radical Center of the Lucas Circles are perspective.

The Hexyl Triangle and the Half-Anticevian Triangle of the Mittenpunkt are perspective.

The Johnson Triangle and the Half-Anticevian Triangle of the Nine-Point Center are perspective.

The Inner Johnson-Yff Triangle and the Half-Anticevian Triangle of the Incenter are perspective.

The Inner Johnson-Yff Triangle and the Half-Anticevian Triangle of the Mittenpunkt are perspective.

The Outer Johnson-Yff Triangle and the Half-Anticevian Triangle of the Incenter are perspective.

The Outer Johnson-Yff Triangle and the Half-Anticevian Triangle of the Mittenpunkt are perspective.

The Apollonius Triangle and the Half-Anticevian Triangle of the Spieker Center are perspective.

The Apollonius Triangle and the Half-Anticevian Triangle of the Apollonius Point are perspective.

The Outer Fermat Triangle and the Half-Anticevian Triangle of the Outer Fermat Point are perspective.

The Inner Fermat Triangle and the Half-Anticevian Triangle of the Inner Fermat Point are perspective.

The Outer Napoleon Triangle and the Half-Anticevian Triangle of the Outer Napoleon Point are perspective.

The Inner Napoleon Triangle and the Half-Anticevian Triangle of the Inner Napoleon Point are perspective.

The Outer Vecten Triangle and the Half-Anticevian Triangle of the Outer Vecten Point are perspective.

The Outer Vecten Triangle and the Half-Anticevian Triangle of the Malfatti-Moses Point are perspective.

The Inner Vecten Triangle and the Half-Anticevian Triangle of the Inner Vecten Point are perspective.

We specify the perspectors provided they between the basic points. The other perspectors could be described upon reauest.

The Circum-Orthic Triangle and the Half-Anticevian Triangle of the Orthocenter are perspective with perspector the Orthocenter.

The Half-Altitude Triangle and the Half-Anticevian Triangle of the Circumcenter are perspective with perspector the Nine-Point Center.

The Half-Altitude Triangle and the Half-Anticevian Triangle of the Nine-Point Center are perspective with perspector the Center of the Taylor Circle.

The Euler Triangle and the Half-Anticevian Triangle of the Orthocenter are perspective with perspector the Orthocenter.

The Feuerbach Triangle and the Half-Anticevian Triangle of the Second Feuerbach Point are perspective with perspector the Second Feuerbach Point.

The Mixtilinear Triangle and the Half-Anticevian Triangle of the Incenter are perspective with perspector the Incenter.

The Mid-Arc Triangle and the Half-Anticevian Triangle of the Incenter are perspective with perspector the Incenter.

The Reflection Triangle and the Half-Anticevian Triangle of the Orthocenter are perspective with perspector the Orthocenter.

The Second Brocard Triangle and the Half-Anticevian Triangle of the Symmedian Point are perspective with perspector the Symmedian Point.

The Third Brocard Triangle and the Half-Anticevian Triangle of the Third Power Point are perspective with perspector the Third Power Point.

The de Villiers Triangle and the Half-Anticevian Triangle of the First de Villiers Point are perspective with perspector the First de Villiers Point.

The Malfatti Central Triangle and the Half-Anticevian Triangle of the Incenter are perspective with perspector the Incenter.

The Lucas Central Triangle and the Half-Anticevian Triangle of the Circumcenter are perspective with perspector the Circumcenter.

The Johnson Triangle and the Half-Anticevian Triangle of the Nine-Point Center are perspective with perspector the Nine-Point Center.

The Inner Johnson-Yff Triangle and the Half-Anticevian Triangle of the Incenter are perspective with perspector the Incenter.

The Outer Johnson-Yff Triangle and the Half-Anticevian Triangle of the Incenter are perspective with perspector the Incenter.

The Apollonius Triangle and the Half-Anticevian Triangle of the Apollonius Point are perspective with perspector the Apollonius Point.

Invitation

The reader is invited to submit a note/paper containing

- synthetic proofs of theorems from this paper,

- or, applications of theorems from this paper,
- or, additional references related to this paper.

Definitions and Conventions

We use the definitions and conventions in accordance with [1 - 6] and papers published in this journal.

The Level

The Machine for Questions and Answers is used to produce results in this paper. Currently the Machine has 6 levels of depths - 0,1,2,3,4,5. We use for this paper the level 0, that is, the Machine produces only elementary results. If we need deeper investigation, we have to use a level bigger than 0. Since the Machine for Questions and Answers produces too many results, it is suitable we to use bigger levels upon request, that is, for specific questions.

Thanks

The figure in this note is produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download in the Web: Rene Grothmann's C.a.R.. It is free and open source. The reader may verify easily the statements of this paper by using C.a.R. Many thanks to Rene Grothmann for his wonderful program.

References

1. Quim Castellsaguer, The Triangles Web, http://www.xtec.es/~qcastell/ttw/ttweng/portada.html
2. D. Dekov, Computer-Generated Encyclopedia of Euclidean Geometry, First Edition, 2006, http://www.dekovsoft.com/
3. D. Dekov, The Ambiguities of the Natural Language, Journal of Computer-Generated Euclidean Geometry, vol. 2 (2007), http://www.dekovsoft.com/j/2007/01/index.htm
4. C. Kimberling, Encyclopedia of Triangle Centers, http://faculty.evansville.edu/ck6/encyclopedia/
5. Eric W. Weisstein, MathWorld - A Wolfram Web Resource. http://mathworld.wolfram.com/
6. Paul Yiu, Introduction to the Geometry of the Triangle, 2001, http://www.math.fau.edu/yiu/geometry.html

Publication Date: 28 November 2007

Dr.Deko Dekov, ddekov@dekovsoft.com.