

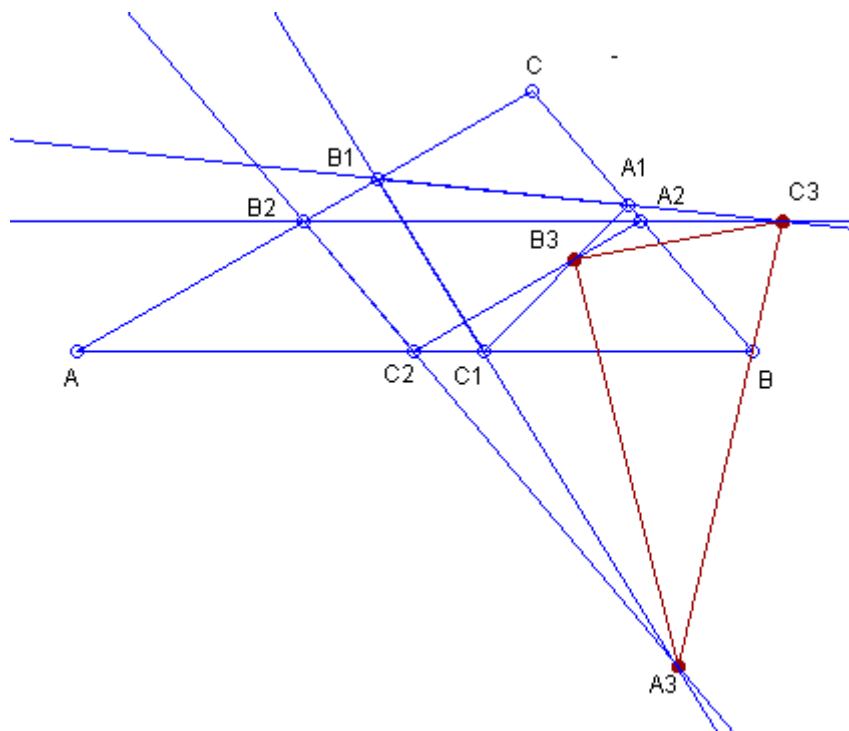
Side Triangles

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Abstract. By using the computer program "Machine for Questions and Answers", we find perspectives of Side Triangles.

Given Triangle Centers P and Q . Let $A_1B_1C_1$ and $A_2B_2C_2$ are the cevian triangles of P and Q , respectively. Let A_3 is the intersection point of lines B_1C_1 and B_2C_2 . Similarly, define B_3 and C_3 . Then triangle $A_3B_3C_3$ is the *Side Triangle of P and Q* .

See the Figure:



$A_1B_1C_1$ - cevian triangle of P ;

$A_2B_2C_2$ - cevian triangle of Q ;

A_3 - intersection point of lines B_1C_1 and B_2C_2 ;

B_3 - intersection point of lines C_1A_1 and C_2A_2 ;

C_3 - intersection point of lines A_1B_1 and A_2B_2 ;

$A_3B_3C_3$ - Side Triangle of P and Q .

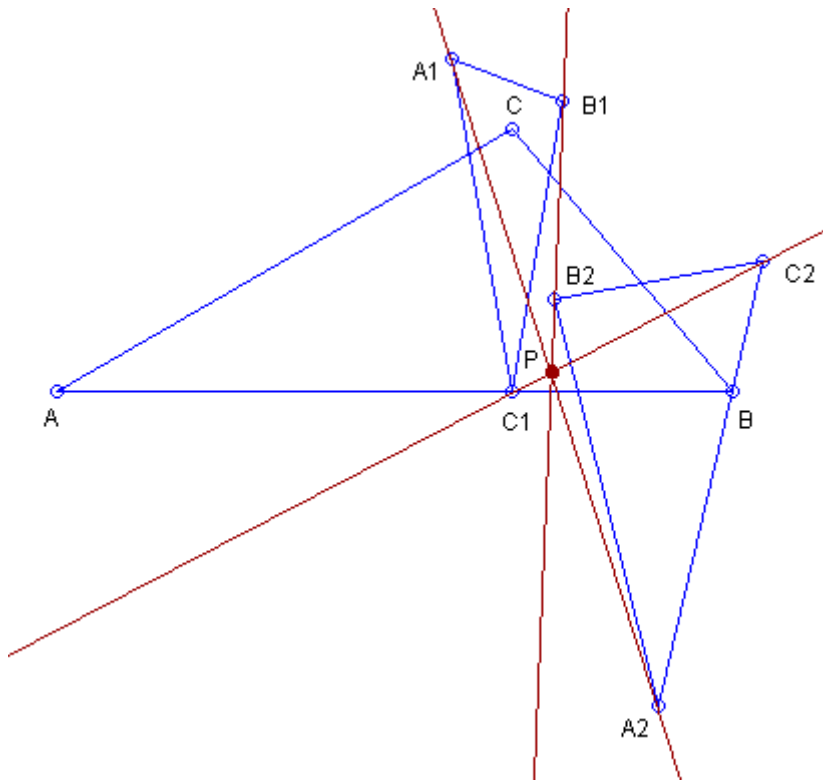
In this Figure:

$A_1B_1C_1$ - cevian triangle of the Incenter = Incentral Triangle;

$A_2B_2C_2$ - cevian triangle of the Centroid = Medial Triangle;
 A_3 - intersection point of lines B_1C_1 and B_2C_2 ;
 B_3 - intersection point of lines C_1A_1 and C_2A_2 ;
 C_3 - intersection point of lines A_1B_1 and A_2B_2 ;
 $A_3B_3C_3$ - Side Triangle of the Incenter and the Centroid.

Given triangles centers P, Q and R, the cevian triangle of R is perspective with the Side Triangle of P and Q.

See the Figure:



$A_1B_1C_1$ - cevian triangle of R;
 $A_2B_2C_2$ - Side Triangle of P and Q;
 P - perspector of triangles $A_1B_1C_1$ and $A_2B_2C_2$.

In this Figure:

$A_1B_1C_1$ - Cevian Triangle of the Orthocenter = Orthic Triangle;
 $A_2B_2C_2$ - Side Triangle of the Incenter and the Centroid;
 P - (First) Feuerbach Point = perspector of triangles $A_1B_1C_1$ and $A_2B_2C_2$.

The above theorem gives us a number of remarkable points - perspectors.

In the Hyacinthos message #4383 of Nov 18, 2001, Tatiana Emelyanova stated the following results:

The Incentral Triangle and the Side Triangle of the Centroid and the Orthocenter are perspective with perspector the First Feuerbach Point.

The Medial Triangle and the Side Triangle of the Incenter and the Orthocenter are

perspective with perspector the First Feuerbach Point.

The Orthic Triangle and the Side Triangle of the Centroid and the Incenter are perspective with perspector the First Feuerbach Point.

The Intouch Triangle and the Side Triangle of the Centroid and the Gergonne Point are perspective with perspector the First Feuerbach Point.

Below we give computer-generated examples.

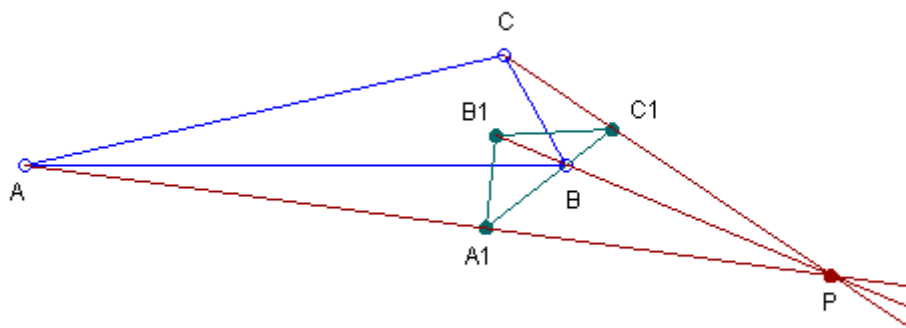
Examples

The Machine for Questions and Answers gives perspectives between given triangles. Examples of perspectives between triangles and side triangles are given below. In the examples below the perspectors are between the basic points.

Triangle ABC and the Side Triangle of the Incenter and the Orthocenter are perspective with perspector the Center of the Stevanovic Circle.

Triangle ABC and the Side Triangle of the Incenter and the Nagel Point are perspective with perspector the Center of the Stevanovic Circle.

See the Figure:



$A_1B_1C_1$ - Side Triangle of the Incenter and the Nagel Point;
P - Center of the Stevanovic Circle = perspector of triangles ABC and $A_1B_1C_1$.

Triangle ABC and the Side Triangle of the Incenter and the Mittenpunkt are perspective with perspector the Center of the Stevanovic Circle.

Triangle ABC and the Side Triangle of the Gergonne Point and the Incenter are perspective with perspector the Center of the Stevanovic Circle.

Triangle ABC and the Side Triangle of the Gergonne Point and the Orthocenter are

perspective with perspector the Center of the Stevanovic Circle.

Triangle ABC and the Side Triangle of the Gergonne Point and the Nagel Point are perspective with perspector the Center of the Stevanovic Circle.

Triangle ABC and the Side Triangle of the Gergonne Point and the Mittenpunkt are perspective with perspector the Center of the Stevanovic Circle.

Triangle ABC and the Side Triangle of the Nagel Point and the Orthocenter are perspective with perspector the Center of the Stevanovic Circle.

Triangle ABC and the Side Triangle of the Mittenpunkt and the Orthocenter are perspective with perspector the Center of the Stevanovic Circle.

Triangle ABC and the Side Triangle of the Mittenpunkt and the Nagel Point are perspective with perspector the Center of the Stevanovic Circle.

The Incentral Triangle and the Side Triangle of the Centroid and the Orthocenter are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Centroid and the Spieker Center are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Centroid and the Outer Fermat Point are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Centroid and the Inner Fermat Point are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Centroid and the Outer Napoleon Point are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Centroid and the Inner Napoleon Point are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Orthocenter and the Spieker Center are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Orthocenter and the Outer Fermat Point are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Orthocenter and the Outer Napoleon Point are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Outer Fermat Point and the Spieker Center are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Outer Fermat Point and the Outer Napoleon Point are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Inner Fermat Point and the Orthocenter are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Inner Fermat Point and the Spieker Center are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Inner Fermat Point and the Outer Fermat Point are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Inner Fermat Point and the Outer Napoleon Point are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Inner Fermat Point and the Inner Napoleon Point are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Outer Napoleon Point and the Spieker Center are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Inner Napoleon Point and the Orthocenter are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Inner Napoleon Point and the Spieker Center are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Inner Napoleon Point and the Outer Fermat Point are perspective with perspector the First Feuerbach Point.

The Incentral Triangle and the Side Triangle of the Inner Napoleon Point and the Outer Napoleon Point are perspective with perspector the First Feuerbach Point.

The Medial Triangle and the Side Triangle of the Incenter and the Orthocenter are perspective with perspector the First Feuerbach Point.

The Medial Triangle and the Side Triangle of the Incenter and the Nagel Point are perspective with perspector the First Feuerbach Point.

The Medial Triangle and the Side Triangle of the Incenter and the Mittenpunkt are perspective with perspector the First Feuerbach Point.

The Medial Triangle and the Side Triangle of the Centroid and the Orthocenter are perspective with perspector the Kiepert Center.

The Medial Triangle and the Side Triangle of the Centroid and the Spieker Center are perspective with perspector the Kiepert Center.

The Medial Triangle and the Side Triangle of the Centroid and the Outer Fermat Point are perspective with perspector the Kiepert Center.

The Medial Triangle and the Side Triangle of the Centroid and the Inner Fermat Point are

perspective with perspector the Kiepert Center.

The Medial Triangle and the Side Triangle of the Centroid and the Outer Napoleon Point are perspective with perspector the Kiepert Center.

The Medial Triangle and the Side Triangle of the Centroid and the Inner Napoleon Point are perspective with perspector the Kiepert Center.

The Medial Triangle and the Side Triangle of the Orthocenter and the Spieker Center are perspective with perspector the Kiepert Center.

The Medial Triangle and the Side Triangle of the Orthocenter and the Outer Fermat Point are perspective with perspector the Kiepert Center.

The Medial Triangle and the Side Triangle of the Orthocenter and the Outer Napoleon Point are perspective with perspector the Kiepert Center.

The Medial Triangle and the Side Triangle of the Gergonne Point and the Incenter are perspective with perspector the First Feuerbach Point.

The Medial Triangle and the Side Triangle of the Gergonne Point and the Orthocenter are perspective with perspector the First Feuerbach Point.

The Medial Triangle and the Side Triangle of the Gergonne Point and the Nagel Point are perspective with perspector the First Feuerbach Point.

The Medial Triangle and the Side Triangle of the Gergonne Point and the Mittenpunkt are perspective with perspector the First Feuerbach Point.

The Medial Triangle and the Side Triangle of the Nagel Point and the Orthocenter are perspective with perspector the First Feuerbach Point.

The Medial Triangle and the Side Triangle of the Mittenpunkt and the Orthocenter are perspective with perspector the First Feuerbach Point.

The Medial Triangle and the Side Triangle of the Mittenpunkt and the Nagel Point are perspective with perspector the First Feuerbach Point.

The Medial Triangle and the Side Triangle of the Outer Fermat Point and the Spieker Center are perspective with perspector the Kiepert Center.

The Medial Triangle and the Side Triangle of the Outer Fermat Point and the Outer Napoleon Point are perspective with perspector the Kiepert Center.

The Medial Triangle and the Side Triangle of the Inner Fermat Point and the Orthocenter are perspective with perspector the Kiepert Center.

The Medial Triangle and the Side Triangle of the Inner Fermat Point and the Spieker Center are perspective with perspector the Kiepert Center.

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The Medial Triangle and the Side Triangle of the Inner Napoleon Point and the Orthocenter are perspective with perspector the Kiepert Center.

The Medial Triangle and the Side Triangle of the Inner Napoleon Point and the Spieker Center are perspective with perspector the Kiepert Center.

The Medial Triangle and the Side Triangle of the Inner Napoleon Point and the Outer Fermat Point are perspective with perspector the Kiepert Center.

The Medial Triangle and the Side Triangle of the Inner Napoleon Point and the Outer Napoleon Point are perspective with perspector the Kiepert Center.

The Orthic Triangle and the Side Triangle of the Centroid and the Incenter are perspective with perspector the First Feuerbach Point.

The Orthic Triangle and the Side Triangle of the Centroid and the Symmedian Point are perspective with perspector the Kiepert Center.

The Symmedian Triangle and the Side Triangle of the Nine-Point Center and the Orthocenter are perspective with perspector the Kiepert Center.

The Intouch Triangle and the Side Triangle of the Centroid and the Gergonne Point are perspective with perspector the First Feuerbach Point.

The Extouch Triangle and the Side Triangle of the Centroid and the Nagel Point are perspective with perspector the First Feuerbach Point.

The Feuerbach Triangle and the Side Triangle of the Centroid and the Orthocenter are perspective with perspector the First Feuerbach Point.

The Feuerbach Triangle and the Side Triangle of the Centroid and the Spieker Center are perspective with perspector the First Feuerbach Point.

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The Feuerbach Triangle and the Side Triangle of the Centroid and the Inner Fermat Point

are perspective with perspector the First Feuerbach Point.

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The Feuerbach Triangle and the Side Triangle of the Inner Napoleon Point and the Outer Napoleon Point are perspective with perspector the First Feuerbach Point.

Invitation

The reader is invited to submit a note/paper containing

- synthetic proofs of theorems from this paper,
- or, applications of theorems from this paper,
- or, additional references related to this paper.

Definitions and Conventions

We use the definitions and conventions in accordance with [1 - 6] and papers published in this journal.

The Level

The Machine for Questions and Answers is used to produce results in this paper. Currently the Machine has 6 levels of depths - 0,1,2,3,4,5. We use for this paper the level 0, that is, the Machine produces only elementary results. If we need deeper investigation, we have to use a level bigger than 0. Since the Machine for Questions and Answers produces too many results, it is suitable we to use bigger levels upon request, that is, for specific questions.

Thanks

The figures in this note are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download in the Web: [Rene Grothmann's C.a.R.](#). It is free and open source. The reader may verify easily the statements of this paper by using C.a.R. Many thanks to Rene Grothmann for his wonderful program.

Note added on 18 December 2007

The paper

Cosmin Pohoata and Paul Yiu, On a Product of Two Points Induced by Their Cevian Triangles, Forum Geometricorum, volume 7, 2007, pp.169-180, <http://forumgeom.fau.edu/>

contains important results on side triangles.

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Publication Date: 7 December 2007

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