

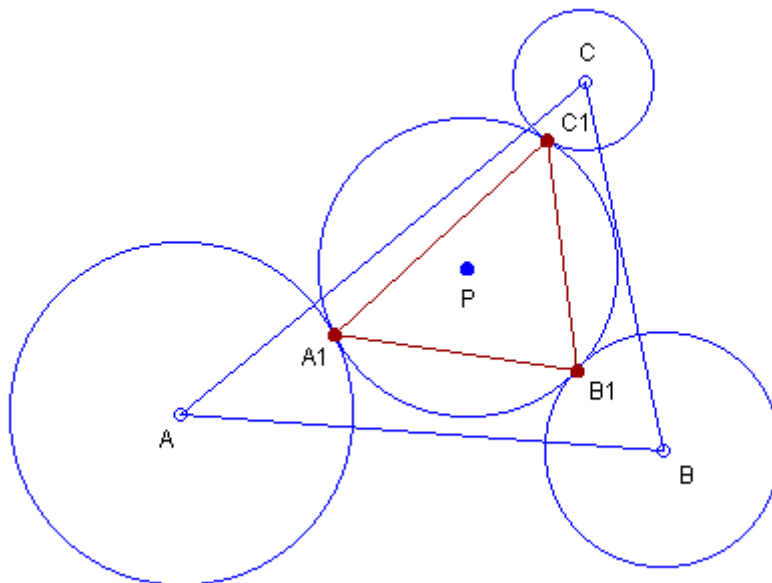
## Apollonius Triangles

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**Abstract.** By using the computer program "Machine for Questions and Answers", we find perspectives of Apollonius Triangles.

Given three circles (A), (B) and (C) with noncollinear centers A, B and C, respectively. Let (P) be the Inner Apollonius Circle of circles (A), (B) and (C), that is, the circle tangent externally to these circles. Let  $A_1$  be the tangent point of circles of (P) and (A). Similarly define  $B_1$  and  $C_1$ . The triangle  $A_1B_1C_1$  is called the *Inner Apollonius Triangle of circles (A), (B), (C)*.

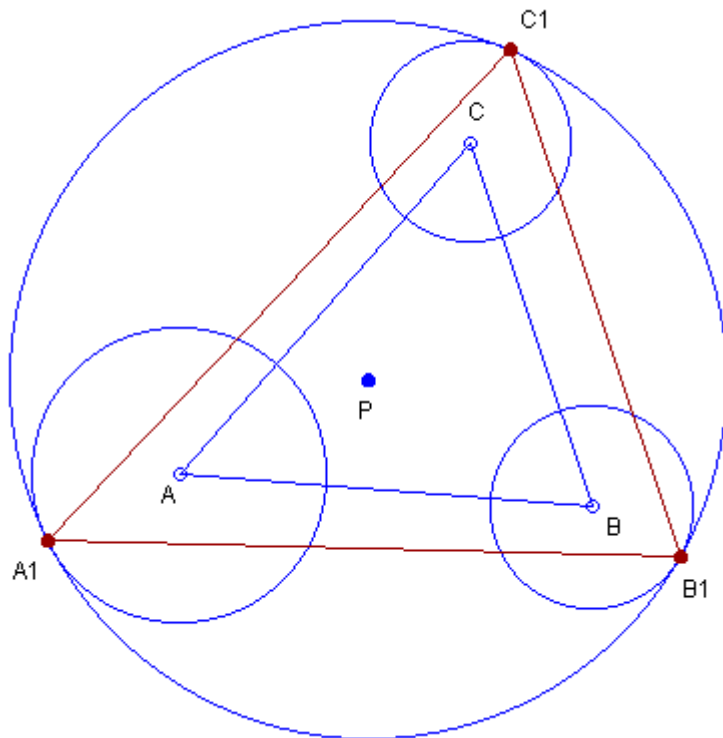
See the Figure:



- (A), (B), (C) - circles;
- (P) - Inner Apollonius circle of circles (A), (B), (C);
- $A_1$  - tangent point of circles (P) and (A);
- $B_1$  - tangent point of circles (P) and (B);
- $C_1$  - tangent point of circles (P) and (C);
- $A_1B_1C_1$  - Inner Apollonius Triangle of circles (A), (B), (C).

Similarly, define the *Outer Apollonius Triangle of circles (A), (B), (C)*.

See the Figure:



(A), (B), (C) - circles;

(P) - Outer Apollonius circle of circles (A), (B), (C);

$A_1$  - tangent point of circles (P) and (A);

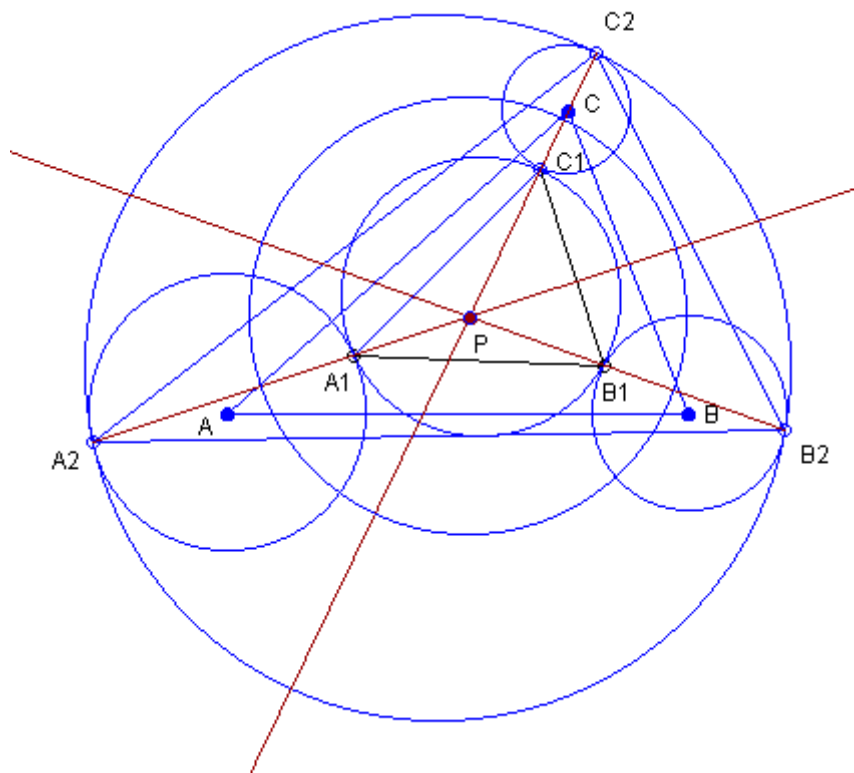
$B_1$  - tangent point of circles (P) and (B);

$C_1$  - tangent point of circles (P) and (C);

$A_1B_1C_1$  - Outer Apollonius Triangle of circles (A), (B), (C).

The Inner and the Outer Apollonius triangles of given three circles are perspective. The perspector is the Radical Center of the given three circles.

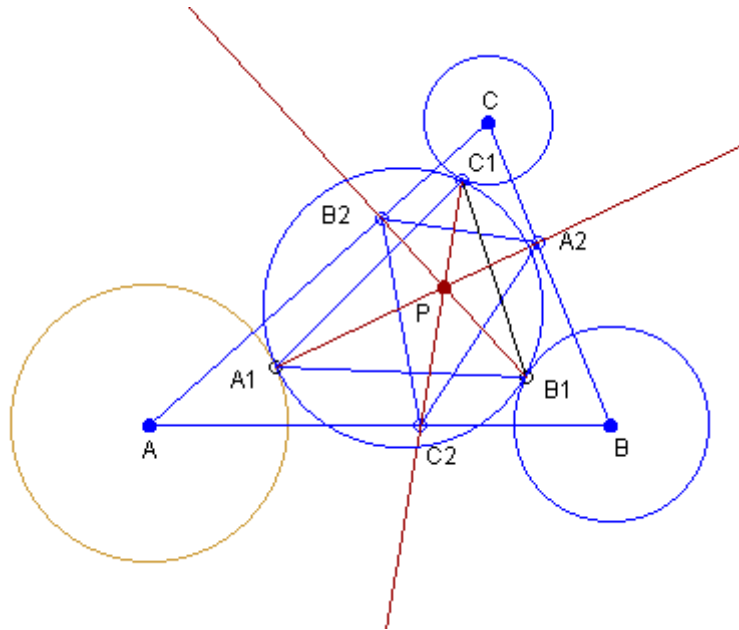
See the Figure:



- (A), (B), (C) - circles;
- (P) - Radical circle of circles (A), (B), (C);
- $A_1B_1C_1$  - Inner Apollonius Triangle of circles (A), (B), (C);
- $A_2B_2C_2$  - Outer Apollonius Triangle of circles (A), (B), (C);
- P - Radical Center of circles (A), (B), (C) = perspector of triangles  $A_1B_1C_1$  and  $A_2B_2C_2$ .

The Inner Apollonius triangle and the Inner Johnson triangle of given three circles are perspective.

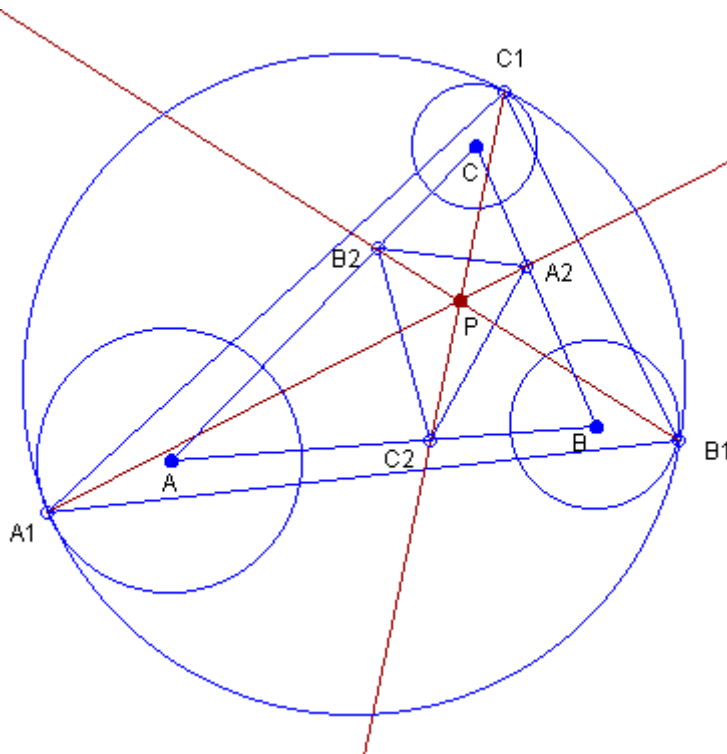
See the Figure:



(A), (B), (C) - circles;  
 $A_1B_1C_1$  - Inner Apollonius Triangle of circles (A), (B), (C);  
 $A_2B_2C_2$  - Inner Johnson Triangle of circles (A), (B), (C);  
 P - perspector of triangles  $A_1B_1C_1$  and  $A_2B_2C_2$ .

Similarly, The Outer Apollonius triangle and the Inner Johnson triangle of given three circles are perspective.

See the Figure:



(A), (B), (C) - circles;

$A_1B_1C_1$  - Outer Apollonius Triangle of circles (A), (B), (C);  
 $A_2B_2C_2$  - Inner Johnson Triangle of circles (A), (B), (C);  
P - perspector of triangles  $A_1B_1C_1$  and  $A_2B_2C_2$ .

The Inner Apollonius circle of the Excircles is the Nine-Point Circle, and the Inner Apollonius triangle of the Excircles is the Feuerbach triangle. The Outer Apollonius circle of the Excircles is the Apollonius Circle.

### Examples

The Machine for Questions and Answers gives perspectives between triangles. Examples of perspectives between triangles and Apollonius triangles are given below.

Triangle ABC and the Inner Apollonius Triangle of the Malfatti Circles are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Malfatti Circles are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Lucas Circles are perspective.

Triangle ABC and the Feuerbach Triangle of the Yff Central Triangle are perspective.

Triangle ABC and the Apollonius Triangle of the Yff Central Triangle are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Soddy Circles of the Malfatti Central Triangle are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Soddy Circles of the Malfatti Central Triangle are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Soddy Circles of the Lucas Central Triangle are perspective.

The Symmedial Triangle and the Inner Apollonius Triangle of the Lucas Circles are perspective.

The Symmedial Triangle and the Inner Apollonius Triangle of the Soddy Circles of the Lucas Central Triangle are perspective.

The Intouch Triangle and the Inner Apollonius Triangle of the Soddy Circles are perspective.

The Intouch Triangle and the Outer Apollonius Triangle of the Soddy Circles are perspective.

The Excentral Triangle and the Inner Apollonius Triangle of the Neuberg Circles are perspective.

The Excentral Triangle and the Outer Apollonius Triangle of the Neuberg Circles are perspective.

The Excentral Triangle and the Inner Apollonius Triangle of the Reflected Neuberg Circles are perspective.

The Excentral Triangle and the Outer Apollonius Triangle of the Reflected Neuberg Circles are perspective.

The Excentral Triangle and the Feuerbach Triangle of the Johnson Triangle are perspective.

The Excentral Triangle and the Inner Apollonius Triangle of the Neuberg Circles of the First Brocard Triangle are perspective.

The Excentral Triangle and the Outer Apollonius Triangle of the Neuberg Circles of the First Brocard Triangle are perspective.

The Excentral Triangle and the Inner Apollonius Triangle of the Neuberg Circles of the Inner Gallatly-Kiepert Triangle are perspective.

The Excentral Triangle and the Outer Apollonius Triangle of the Neuberg Circles of the Inner Gallatly-Kiepert Triangle are perspective.

The Tangential Triangle and the Inner Apollonius Triangle of the Lucas Circles are perspective.

The Tangential Triangle and the Feuerbach Triangle of the Excentral Triangle are perspective.

The Tangential Triangle and the Feuerbach Triangle of the Anticomplementary Triangle are perspective.

The Circum-Incentral Triangle and the Outer Apollonius Triangle of the Incenter-Excenter Circles are perspective.

The Feuerbach Triangle and the Feuerbach Triangle of the Medial Triangle are homothetic.

The Feuerbach Triangle and the Feuerbach Triangle of the Anticomplementary Triangle are homothetic.

The Feuerbach Triangle and the Feuerbach Triangle of the Pedal Triangle of the Circumcenter are homothetic.

The Feuerbach Triangle and the Feuerbach Triangle of the Circumcevian Triangle of the Circumcenter are homothetic.

The Feuerbach Triangle and the Feuerbach Triangle of the Euler Triangle are homothetic.

The Feuerbach Triangle and the Feuerbach Triangle of the Johnson Triangle are homothetic.

The Feuerbach Triangle and the Feuerbach Triangle of the Inner Johnson-Yff Triangle are homothetic.

The Feuerbach Triangle and the Feuerbach Triangle of the Outer Johnson-Yff Triangle are homothetic.

The Yff Central Triangle and the Outer Apollonius Triangle of the Soddy Circles of the Antipedal Triangle of the Center of the Inner Soddy Circle are homothetic.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the First Droz-Farny Circles of the Triangulation Triangles of the Orthocenter are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the First Droz-Farny Circles of the Triangulation Triangles of the Orthocenter are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Circumcircles of the Triangulation Triangles of the First Isodynamic Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Radical Circles of the Lucas Circles of the Triangulation Triangles of the First Isodynamic Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Radical Circles of the Lucas Circles of the Triangulation Triangles of the First Isodynamic Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Incenter are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Incenter are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Nine-Point Center are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Nine-Point Center are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Symmedian Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Symmedian Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Gergonne Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner

Triangles of the Gergonne Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Nagel Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Nagel Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Mittenpunkt are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Mittenpunkt are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Spieker Center are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Spieker Center are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the First Feuerbach Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the First Feuerbach Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Second Feuerbach Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Second Feuerbach Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Outer Fermat Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Outer Fermat Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the First Isodynamic Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the First Isodynamic Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Outer Napoleon Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Outer Napoleon Point are perspective.



Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Schiffler Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Schiffler Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Gibert Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Gibert Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Second Power Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Second Power Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Third Power Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Third Power Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Moses Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Moses Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Grinberg Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Grinberg Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Brocard Midpoint are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Brocard Midpoint are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Internal Center of Similitude of the Incircle and the Circumcircle are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Internal Center of Similitude of the Incircle and the Circumcircle are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the External Center of Similitude of the Incircle and the Circumcircle are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the External Center of Similitude of the Incircle and the Circumcircle are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Prasolov Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Prasolov Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Kiepert Center are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Kiepert Center are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Congruent Isoscelizers Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Congruent Isoscelizers Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Yff Center of Conguence are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Yff Center of Conguence are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Center of the Inner Soddy Circle are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Center of the Inner Soddy Circle are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the First Ajima-Malfatti Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the First Ajima-Malfatti Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Apollonius Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Apollonius Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Center of the Brocard Circle are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Center of the Brocard Circle are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Equal Parallelians Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Equal Parallelians Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Weill Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Weill Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Inner Kenmotu Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Inner Kenmotu Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Center of the Taylor Circle are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Center of the Taylor Circle are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Outer Eppstein Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Outer Eppstein Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Inner Eppstein Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Inner Eppstein Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Radical Center of the Malfatti Circles are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Radical Center of the Malfatti Circles are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner

Triangles of the Outer Vecten Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Outer Vecten Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Inner Vecten Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Inner Vecten Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Johnson Midpoint are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Johnson Midpoint are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Miquel Point of the Incenter are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Miquel Point of the Incenter are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Fourth Power Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Fourth Power Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the First de Villiers Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the First de Villiers Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Second de Villiers Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Second de Villiers Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the First Malfatti-Rabinowitz Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the First Malfatti-Rabinowitz Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Second Malfatti-Rabinowitz Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Second Malfatti-Rabinowitz Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Radical Center of the Lucas Circles are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Radical Center of the Lucas Circles are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Center of the van Lamoen Circle are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Center of the van Lamoen Circle are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Center of the Inner Johnson-Yff Circle are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Center of the Inner Johnson-Yff Circle are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Center of the Outer Johnson-Yff Circle are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Center of the Outer Johnson-Yff Circle are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Danneels-Apollonius Prespector are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Danneels-Apollonius Prespector are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Malfatti-Moses Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Malfatti-Moses Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the First Jerabek Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the First Jerabek Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Second Jerabek Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner

Triangles of the Second Jerabek Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the First Brocard Point are perspective.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the First Brocard Point are perspective.

Triangle ABC and the Inner Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Second Brocard Point are perspective.

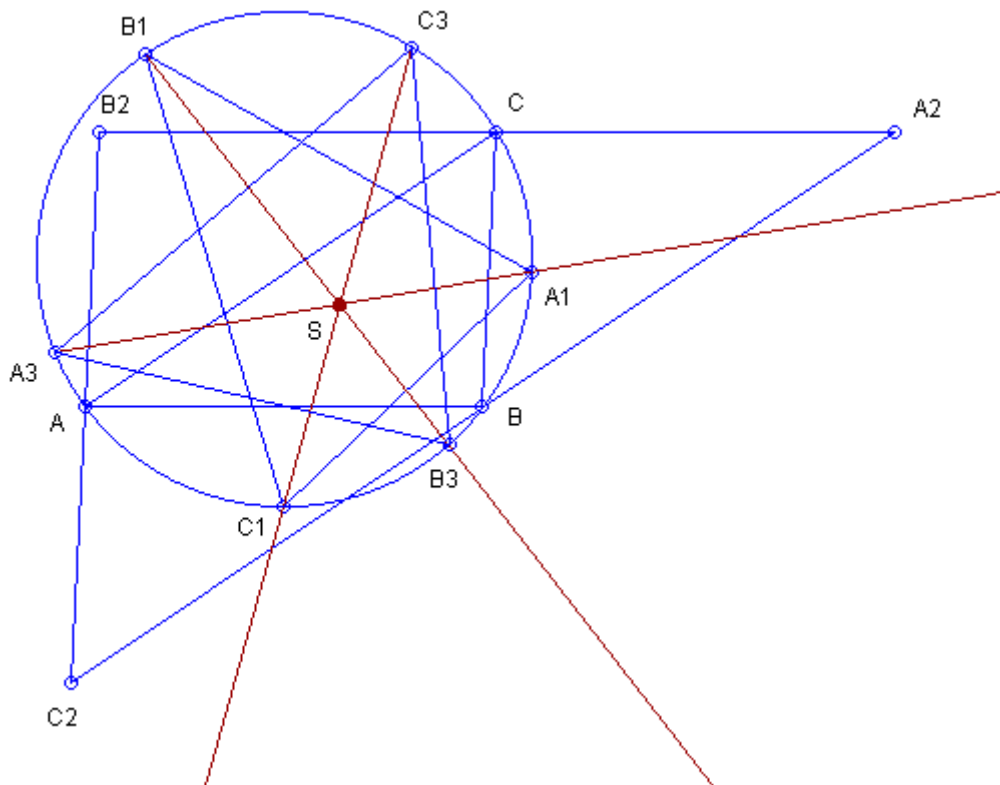
Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Corner Triangles of the Second Brocard Point are perspective.

### Perspectors

The Machine for Questions and Answers could identify perspectors upon request. A few examples are given below. In these examples the perspectors are between the basic points.

The Circum-Incentral Triangle and the Feuerbach Triangle of the Anticomplementary Triangle are perspective with perspector the Schiffler Point.

See the Figure:

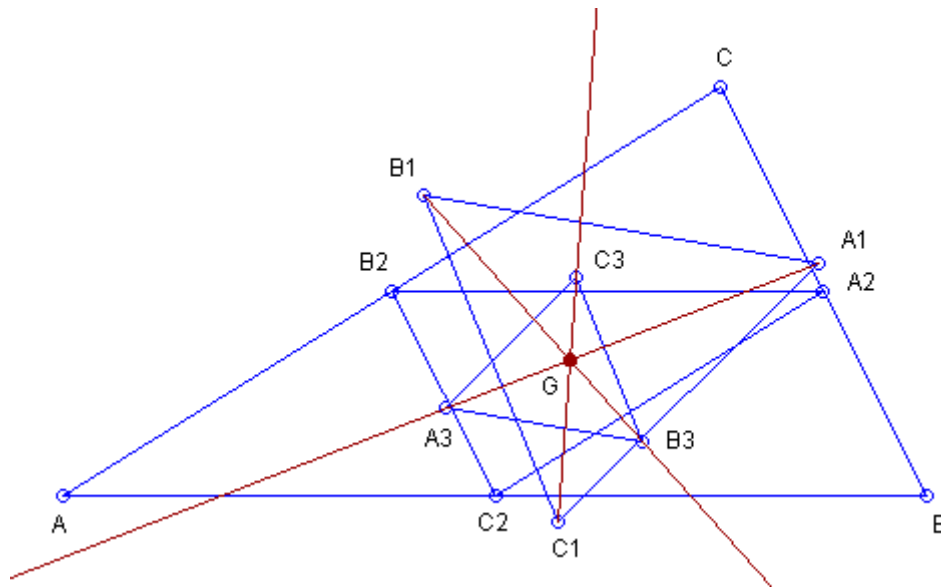


$A_1B_1C_1$  - Circum-Incentral Triangle;  
 $A_2B_2C_2$  - Anticomplementary Triangle;

$A_3B_3C_3$  - Feuerbach Triangle of the Anticomplementary Triangle;  
 $S$  - Schiffler Point = perspector of triangles  $A_1B_1C_1$  and  $A_3B_3C_3$ .

The Feuerbach Triangle and the Feuerbach Triangle of the Medial Triangle are homothetic with homothetic center the Centroid.

See the Figure:



The Feuerbach Triangle and the Feuerbach Triangle of the Anticomplementary Triangle are homothetic with homothetic center the Centroid.

The Feuerbach Triangle and the Feuerbach Triangle of the Circumcevian Triangle of the Circumcenter are homothetic with homothetic center the Circumcenter.

The Feuerbach Triangle and the Feuerbach Triangle of the Euler Triangle are homothetic with homothetic center the Orthocenter.

The Feuerbach Triangle and the Feuerbach Triangle of the Johnson Triangle are homothetic with homothetic center the Nine-Point Center.

The Feuerbach Triangle and the Feuerbach Triangle of the Inner Johnson-Yff Triangle are homothetic with homothetic center the Incenter.

The Feuerbach Triangle and the Feuerbach Triangle of the Outer Johnson-Yff Triangle are homothetic with homothetic center the Incenter.

The Apollonius Triangle and the Apollonius Triangle of the Medial Triangle are homothetic with homothetic center the Centroid.

The Apollonius Triangle and the Apollonius Triangle of the Anticomplementary Triangle are homothetic with homothetic center the Centroid.

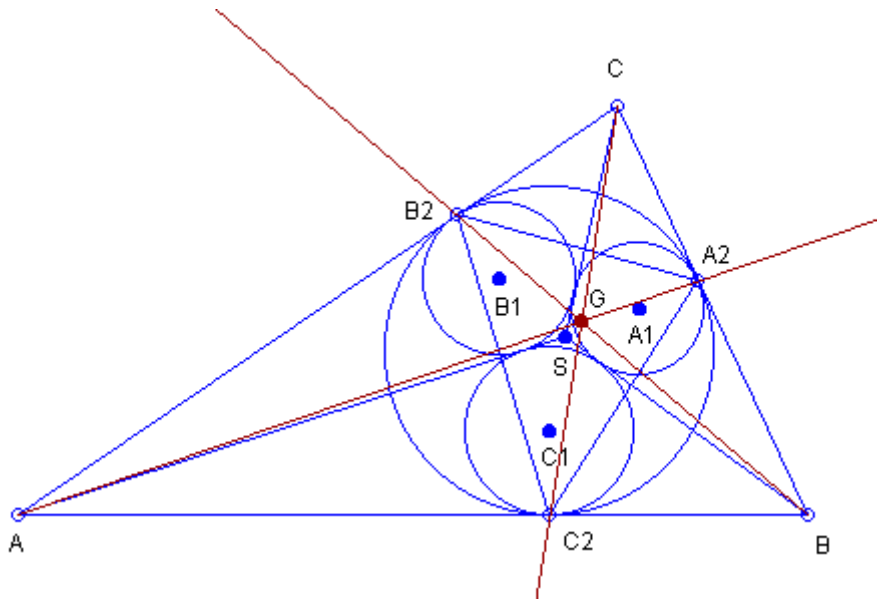
The Apollonius Triangle and the Apollonius Triangle of the Euler Triangle are homothetic

with homothetic center the Orthocenter.

The Apollonius Triangle and the Apollonius Triangle of the Johnson Triangle are homothetic with homothetic center the Nine-Point Center.

Triangle ABC and the Outer Apollonius Triangle of the Triad of the Incircles of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Gergonne Point.

See the Figure:



S - Center of the Inner Soddy Circle;

(A<sub>1</sub>) - Incircle of triangle BCS;

(B<sub>1</sub>) - Incircle of triangle CAS;

(C<sub>1</sub>) - Incircle of triangle ABS;

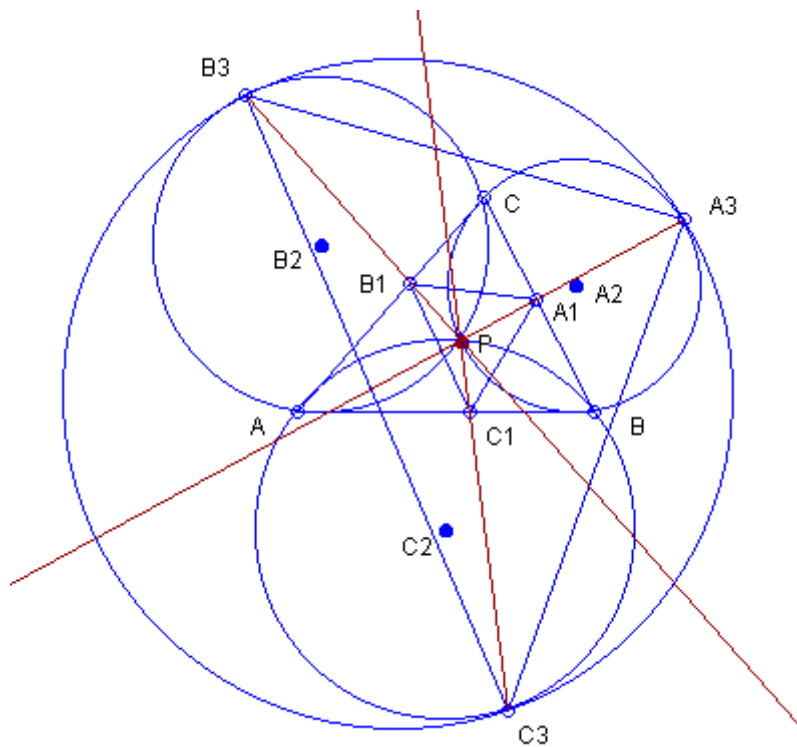
A<sub>2</sub>B<sub>2</sub>C<sub>2</sub> - Outer Apollonius Triangle of circles (A<sub>1</sub>), (B<sub>1</sub>), (C<sub>1</sub>) = Outer Apollonius Triangle of the Triad of the Incircles of the Triangulation Triangles of the Center of the Inner Soddy Circle;

G - Gergonne Point = perspector of triangles ABC and A<sub>2</sub>B<sub>2</sub>C<sub>2</sub>.

The Symmedial Triangle and the Outer Apollonius Triangle of the Triad of the Circumcircles of the Triangulation Triangles of the First Isodynamic Point are perspective with perspector the First Isodynamic Point.

See the Figure:





P - First Isodynamic Point;  
 $A_1B_1C_1$  - Symmedial Triangle;  
 $(A_2)$  - Circumcircle of triangle BCP;  
 $(B_2)$  - Circumcircle of triangle CAP;  
 $(C_2)$  - Circumcircle of triangle ABP;  
 $A_3B_3C_3$  - Outer Apollonius Triangle of the circles  $(A_2)$ ,  $(B_2)$ ,  $(C_2)$  = Outer Apollonius Triangle of the Triad of the Circumcircles of the Triangulation Triangles of the First Isodynamic Point ;  
 P - First Isodynamic Point = perspector of triangles  $A_1B_1C_1$  and  $A_3B_3C_3$ .

The Intouch Triangle and the Inner Apollonius Triangle of the Triad of the Incircles of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Center of the Inner Soddy Circle.

The Excentral Triangle and the Outer Apollonius Triangle of the Triad of the Incircles of the Triangulation Triangles of the Center of the Inner Soddy Circle are homothetic with homothetic center the Isogonal Conjugate of the Mittenpunkt.

The Anticomplementary Triangle and the Outer Apollonius Triangle of the Triad of the Incircles of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Nagel Point of the Anticomplementary Triangle.

The Tangential Triangle and the Outer Apollonius Triangle of the Triad of the Radical Circles of the Lucas Circles of the Triangulation Triangles of the First Isodynamic Point are perspective with perspector the Inner Kenmotu Point.

The Tangential Triangle and the Outer Apollonius Triangle of the Triad of the Incircles of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with

perspector the Perspector of the Intouch Triangle and the Tangential Triangle.

The Circum-Incentral Triangle and the Outer Apollonius Triangle of the Triad of the Incircles of the Triangulation Triangles of the Center of the Inner Soddy Circle are homothetic with homothetic center the External Center of Similitude of the Incircle and the Circumcircle.

The Half-Altitude Triangle and the Outer Apollonius Triangle of the Triad of the Incircles of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Isogonal Conjugate of the Mittenpunkt.

The Intangents Triangle and the Outer Apollonius Triangle of the Triad of the Incircles of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Incenter.

The Mid-Arc Triangle and the Outer Apollonius Triangle of the Triad of the Incircles of the Triangulation Triangles of the Center of the Inner Soddy Circle are perspective with perspector the Incenter of the Intouch Triangle.

The Yff Central Triangle and the Outer Apollonius Triangle of the Triad of the Incircles of the Triangulation Triangles of the Center of the Inner Soddy Circle are homothetic with homothetic center the Yff Center of Congruence.

The Lucas Central Triangle and the Outer Apollonius Triangle of the Triad of the Radical Circles of the Lucas Circles of the Triangulation Triangles of the First Isodynamic Point are perspective with perspector the Inner Kenmotu Point.

The Hexyl Triangle and the Outer Apollonius Triangle of the Triad of the Incircles of the Triangulation Triangles of the Center of the Inner Soddy Circle are homothetic with homothetic center the Incenter.

## **Invitation**

The reader is invited to submit a note/paper containing

- synthetic proofs of theorems from this paper,
- or, applications of theorems from this paper,
- or, additional references related to this paper.

## **Definitions and Conventions**

We use the definitions and conventions in accordance with [1 - 6] and papers published in this journal.

## **The Level**

The Machine for Questions and Answers is used to produce results in this paper. Currently the Machine has 6 levels of depths - 0,1,2,3,4,5. We use for this paper the level 0, that is, the Machine produces only elementary results. If we need deeper investigation, we have to use a level bigger than 0. Since the Machine for Questions and Answers produces too many

results, it is suitable we to use bigger levels upon request, that is, for specific questions.

## Thanks

The figures in this note are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download in the Web: [Rene Grothmann's C.a.R.](#). It is free and open source. The reader may verify easily the statements of this paper by using C.a.R. Many thanks to Rene Grothmann for his wonderful program.

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