Journal of Computer-Generated Euclidean Geometry

Equilateral Triangles

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Abstract. By using the computer program "Machine for Questions and Answers", we find equilateral triangles.

The Machine for Questions and Answers finds equilateral triangles (as well as isosceles, right triangles, etc.). A few examples of equlateral triangles are given below.

The Pedal Triangle of the First Isodynamic Point is equilateral.

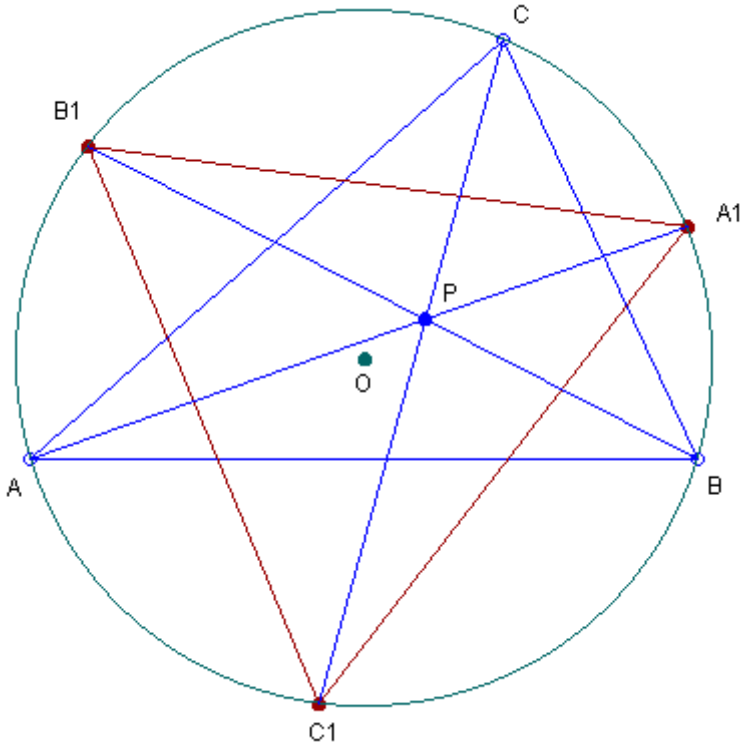
The Pedal Triangle of the Second Isodynamic Point is equilateral.

The Antipedal Triangle of the Outer Fermat Point is equilateral.

The Antipedal Triangle of the Inner Fermat Point is equilateral.

The Circumcevian Triangle of the First Isodynamic Point is equilateral.

See the Figure:



(O) - Circumcircle;
P - First Isodynamic Point;
$A_1B_1C_1$ - equilateral triangle - Circumcevian Triangle of the First Isodynamic Point.

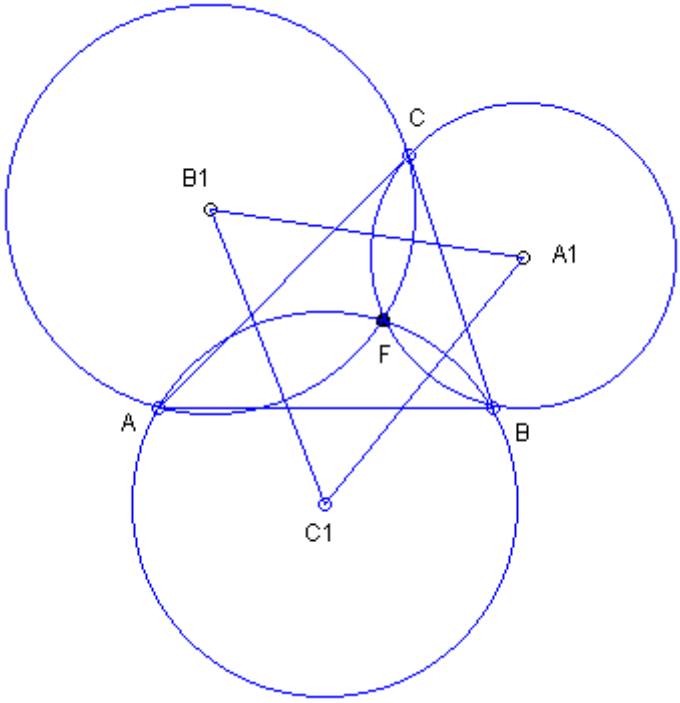
The Circumcevian Triangle of the Second Isodynamic Point is equilateral.

The Outer Napoleon Triangle is equilateral.

The Inner Napoleon Triangle is equilateral.

The Triangle of the Circumcenters of the Triangulation Triangles of the Outer Fermat Point is equilateral.

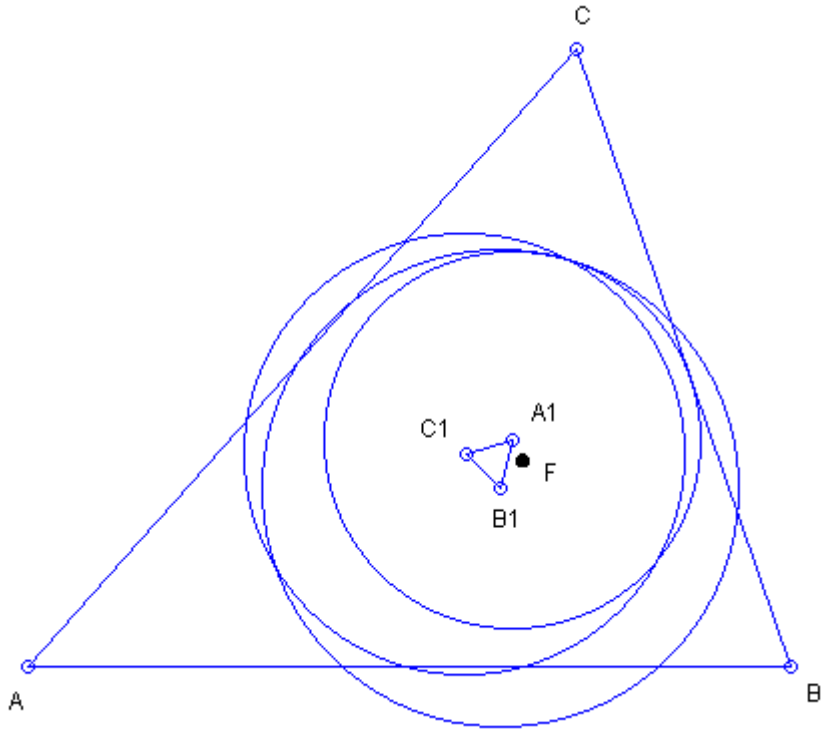
See the Figure:



F - Outer Fermat Point;
(A_1) - Circumcircle of triangle BCF;
(B_1) - Circumcircle of triangle CAF;
(C_1) - Circumcircle of triangle ABF;
$A_1B_1C_1$ - equilateral triangle - Triangle of the Circumcenters of the Triangulation Triangles of the Outer Fermat Point.

The Triangle of the Nine-Point Centers of the Triangulation Triangles of the Outer Fermat Point is equilateral.

See the Figure:



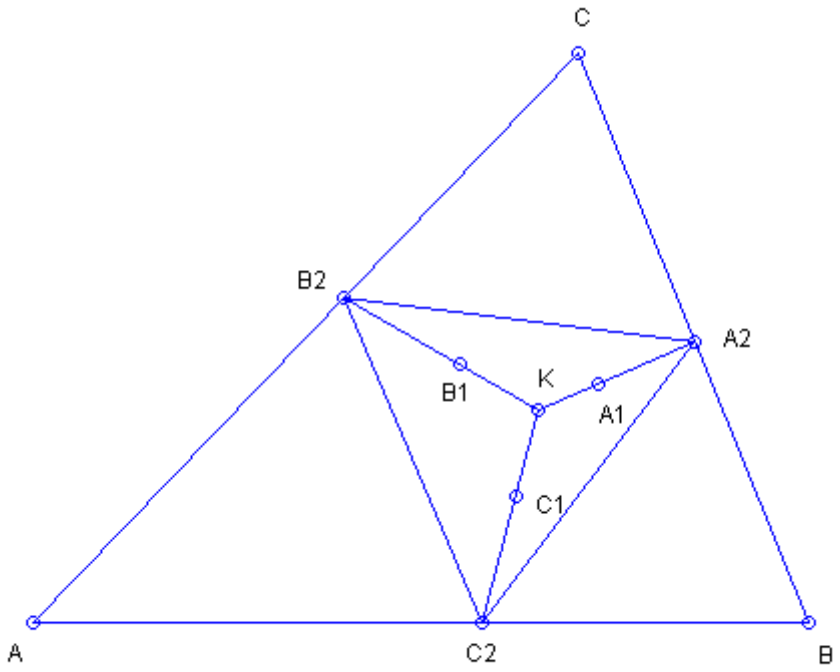
F - Outer Fermat Point;
(A_1) - Nine-Point Circle of triangle BCF;
(B_1) - Nine-Point Circle of triangle CAF;
(C_1) - Nine-Point Circle of triangle ABF;
$A_1B_1C_1$ - equilateral triangle - Triangle of the Nine-Point Centers of the Triangulation Triangles of the Outer Fermat Point.

The Triangle of the Circumcenters of the Triangulation Triangles of the Inner Fermat Point is equilateral.

The Triangle of the Nine-Point Centers of the Triangulation Triangles of the Inner Fermat Point is equilateral.

The Stevanovic Triangle of the Outer Napoleon Points of the Triangulation triangles of the Symmedian Point is equilateral.

See the Figure:



K - Symmedian Point;
A_1 - Outer Napoleon Point of triangle BCK;
B_1 - Outer Napoleon Point of triangle CAK;
C_1 - Outer Napoleon Point of triangle ABK;
$A_2B_2C_2$ - equilateral triangle - Stevanovic Triangle of the Outer Napoleon Points of the Triangulation triangles of the Symmedian Point.

The Stevanovic Triangle of the Inner Napoleon Points of the Triangulation triangles of the Symmedian Point is equilateral.

The Stevanovic Triangle of the Orthocenters of the Triangulation triangles of the First Isodynamic Point is equilateral.

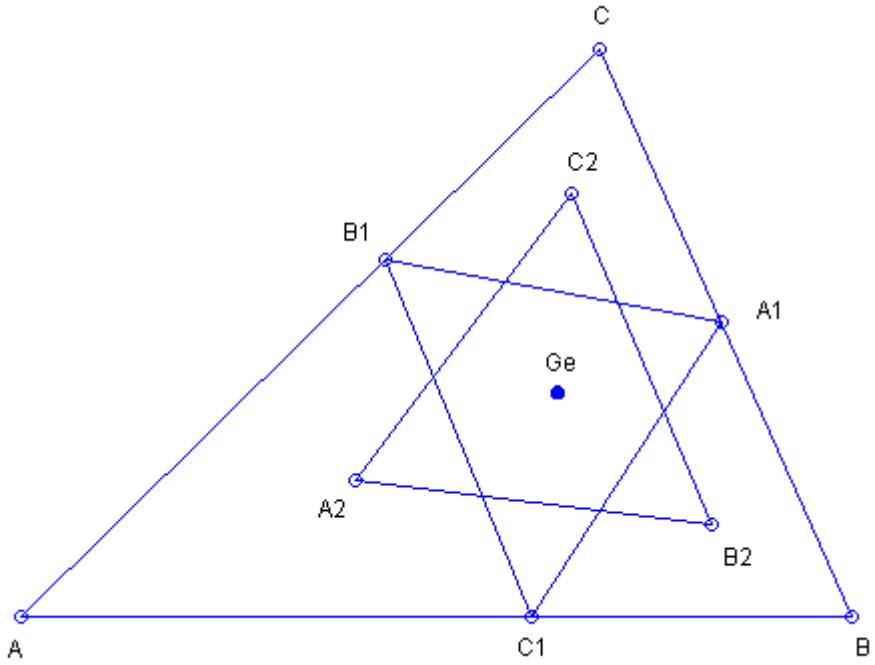
The Stevanovic Triangle of the Orthocenters of the Triangulation triangles of the Second Isodynamic Point is equilateral.

The Triangle of the First Isodynamic Points of the Corner Triangles of the Orthocenter is equilateral.

The Triangle of the Second Isodynamic Points of the Corner Triangles of the Orthocenter is equilateral.

The Triangle of the Outer Fermat Points of the Corner Triangles of the Gergonne Point is equilateral.

See the Figure:

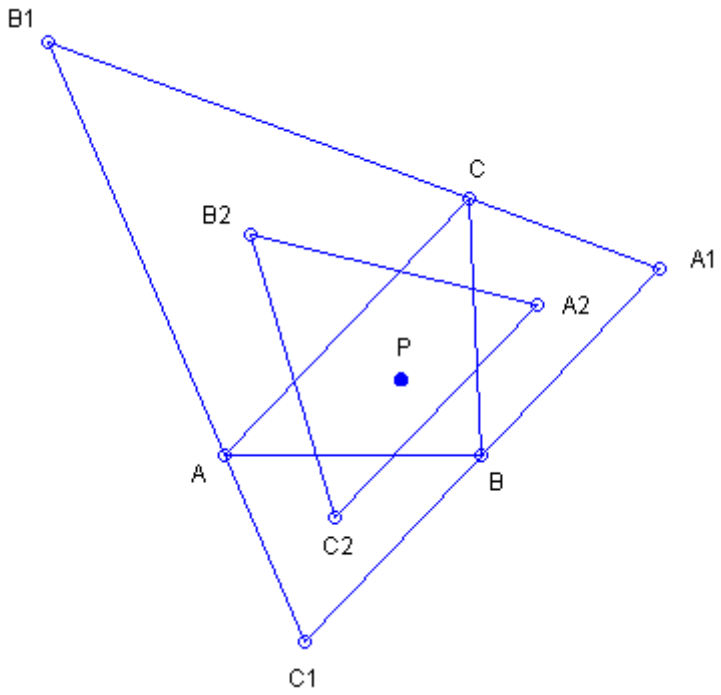


Ge - Gergonne Point;
$A_1B_1C_1$ - Cevian Triangle of the Gergonne Point = Intouch Triangle;
A_2 - Outer Fermat Point of triangle AB_1C_1;
B_2 - Outer Fermat Point of triangle BC_1A_1;
C_2 - Outer Fermat Point of triangle CA_1B_1;
$A_2B_2C_2$ - equilateral triangle - Triangle of the Outer Fermat Points of the Corner Triangles of the Gergonne Point.

The Triangle of the Inner Fermat Points of the Corner Triangles of the Gergonne Point is equilateral.

The Triangle of the First Isodynamic Points of the Anticevian Corner Triangles of the Incenter is equilateral.

See the Figure:



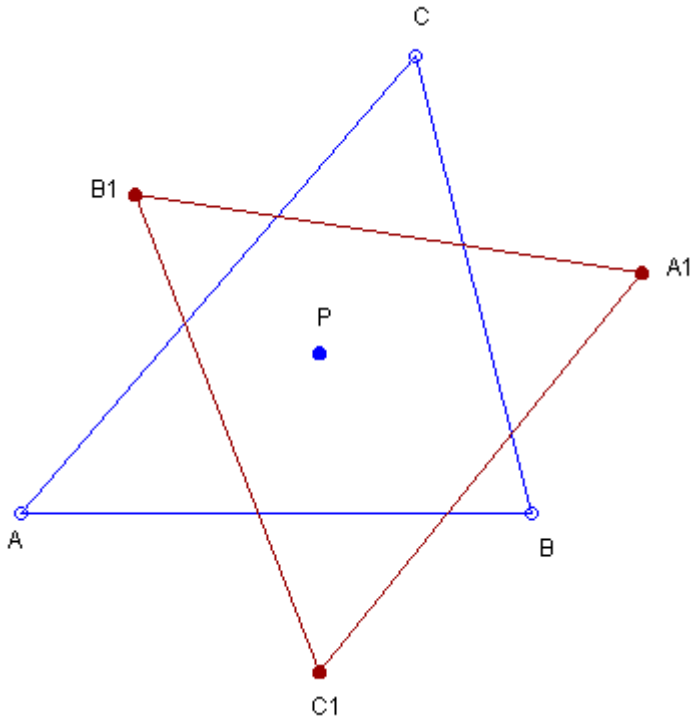
P - Incenter;
$A_1B_1C_1$ - Anticevian Triangle of the Incenter = Excentral Triangle;
A_2 - First Isodynamic Point of triangle A_1BC;
B_2 - First Isodynamic Point of triangle B_1CA;
C_2 - First Isodynamic Point of triangle C_1AB;
$A_2B_2C_2$ - equilateral triangle - Triangle of the First Isodynamic Points of the Anticevian Corner Triangles of the Incenter.

The Triangle of the Second Isodynamic Points of the Anticevian Corner Triangles of the Incenter is equilateral.

The Triangle of the Inner Fermat Points of the Anticevian Corner Triangles of the Second Isodynamic Point is equilateral.

The Triangle of the reflections of the First Isodynamic Point in the sides of Triangle ABC is equilateral.

See the Figure:



P - First Isodynamic Point;
A_1 - reflection of P in line BC;
B_1 - reflection of P in line CA;
C_1 - reflection of P in line AB;
$A_1B_1C_1$ - equilateral triangle - Triangle of the reflections of the First Isodynamic Point in the sides of Triangle ABC.

The Triangle of the reflections of the Second Isodynamic Point in the sides of Triangle ABC is equilateral.

Invitation

The reader is invited to submit a note/paper containing

- synthetic proofs of theorems from this paper,
- or, applications of theorems from this paper,
- or, additional references related to this paper.

Definitions

We use the definitions in accordance with [1 - 5] and papers published in this journal.

The Level

The Machine for Questions and Answers is used to produce results in this paper. Currently the Machine has 6 levels of depths - 0,1,2,3,4,5. We use for this paper the level 0, that is, the Machine produces only elementary results. If we need deeper investigation, we have to use a level bigger than 0. Since the Machine for Questions and Answers produces too many

results, it is suitable we to use bigger levels upon request, that is, for specific questions.

Thanks

The figures in this note are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download in the Web: Rene Grothmann's C.a.R.. It is free and open source. The reader may verify easily the statements of this paper by using C.a.R. Many thanks to Rene Grothmann for his wonderful program.

References

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Publication Date: 24 December 2007

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