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Construction of the Outer Gallatly-Kiepert Triangle

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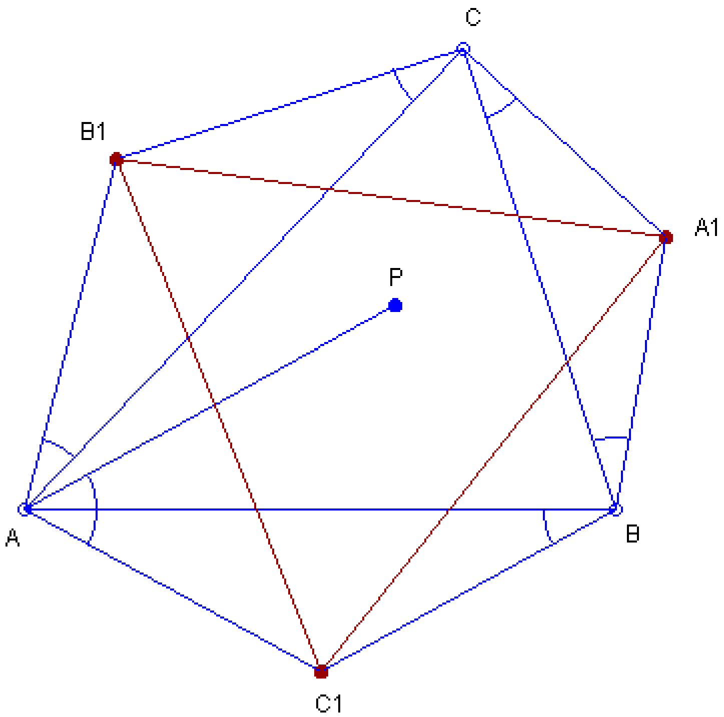
Abstract. By using the computer program "Machine for Questions and Answers", we find 28 different ways to construct the Outer Gallatly-Kiepert Triangle.

The Outer Gallatly-Kiepert Triangle is defined and studied in [3].

We can construct the Outer Gallatly-Kiepert Triangle by using the definition. We construct the Brocard angle *omega* of the given triangle ABC and then we construct isosceles triangles with base angle *omega* on the outside of the given triangle ABC. The vertices of the constructed isosceles triangles form the Outer Gallatly-Kiepert Triangle. We could construct the Brocard angle *omega* e.g. as follow. We could construct the First Brocard Point P, then angle *omega* = angle BAP.

See the Figure:



P - First Brocard Point;
angle *omega* = angle BAP
= angle BCA_1 = angle CBA_1 = angle CAB_1 = angle ACB_1 = angle ABC_1 = angle BAC_1;

$A_1B_1C_1$ - Outer Gallatly-Kiepert Triangle.

We use the Machine for Questions and Answers to find 28 additional ways how to construct the Outer Gallatly-Kiepert Triangle. In these ways we do not need to construct the Brocard angle *omega*.

We use the following method. We can construct a triangle, if we can construct

- A triangle perspective to the triangle, and the perspector,
- and, a second triangle perspective to the triangle, and the second perspector.

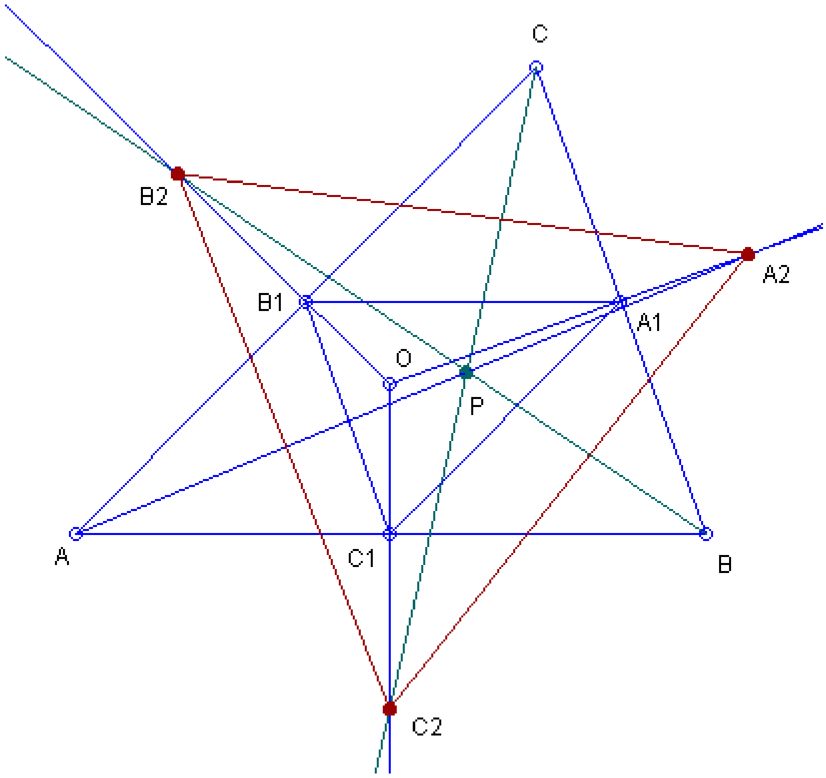
We use the Machine for Questions and Answers to specify a few theorems given in the paper [3]. We obtain the following theorems (the list below could be extended by the reader by adding additional similar theorems):

1. The Outer Gallatly-Kiepert Triangle and Triangle ABC are perspective with perspector the Isogonal Conjugate of the Brocard Midpoint.
2. The Outer Gallatly-Kiepert Triangle and the Medial Triangle are perspective with perspector the Circumcenter.
3. The Outer Gallatly-Kiepert Triangle and the Pedal Triangle of the Center of the Brocard Circle are homothetic with homothetic center the Symmedian Point.
4. The Outer Gallatly-Kiepert Triangle and the Anticevian Triangle of the Center of the Brocard Circle are perspective with perspector the Third Power Point.
5. The Outer Gallatly-Kiepert Triangle and the Circumcevian Triangle of the Symmedian Point are perspective with perspector the Steiner Point.
6. The Outer Gallatly-Kiepert Triangle and the Anticomplementary Triangle are perspective with perspector the Perspector of the Symmedial Triangle and the Anticomplementary Triangle.
7. The Outer Gallatly-Kiepert Triangle and the Pedal Triangle of the Center of the Brocard Circle of the Anticomplementary Triangle are homothetic with homothetic center the Symmedian Point of the Medial Triangle.
8. The Outer Gallatly-Kiepert Triangle and the Desmic Mate the Reflected Neuberg Triangle are perspective with perspector the Orthocenter.

We use the above theorems to obtain 28 ways how to construct the Outer Gallatly-Kiepert Triangle: The above eight perspectives give 28 ways (Clearly, if the reader extends the list of the perspectives, he will obtain additional ways.)

Solution 1

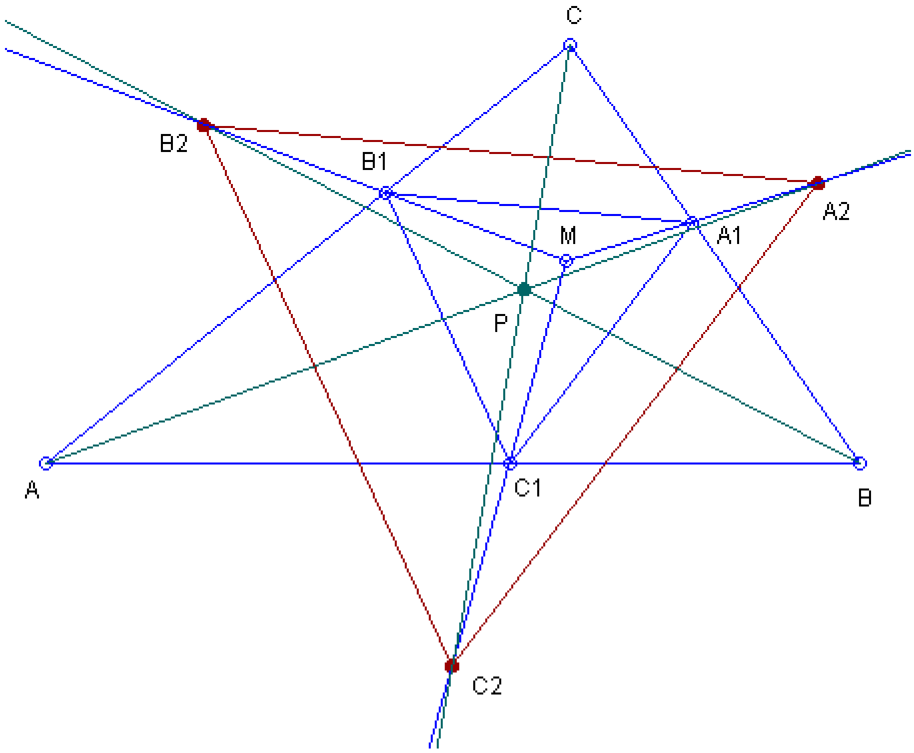
We use Theorems 1 and 2. See the Figure:



P - Gallatly-Kipepert Point = Isogonal Conjugate of the Brocard Midpoint;
O - Circumcenter;
$A_1B_1C_1$ - Medial Triangle;
A_2 - intersection point of lines PA and OA_1;
B_2 - intersection point of lines PB and OB_1;
C_2 - intersection point of lines PC and OC_1;
$A_2B_2C_2$ - Outer Gallatly-Kiepert Triangle.

Solution 2

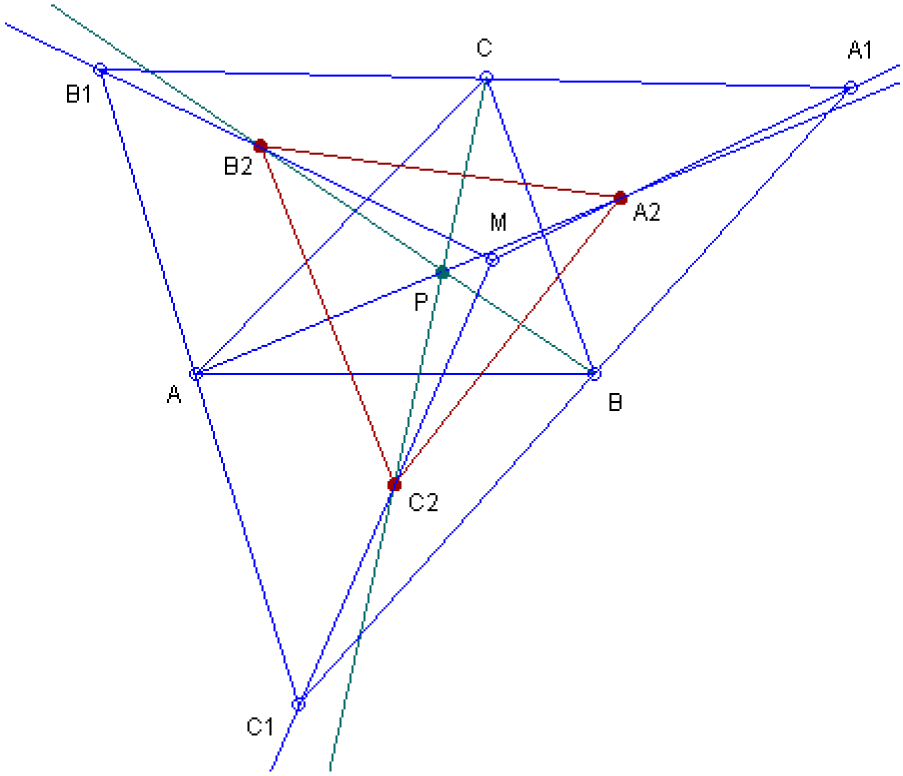
We use Theorems 1 and 3. See the Figure:



P - Gallatly-Kipepert Point = Isogonal Conjugate of the Brocard Midpoint;
M - Symmedian Point;
$A_1B_1C_1$ - Pedal Triangle of the Center of the Brocard Circle;
A_2 - intersection point of lines PA and MA_1;
B_2 - intersection point of lines PB and MB_1;
C_2 - intersection point of lines PC and MC_1;
$A_2B_2C_2$ - Outer Gallatly-Kiepert Triangle.

Solution 3

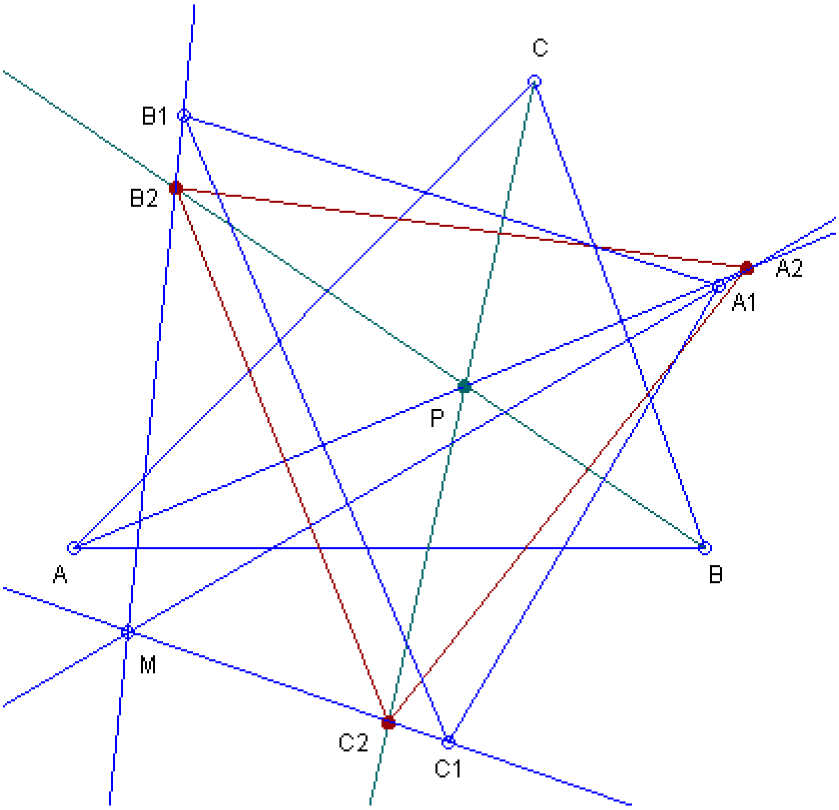
We use Theorems 1 and 4. See the Figure:



P - Gallatly-Kipepert Point = Isogonal Conjugate of the Brocard Midpoint;
M - Third Power Point;
$A_1B_1C_1$ - Anticevian Triangle of the Center of the Brocard Circle;
A_2 - intersection point of lines PA and MA_1;
B_2 - intersection point of lines PB and MB_1;
C_2 - intersection point of lines PC and MC_1;
$A_2B_2C_2$ - Outer Gallatly-Kiepert Triangle.

Solution 4

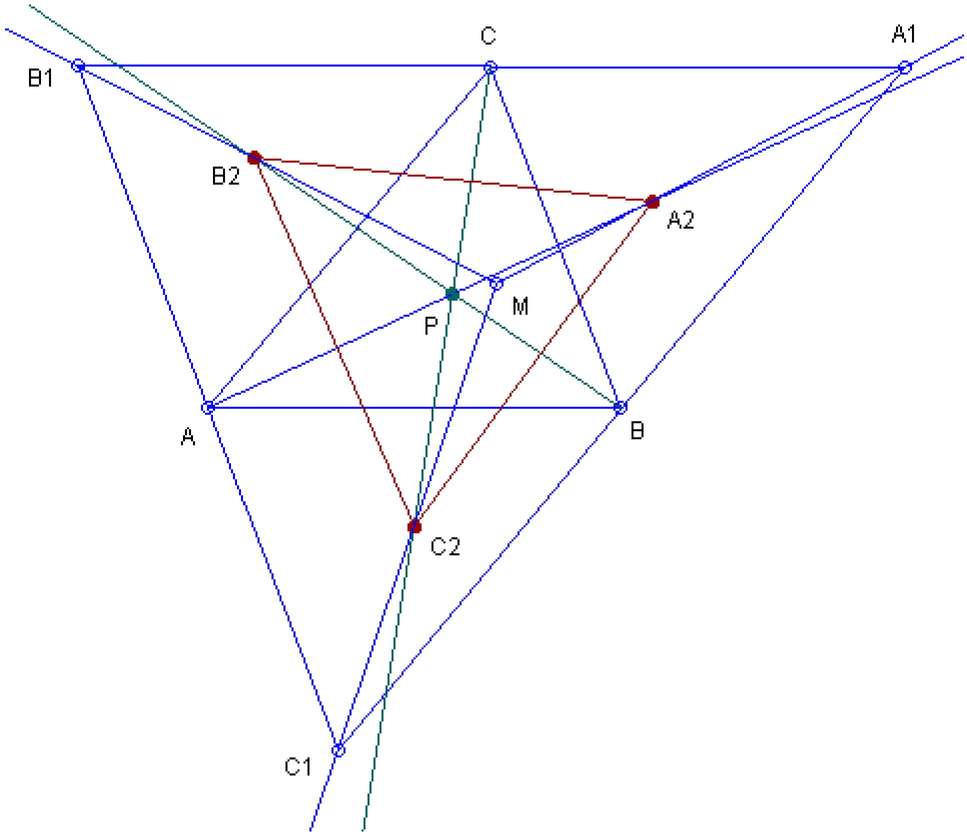
We use Theorems 1 and 5. See the Figure:



P - Gallatly-Kipepert Point = Isogonal Conjugate of the Brocard Midpoint;
M - Steiner Point;
$A_1B_1C_1$ - Circumcevian Triangle of the Symmedian Point;
A_2 - intersection point of lines PA and MA_1;
B_2 - intersection point of lines PB and MB_1;
C_2 - intersection point of lines PC and MC_1;
$A_2B_2C_2$ - Outer Gallatly-Kiepert Triangle.

Solution 5

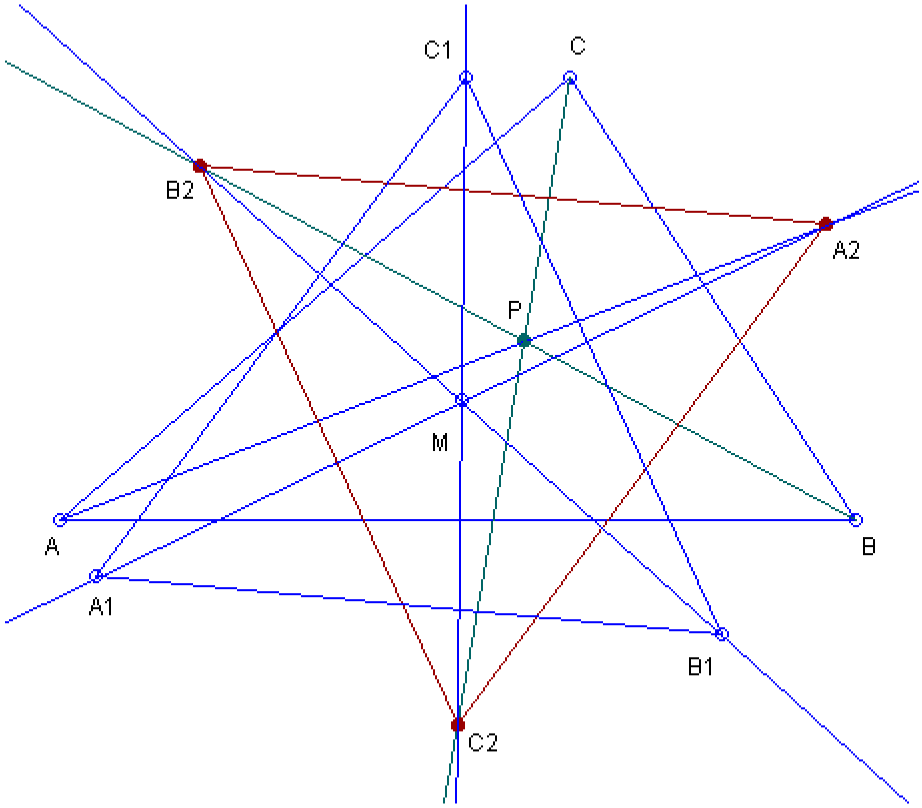
We use Theorems 1 and 6. See the Figure:



P - Gallatly-Kipepert Point = Isogonal Conjugate of the Brocard Midpoint;
M - Perspector of the Symmedial Triangle and the Anticomplementary Triangle;
$A_1B_1C_1$ - Anticomplementary Triangle;
A_2 - intersection point of lines PA and MA_1;
B_2 - intersection point of lines PB and MB_1;
C_2 - intersection point of lines PC and MC_1;
$A_2B_2C_2$ - Outer Gallatly-Kiepert Triangle.

Solution 6

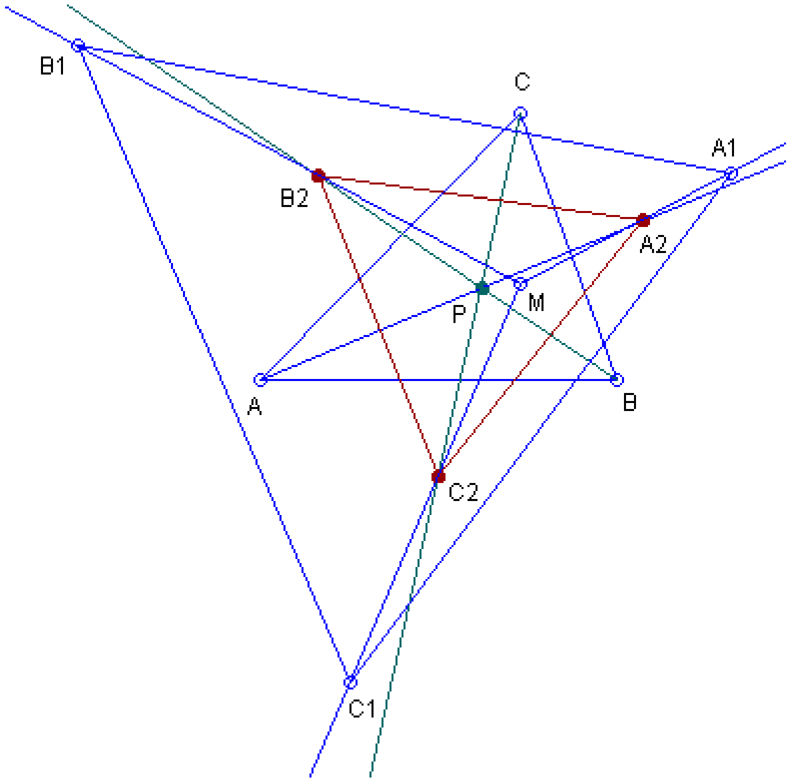
We use Theorems 1 and 7. See the Figure:



P - Gallatly-Kipepert Point = Isogonal Conjugate of the Brocard Midpoint;
M - Symmedian Point of the Medial Triangle;
$A_1B_1C_1$ - Pedal Triangle of the Center of the Brocard Circle of the Anticomplementary Triangle;
A_2 - intersection point of lines PA and MA_1;
B_2 - intersection point of lines PB and MB_1;
C_2 - intersection point of lines PC and MC_1;
$A_2B_2C_2$ - Outer Gallatly-Kiepert Triangle.

Solution 7

We use Theorems 1 and 8. See the Figure:



P - Gallatly-Kipepert Point = Isogonal Conjugate of the Brocard Midpoint;
M - Orthocenter;
$A_1B_1C_1$ - Desmic Mate the Reflected Neuberg Triangle;
A_2 - intersection point of lines PA and MA_1;
B_2 - intersection point of lines PB and MB_1;
C_2 - intersection point of lines PC and MC_1;
$A_2B_2C_2$ - Outer Gallatly-Kiepert Triangle.

We leave the other solutions to the reader. To obtain the other solutions, we have to use: Theorems 2 and 3, Theorems 2 and 4, etc.

Note

It is clear that if we have n perspectives, we obtain n(n-1)/2 different ways how to construct the triangle.

Invitation

The reader is invited to submit a note/paper containing

- synthetic proofs of theorems from this paper,
- or, applications of theorems from this paper,
- or, additional references related to this paper.

Definitions

We use the definitions in accordance with [1 - 6] and papers published in this journal.

The Level

The Machine for Questions and Answers is used to produce results in this paper. Currently the Machine has 6 levels of depths - 0,1,2,3,4,5. We use for this paper the level 0, that is, the Machine produces only elementary results. If we need deeper investigation, we have to use a level bigger than 0. Since the Machine for Questions and Answers produces too many results, it is suitable we to use bigger levels upon request, that is, for specific questions.

Thanks

The figures in this note are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download in the Web: Rene Grothmann's C.a.R.. It is free and open source. The reader may verify easily the statements of this paper by using C.a.R. Many thanks to Rene Grothmann for his wonderful program.

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