

Gallatly-Kiepert Points

Deko Dekov

Abstract. By using the computer program "Machine for Questions and Answers", we find properties of the Gallatly-Kiepert Points.

The *Outer Gallatly-Kiepert Point* is the perspector of triangle ABC and the Outer Gallatly-Kiepert Triangle.

The *Inner Gallatly-Kiepert Point* is the perspector of triangle ABC and the Inner Gallatly-Kiepert Triangle. The Inner Gallatly-Kiepert Point is known also as the Third Brocard Point.

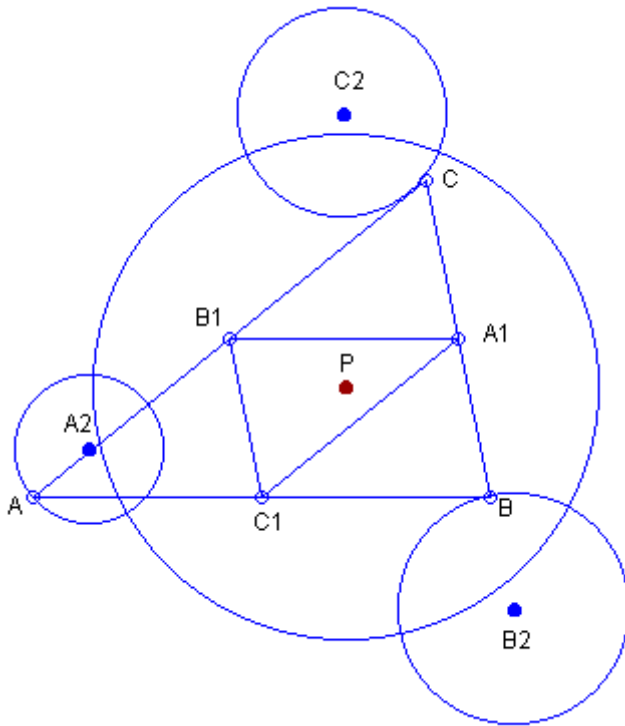
Outer Gallatly-Kiepert Point

Given a point, the Machine for Questions and Answers produces theorems related to properties of the point. The Machine for Questions and Answers produces theorems related to properties of the Outer Gallatly-Kiepert Point:

Outer Gallatly-Kiepert Point = Internal Center of Similitude of the 2ω Tucker Circle and the Nine-Point Circle.

Outer Gallatly-Kiepert Point = Radical Center of the Reflected Neuberg Circles of the Medial Triangle.

See the Figure:



$A_1B_1C_1$ - Medial Triangle;

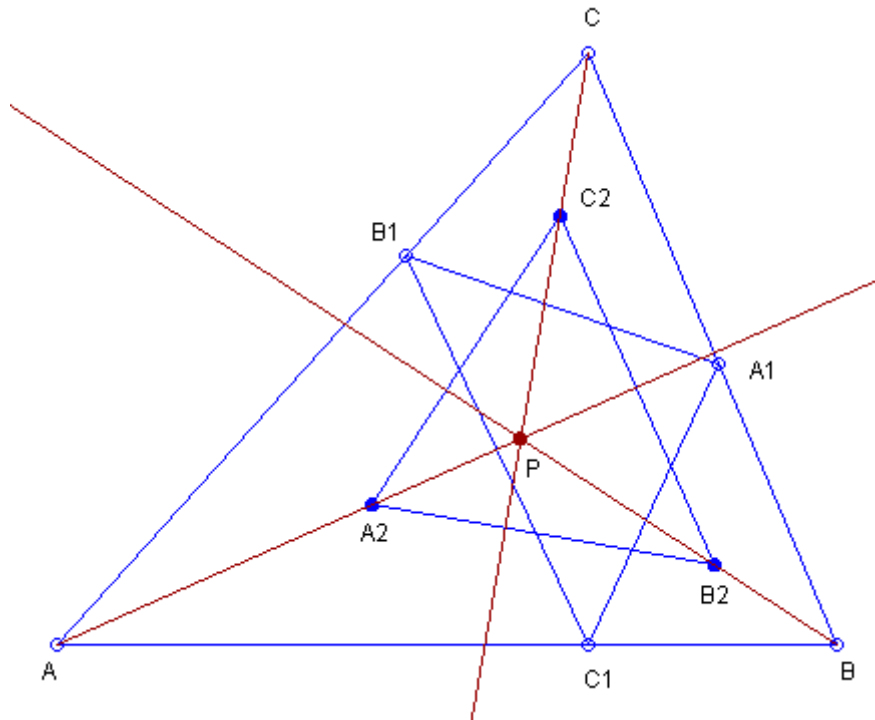
(A_2) , (B_2) , (C_2) - Reflected Neuberg Circles of the Medial Triangle;

(P) - Radical Circle of the Reflected Neuberg Circles of the Medial Triangle;

P - Outer Gallatly-Kiepert Point = center of circle (P) .

Outer Gallatly-Kiepert Point = Perspector of Triangle ABC and the Triangle of the Brocard Midpoints of the Corner Triangles of the Orthocenter.

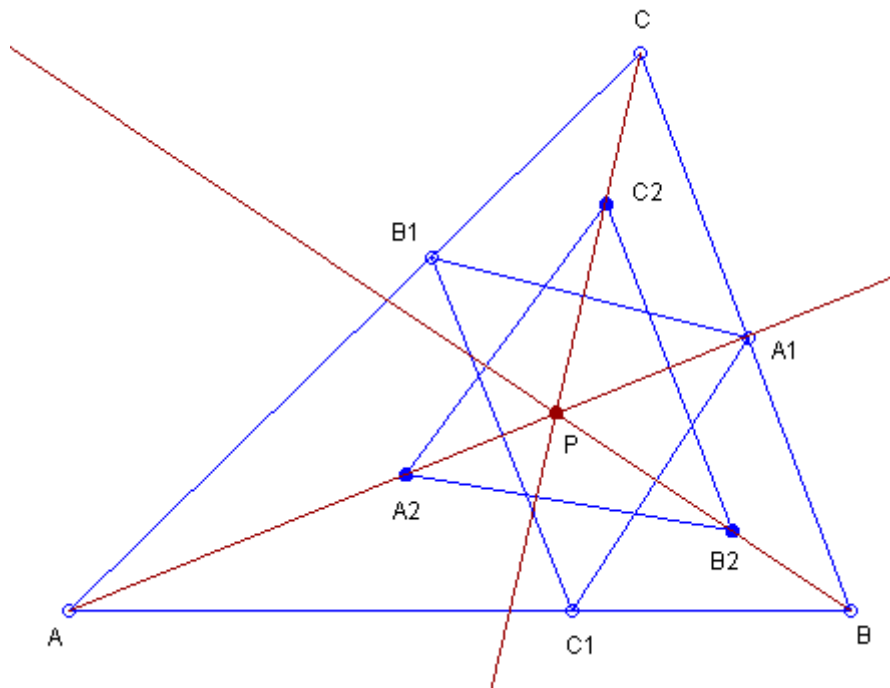
See the Figure:



- $A_1B_1C_1$ - Orthic Triangle;
- A_2 - Brocard Midpoint of triangle AB_1C_1 ;
- B_2 - Brocard Midpoint of triangle BC_1A_1 ;
- C_2 - Brocard Midpoint of triangle CA_1B_1 ;
- $A_2B_2C_2$ - Triangle of the Brocard Midpoints of the Corner Triangles of the Orthocenter;
- P - Outer Gallatly-Kiepert Point = Perspector of triangles ABC and $A_2B_2C_2$.

Outer Gallatly-Kiepert Point = Perspector of Triangle ABC and the Triangle of the Symmedian Points of the Corner Triangles of the Symmedian Point.

See the Figure:



- $A_1B_1C_1$ - Symmedial Triangle;
- A_2 - Symmedian Point of triangle AB_1C_1 ;
- B_2 - Symmedian Point of triangle BC_1A_1 ;
- C_2 - Symmedian Point of triangle CA_1B_1 ;
- $A_2B_2C_2$ - Triangle of the Symmedian Points of the Corner Triangles of the Symmedian Point;
- P - Outer Gallatly-Kiepert Point = Perspector of triangles ABC and $A_2B_2C_2$.

Outer Gallatly-Kiepert Point = Perspector of Triangle ABC and the Triangle of the reflections of the Brocard Midpoint in the sides of the Excentral Triangle.

Outer Gallatly-Kiepert Point = Isotomic Conjugate of the Complement of the Symmedian Point.

Outer Gallatly-Kiepert Point = Isogonal Conjugate of the Brocard Midpoint.

Outer Gallatly-Kiepert Point = Isotomic Conjugate of the Symmedian Point of the Medial Triangle.

Outer Gallatly-Kiepert Point = Isotomic Conjugate of the Kiepert Center of the First Brocard Triangle.

Outer Gallatly-Kiepert Point = Isogonal Conjugate of the Internal Center of Similitude of the Gallatly Circle and the Moses Circle.

Outer Gallatly-Kiepert Point = Isogonal Conjugate of the Internal Center of Similitude of the Gallatly Circle and the Half-Moses Circle.

Outer Gallatly-Kiepert Point = Isogonal Conjugate of the External Center of Similitude of

the Half-Moses Circle and the Moses Circle.

Outer Gallatly-Kiepert Point = Isogonal Conjugate of the External Center of Similitude of the Gallatly Circle and the Moses Circle.

Outer Gallatly-Kiepert Point = Isogonal Conjugate of the External Center of Similitude of the Gallatly Circle and the Half-Moses Circle.

Outer Gallatly-Kiepert Point = Isogonal Conjugate of the Inverse of the Third Power Point in the Brocard Circle.

Outer Gallatly-Kiepert Point = Isogonal Conjugate of the Perspector of the Cevian Triangle of the Brocard Midpoint and the Circumcevian Triangle of the Brocard Midpoint.

Outer Gallatly-Kiepert Point = Isogonal Conjugate of the Perspector of the Anticevian Triangle of the Brocard Midpoint and the Circumcevian Triangle of the Brocard Midpoint.

Outer Gallatly-Kiepert Point = Complement of the Perspector of the Anticomplementary Triangle and the First Brocard Triangle.

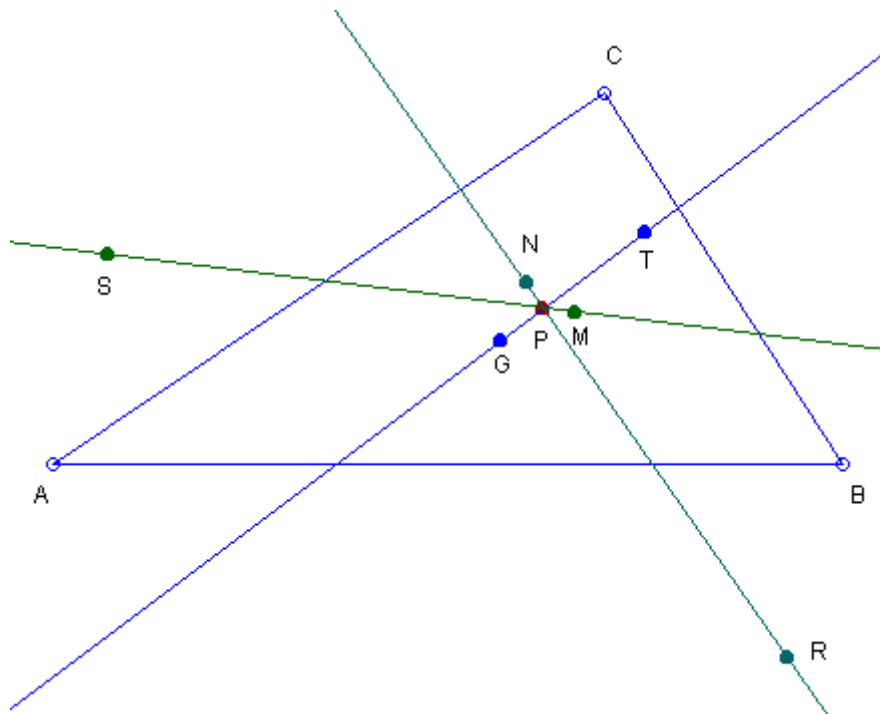
Outer Gallatly-Kiepert Point = Complement of the Perspector of the Anticomplementary Triangle and the Inner Gallatly-Kiepert Triangle.

The Outer Gallatly-Kiepert Point lies on the Line through the Centroid and the Third Power Point.

The Outer Gallatly-Kiepert Point lies on the Line through the Nine-Point Center and the Tarry Point.

The Outer Gallatly-Kiepert Point lies on the Line through the Brocard Midpoint and the Steiner Point.

For the last three theorems, See the Figure:



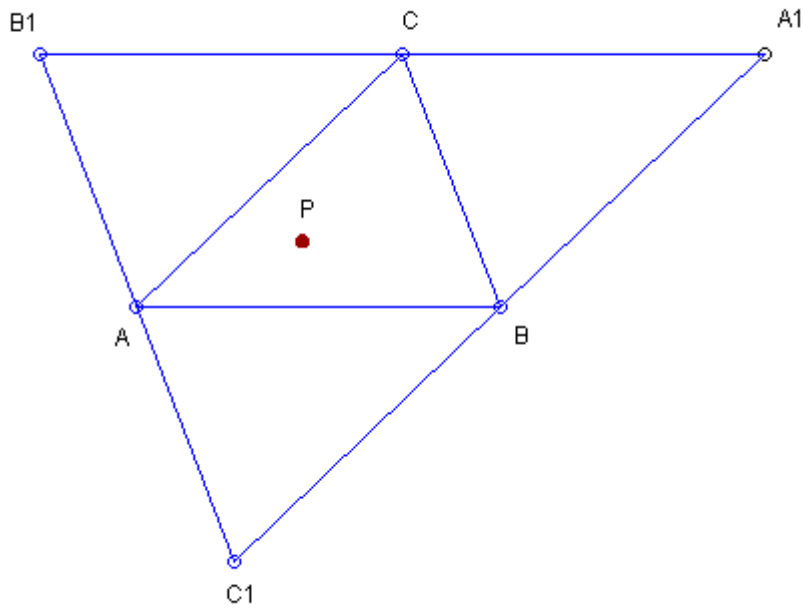
- G - Centroid;
- T - Third Power Point;
- N - Nine-Point Center;
- R - Tarry Point;
- M - Brocard Midpoint;
- S - Steiner Point;
- P - Outer Gallatly-Kiepert Point - lies on the lines GT, NR and MS.

Inner Gallatly-Kiepert Point

Given a point, the Machine for Questions and Answers produces theorems related to properties of the point. The Machine for Questions and Answers produces theorems related to properties of the Inner Gallatly-Kiepert Point:

Inner Gallatly-Kiepert Point = Brocard Midpoint of the Anticomplementary Triangle.

See the Figure:



$A_1B_1C_1$ - Anticomplementary Triangle;

P - Inner Gallatly-Kiepert Point = Brocard Midpoint of triangle $A_1B_1C_1$.

Inner Gallatly-Kiepert Point = Inner $2 \cdot \omega$ Kiepert Point of the First Brocard Triangle.

Inner Gallatly-Kiepert Point = Inner $2 \cdot \omega$ Kiepert Point of the Inner Gallatly-Kiepert Triangle.

Inner Gallatly-Kiepert Point = Center of the Moses Circle of the Anticomplementary Triangle.

Inner Gallatly-Kiepert Point = Center of the Half-Moses Circle of the Anticomplementary Triangle.

Inner Gallatly-Kiepert Point = Center of the Gallatly Circle of the Anticomplementary Triangle.

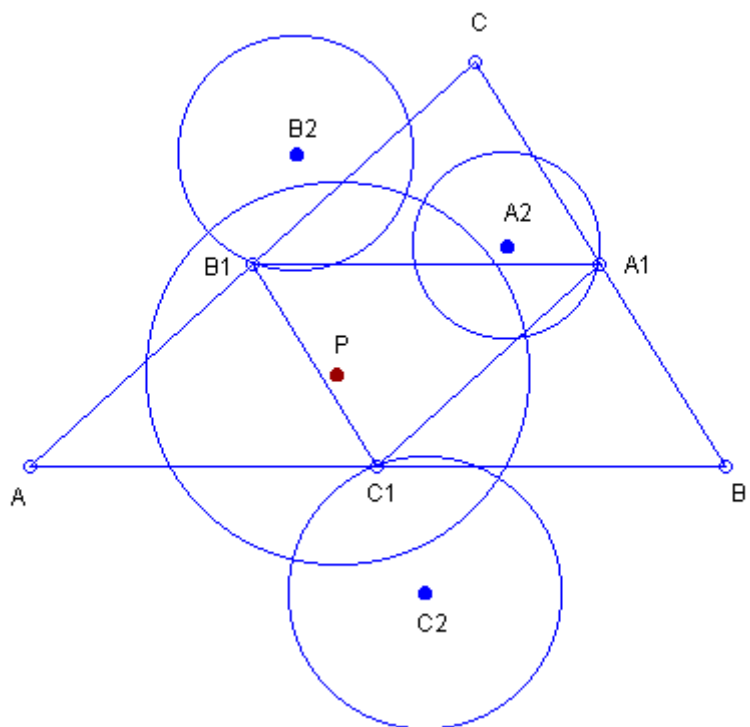
Inner Gallatly-Kiepert Point = Product of the Isotomic Conjugate of the Circumcenter and the Symmedian Point of the Anticomplementary Triangle.

Inner Gallatly-Kiepert Point = External Center of Similitude of the Moses Circle and the Moses Circle of the Medial Triangle.

Inner Gallatly-Kiepert Point = External Center of Similitude of the Half-Moses Circle and the Half-Moses Circle of the Medial Triangle.

Inner Gallatly-Kiepert Point = Radical Center of the Neuberg Circles of the Medial Triangle.

See the Figure:



$A_1B_1C_1$ - Medial Triangle;

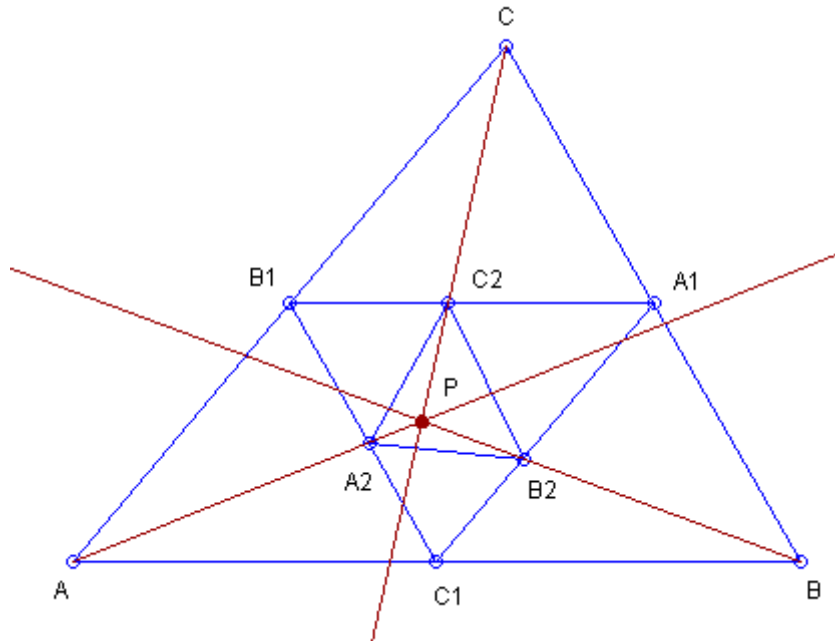
(A_2) , (B_2) , (C_2) - Neuberg Circles of the Medial Triangle;

(P) - Radical Circle of the Neuberg Circles of the Medial Triangle;

P - Inner Gallatly-Kiepert Point = center of circle (P) .

Inner Gallatly-Kiepert Point = Perspector of Triangle ABC and the Symmedial Triangle of the Medial Triangle.

See the Figure:



$A_1B_1C_1$ - Medial Triangle;

$A_2B_2C_2$ - Symmedial Triangle of the Medial Triangle;

P - Inner Gallatly-Kiepert Point = Perspector of triangles ABC and $A_2B_2C_2$.

Inner Gallatly-Kiepert Point = Perspector of Triangle ABC and the Triangle of the Third Power Points of the Corner Triangles of the Orthocenter.

Inner Gallatly-Kiepert Point = Perspector of Triangle ABC and the Triangle of the reflections of the Third Power Point in the sides of the Excentral Triangle.

Inner Gallatly-Kiepert Point = Perspector of Triangle ABC and the Triangle of the reflections of the vertices of the Symmedial Triangle in the Brocard Midpoint.

Inner Gallatly-Kiepert Point = Homothetic Center of Triangle ABC and the Triangle of the reflections of the vertices of the Anticomplementary Triangle in the Brocard Midpoint.

Inner Gallatly-Kiepert Point = Homothetic Center of the Anticomplementary Triangle and the Triangle of the Brocard Midpoints of the Anticevian Corner Triangles of the Centroid.

Inner Gallatly-Kiepert Point = Homothetic Center of the Anticomplementary Triangle and the Triangle of the reflections of the Brocard Midpoint in the vertices of the Medial Triangle.

Inner Gallatly-Kiepert Point = Perspector of the First Brocard Triangle and the Triangle of the Third Power Points of the Corner Triangles of the Orthocenter.

Inner Gallatly-Kiepert Point = Perspector of the First Brocard Triangle and the Triangle of the reflections of the Third Power Point in the sides of the Excentral Triangle.

Inner Gallatly-Kiepert Point = Square of the Complement of the Equal Parallelians Point.

Inner Gallatly-Kiepert Point = Anticomplement of the Brocard Midpoint.

Inner Gallatly-Kiepert Point = Square of the Anticomplement of the Grinberg Point.

Inner Gallatly-Kiepert Point = Isogonal Conjugate of the Third Power Point.

Inner Gallatly-Kiepert Point = Isotomic Conjugate of the Isogonal Conjugate of the Centroid.

Inner Gallatly-Kiepert Point = Square of the Isogonal Conjugate of the Second Power Point.

Inner Gallatly-Kiepert Point = Isotomic Conjugate of the Symmedian Point.

Inner Gallatly-Kiepert Point = Square of the Isotomic Conjugate of the Incenter.

Inner Gallatly-Kiepert Point = Isogonal Conjugate of the Square of the Symmedian Point.

Inner Gallatly-Kiepert Point = Isotomic Conjugate of the Square of the Incenter.

Inner Gallatly-Kiepert Point = Isogonal Conjugate of the Anticomplement of the Third Power Point of the Medial Triangle.

Inner Gallatly-Kiepert Point = Isotomic Conjugate of the Anticomplement of the Symmedian Point of the Medial Triangle.

Inner Gallatly-Kiepert Point = Isotomic Conjugate of the Isogonal Conjugate of the Centroid of the Medial Triangle.

Inner Gallatly-Kiepert Point = Square of the Equal Parallelians Point of the Medial Triangle.

Inner Gallatly-Kiepert Point = Square of the Grinberg Point of the Anticomplementary Triangle.

Inner Gallatly-Kiepert Point = Isotomic Conjugate of the Anticomplement of the Kiepert Center of the First Brocard Triangle.

Inner Gallatly-Kiepert Point = Two Permutatons Point of the First Brocard Point.

Inner Gallatly-Kiepert Point = One Permutation Point of the Second Brocard Point.

The Inner Gallatly-Kiepert Point lies on the Line through the Circumcenter and the Tarry Point.

The Inner Gallatly-Kiepert Point lies on the Line through the Circumcenter and the Steiner Point.

The Inner Gallatly-Kiepert Point lies on the Line through the Brocard Midpoint and the

Centroid.

The Inner Gallatly-Kiepert Point lies on the Line through the Steiner Point and the Tarry Point.

The Inner Gallatly-Kiepert Point lies on the Line through the Orthocenter and the Symmedian Point of the Anticomplementary Triangle.

The Inner Gallatly-Kiepert Point lies on the Line through the Centroid and the Perspector of the Symmedian Triangle and the Anticomplementary Triangle.

The Inner Gallatly-Kiepert Point lies on the Line through the Brocard Midpoint and the Perspector of the Symmedian Triangle and the Anticomplementary Triangle.

The Inner Gallatly-Kiepert Point lies on the Line through the Isotomic Conjugate of the Circumcenter and the Symmedian Point of the Anticomplementary Triangle.

Invitation

The reader is invited to submit a note/paper containing

- synthetic proofs of theorems from this paper,
- or, applications of theorems from this paper,
- or, additional references related to this paper.

Definitions

We use the definitions in accordance with [1 - 5] and papers published in this journal.

The Level

The Machine for Questions and Answers is used to produce results in this paper. Currently the Machine has 6 levels of depths - 0,1,2,3,4,5. We use for this paper the level 0, that is, the Machine produces only elementary results. If we need deeper investigation, we have to use a level bigger than 0. Since the Machine for Questions and Answers produces too many results, it is suitable we to use bigger levels upon request, that is, for specific questions.

Thanks

The figures in this note are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download in the Web: [Rene Grothmann's C.a.R.](http://www.xtec.es/~qcastell/tw/ttweng/portada.html). It is free and open source. The reader may verify easily the statements of this paper by using C.a.R. Many thanks to Rene Grothmann for his wonderful program.

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Dr.Deko Dekov, ddekov@dekovsoft.com.