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Lemoine-Kiepert Points

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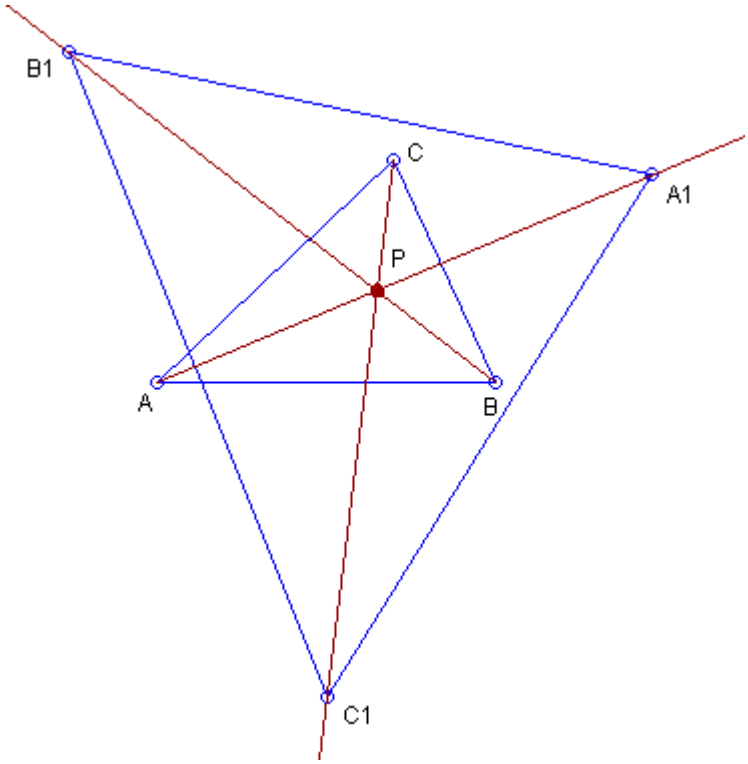
Abstract. By using the computer program "Machine for Questions and Answers", we find properties of the Lemoine-Kiepert Points.

The *Outer Lemoine-Kiepert Point* is the perspector of triangle ABC and the Outer Lemoine-Kiepert Triangle.

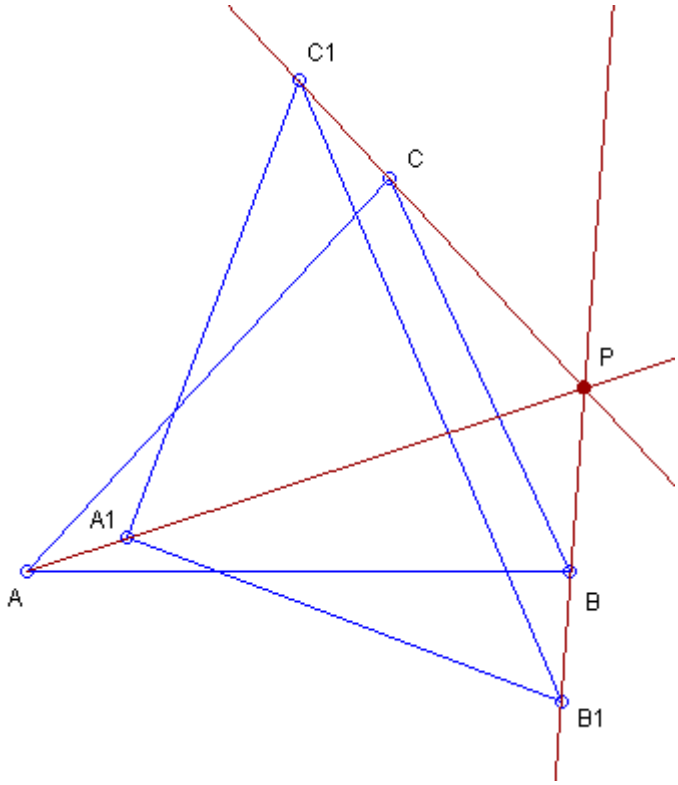
See the Figure:



$A_1B_1C_1$ - Outer Lemoine-Kiepert Triangle;
P - Outer Lemoine-Kiepert Point - perspector of triangles ABC and $A_1B_1C_1$.

The *Inner Lemoine-Kiepert Point* is the perspector of triangle ABC and the Inner Lemoine-Kiepert Triangle. The Inner Lemoine-Kiepert Point coincides with the well known Tarry Point. Hence, it is reasonable here we to list properties of the Tarry Point.

See the Figure:



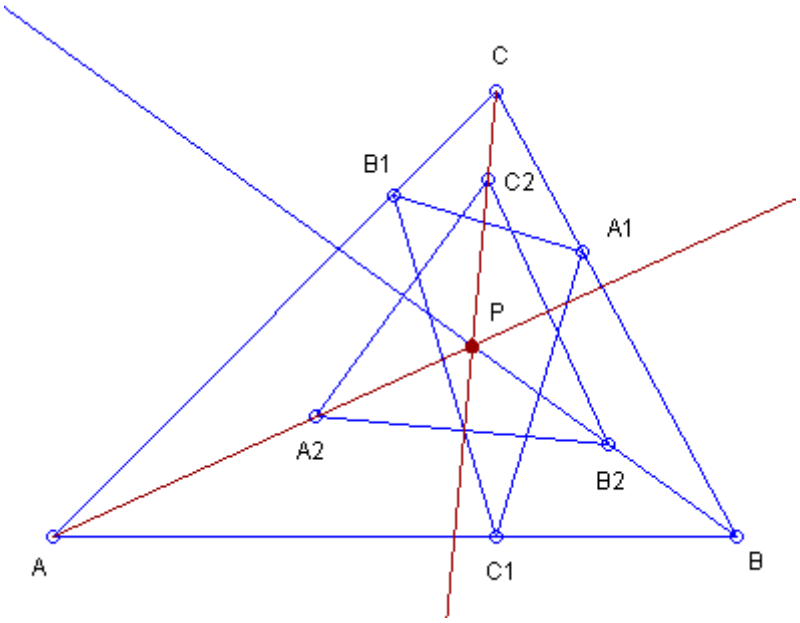
$A_1B_1C_1$ - Inner Lemoine-Kiepert Triangle;
P - Outer Lemoine-Kiepert Point - perspector of triangles ABC and $A_1B_1C_1$.

Outer Lemoine-Kiepert Point

The Machine for Questions and Answers produces theorems related to properties of the Outer Lemoine-Kiepert Point:

Outer Lemoine-Kiepert Point = Perspector of Triangle ABC and the Triangle of the Centers of the Brocard Circles of the Corner Triangles of the Orthocenter.

See the Figure:



$A_1B_1C_1$ - Orthic Triangle = Cevian triangle of the Orthocenter;
A_2 - Centers of the Brocard Circle of triangle AB_1C_1;
B_2 - Centers of the Brocard Circle of triangle BC_1A_1;
C_2 - Centers of the Brocard Circle of triangle CA_1B_1;
$A_2B_2C_2$ - Triangle of the Centers of the Brocard Circles of the Corner Triangles of the Orthocenter;
P - Outer Lemoine-Kiepert Point = Perspector of triangles ABC and $A_2B_2C_2$.

Outer Lemoine-Kiepert Point = Perspector of Triangle ABC and the Triangle of the reflections of the Center of the Brocard Circle in the sides of the Excentral Triangle.

Outer Lemoine-Kiepert Point = Perspector of the Reflected Neuberg Triangle and the Triangle of the Centers of the Brocard Circles of the Corner Triangles of the Orthocenter.

Outer Lemoine-Kiepert Point = Perspector of the Reflected Neuberg Triangle and the Triangle of the reflections of the Center of the Brocard Circle in the sides of the Excentral Triangle.

Outer Lemoine-Kiepert Point = Isogonal Conjugate of the Center of the Brocard Circle.

Outer Lemoine-Kiepert Point = Isogonal Conjugate of the Complement of the Orthocenter of the First Brocard Triangle.

Outer Lemoine-Kiepert Point = Isogonal Conjugate of the Complement of the Symmedian Point of the Johnson Triangle.

Outer Lemoine-Kiepert Point = Isogonal Conjugate of the Anticomplement of the Nine-Point Center of the First Brocard Triangle.

Outer Lemoine-Kiepert Point = Isogonal Conjugate of the Circumcenter of the First Brocard Triangle.

Outer Lemoine-Kiepert Point = Isogonal Conjugate of the Circumcenter of the Second Brocard Triangle.

Outer Lemoine-Kiepert Point = Isogonal Conjugate of the Midpoint between the Circumnter and the Symmedian Point.

Outer Lemoine-Kiepert Point = Isogonal Conjugate of the Internal Center of Similitude of the Brocard Circle and the Lemoine Circle.

Outer Lemoine-Kiepert Point = Isogonal Conjugate of the Internal Center of Similitude of the 2*omega Tucker Circle and the Circumcircle.

Outer Lemoine-Kiepert Point = Isogonal Conjugate of the External Center of Similitude of the Brocard Circle and the Lemoine Circle.

Outer Lemoine-Kiepert Point = Complement of the Perspector of the Anticomplementary Triangle and the Neuberg Triangle.

Outer Lemoine-Kiepert Point = Complement of the Perspector of the Anticomplementary Triangle and the Inner Lemoine-Kiepert Triangle.

Outer Lemoine-Kiepert Point = Complement of the Perspector of the Antipedal Triangle of the Orthocenter and the Neuberg Triangle.

Outer Lemoine-Kiepert Point = Complement of the Perspector of the Antipedal Triangle of the Orthocenter and the Inner Lemoine-Kiepert Triangle.

Outer Lemoine-Kiepert Point = Isogonal Conjugate of the Perspector of the Cevian Triangle of the Center of the Brocard Circle and the Circumcevian Triangle of the Center of the Brocard Circle.

Outer Lemoine-Kiepert Point = Isogonal Conjugate of the Perspector of the Anticevian Triangle of the Center of the Brocard Circle and the Circumcevian Triangle of the Center of the Brocard Circle.

Outer Lemoine-Kiepert Point = Isogonal Conjugate of the Perspector of the Anticevian Triangle of the Third Power Point and the Neuberg Triangle.

Outer Lemoine-Kiepert Point = Isogonal Conjugate of the Perspector of the Anticevian Triangle of the Third Power Point and the Inner Lemoine-Kiepert Triangle.

The Outer Lemoine-Kiepert Point lies on the Line through the Symmedian Point and the Tarry Point.

The Outer Lemoine-Kiepert Point lies on the Line through the Brocard Midpoint and the Orthocenter.

The Outer Lemoine-Kiepert Point lies on the Line through the Centroid and the Centroid of the Orthic Triangle.

Tarry Point

The Machine for Questions and Answers produces theorems related to properties of the Tarry Point:

Tarry Point = Inner Lemoine-Kiepert Point.

Tarry Point = Steiner Point of the Circumcevian Triangle of the Circumcenter.

Tarry Point = Kiepert-Parry Point of the Circumcevian Triangle of the Outer Fermat Point.

Tarry Point = Kiepert-Parry Point of the Circumcevian Triangle of the Inner Fermat Point.

Tarry Point = Kiepert-Parry Point of the Circumcevian Triangle of the Center of the Brocard Circle.

Tarry Point = Reflection of the Orthocenter in the Kiepert Center.

Tarry Point = Reflection of the Steiner Point in the Circumcenter.

Tarry Point = External Center of Similitude of the 2*omega Tucker Circle and the Nine-Point Circle.

Tarry Point = Anticomplement of the Steiner Point of the Euler Triangle.

Tarry Point = Anticomplement of the Kiepert Center of the Johnson Triangle.

Tarry Point = Complement of the Reflection of the de Longchamps Point in the Steiner Point.

Tarry Point = Anticomplement of the Reflection of the Kiepert Center in the Nine-Point Center.

Tarry Point = Complement of the Perspector of the Anticomplementary Triangle and the Reflected Neuberg Triangle.

Tarry Point = Complement of the Perspector of the Anticomplementary Triangle and the Outer Lemoine-Kiepert Triangle.

Tarry Point = Complement of the Perspector of the Antipedal Triangle of the Orthocenter and the Reflected Neuberg Triangle.

Tarry Point = Complement of the Perspector of the Antipedal Triangle of the Orthocenter and the Outer Lemoine-Kiepert Triangle.

The Tarry Point lies on the Circumcircle.

The Tarry Point lies on the Nine-Point Circle of the Excentral Triangle.

The Tarry Point lies on the Nine-Point Circle of the Anticomplementary Triangle.

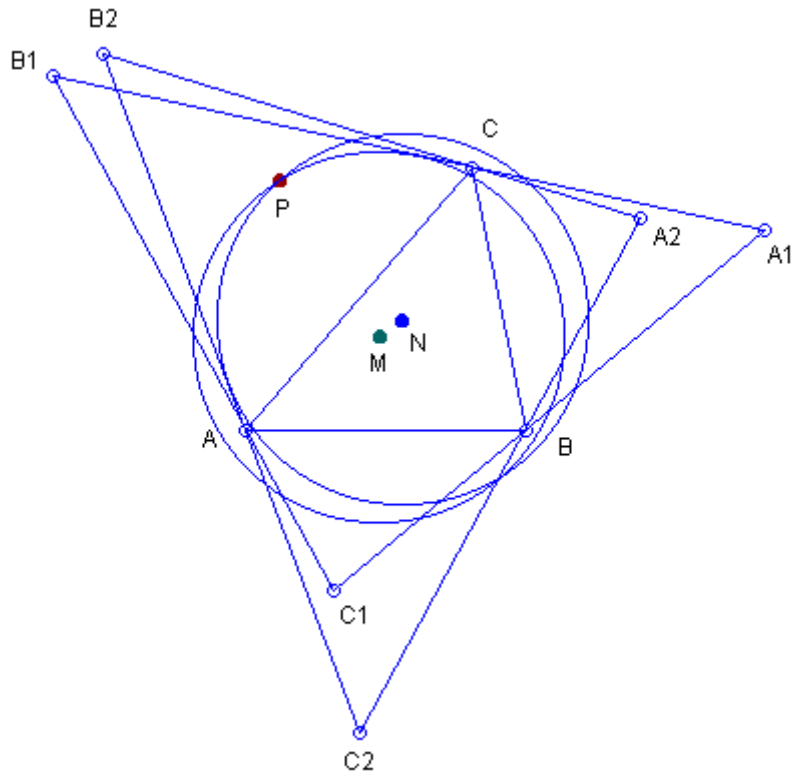
The Tarry Point lies on the Nine-Point Circle of the Antipedal Triangle of the Incenter.

The Tarry Point lies on the Nine-Point Circle of the Antipedal Triangle of the Orthocenter.

The Tarry Point lies on the Nine-Point Circle of the Antipedal Triangle of the First Brocard Point.

The Tarry Point lies on the Nine-Point Circle of the Antipedal Triangle of the Second Brocard Point.

For the last two theorems: See the Figure:



$A_1B_1C_1$ - Antipedal Triangle of the First Brocard Point;
$A_2B_2C_2$ - Antipedal Triangle of the Second Brocard Point;
(M) - Nine-Point Circle of triangle $A_1B_1C_1$;
(N) - Nine-Point Circle of triangle $A_2B_2C_2$;
P - Tarry Point = intersection point of circles (M) and (N).

The Tarry Point lies on the Parry Circle of the Circumcevian Triangle of the Outer Fermat Point.

The Tarry Point lies on the Parry Circle of the Circumcevian Triangle of the Inner Fermat Point.

The Tarry Point lies on the Parry Circle of the Circumcevian Triangle of the Center of the Brocard Circle.

The Tarry Point lies on the Nine-Point Circle of the Hexyl Triangle.

The Tarry Point lies on the Gallatly Circle of the Antipedal Triangle of the First Brocard Point.

The Tarry Point lies on the Gallatly Circle of the Antipedal Triangle of the Second Brocard Point.

The Tarry Point lies on the Circle centered at the Kiepert Center through the Orthocenter.

The Tarry Point lies on the Radical Circle of the Triad of the Parry Circles of the Triangulation Triangles of the Steiner Point.

The Tarry Point lies on the Radical Circle of the Triad of the Stevanovic Circles of the Triangulation Triangles of the Steiner Point.

The Tarry Point lies on the Radical Circle of the Triad of the Parry Circles of the Triangulation Triangles of the Kiepert-Parry Point.

The Tarry Point lies on the Radical Circle of the Triad of the Stevanovic Circles of the Triangulation Triangles of the Kiepert-Parry Point.

The Tarry Point lies on the Radical Circle of the Triad of the Parry Circles of the Triangulation Triangles of the Parry Point.

The Tarry Point lies on the Radical Circle of the Triad of the Stevanovic Circles of the Triangulation Triangles of the Parry Point.

The Tarry Point lies on the Line through the Centroid and the Kiepert-Parry Point.

The Tarry Point lies on the Line through the Circumcenter and the Steiner Point.

The Tarry Point lies on the Line through the Orthocenter and the Third Power Point.

The Tarry Point lies on the Line through the Kiepert Center and the Orthocenter.

The Tarry Point lies on the Line through the Kiepert Center and the Third Power Point.

The Tarry Point lies on the Line through the Circumcenter and the Isotomic Conjugate of the Symmedian Point.

The Tarry Point lies on the Line through the Homothetic Center of the Orthic Triangle and the Tangential Triangle and the Perspector of Triangle ABC and the Symmedial Triangle of the Orthic Triangle.

Invitation

The reader is invited to submit a note/paper containing

- synthetic proofs of theorems from this paper,
- or, applications of theorems from this paper,
- or, additional references related to this paper.

Definitions

We use the definitions in accordance with [1 - 5] and papers published in this journal.

The Level

The Machine for Questions and Answers is used to produce results in this paper. Currently the Machine has 6 levels of depths - 0,1,2,3,4,5. We use for this paper the level 0, that is, the Machine produces only elementary results. If we need deeper investigation, we have to use a level bigger than 0. Since the Machine for Questions and Answers produces too many results, it is suitable we to use bigger levels upon request, that is, for specific questions.

Thanks

The figures in this note are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download in the Web: Rene Grothmann's C.a.R.. It is free and open source. The reader may verify easily the statements of this paper by using C.a.R. Many thanks to Rene Grothmann for his wonderful program.

References

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