Journal of Computer-Generated Euclidean Geometry

New Constructions of the Outer Gallatly-Kiepert Triangle

Hristo Milev

Abstract. Recently Dr.Dekov showed 28 ways how to construct the Outer Gallatly-Kiepert Triangle. Here we show 8 additional ways. Also, we illustrate how the computer program "Machine for Questions and Answers" (the Dekov Machine) helps us to construct geometric objects.

The Outer Gallatly-Kiepert Triangle is defined and studied in [2]. Dr.Dekov [3] showed 28 ways how to construct the Outer Gallatly-Kiepert Triangle.

We call the circumcircle of the Outer Gallatly-Kiepert Triangle the *Outer Gallatly-Kiepert Circle*. We use definitions and conventions in accordance with [1 - 3]. Recall that the Kiepert Center is the midpoint of the Outer and Inner Fermat Points.

We use the computer program Machine for Questions and Answers (further we use the name the *Dekov Machine*) to find 8 additional ways how to construct the Outer Gallatly-Kiepert Triangle. In these ways we do not need to construct the Brocard angle *omega*.

We use the following method. We can construct a triangle, if we can construct

- A triangle perspective to the triangle, and the perspector,
- and, the circumcircle of the triangle.

Construction of the Outer Gallatly-Kiepert Circle

Given a circle, the Dekov Machine produces theorems related to the circle. We use the Dekov Machine to find properties of the Outer Gallatly-Kiepert Circle. We select from the produced list the following three theorems:

1. The Outer Gallatly-Kiepert Circle contains the Reflection of the Symmedian Point in the Kiepert Center.
2. The Center of the Outer Gallatly-Kiepert Circle lies on the Line through the Centroid and the Centroid of the Orthic Triangle.
3. The Outer Gallatly-Kiepert Circle is congruent to the Bevan Circle of the Pedal Triangle of the Center of the Brocard Circle.

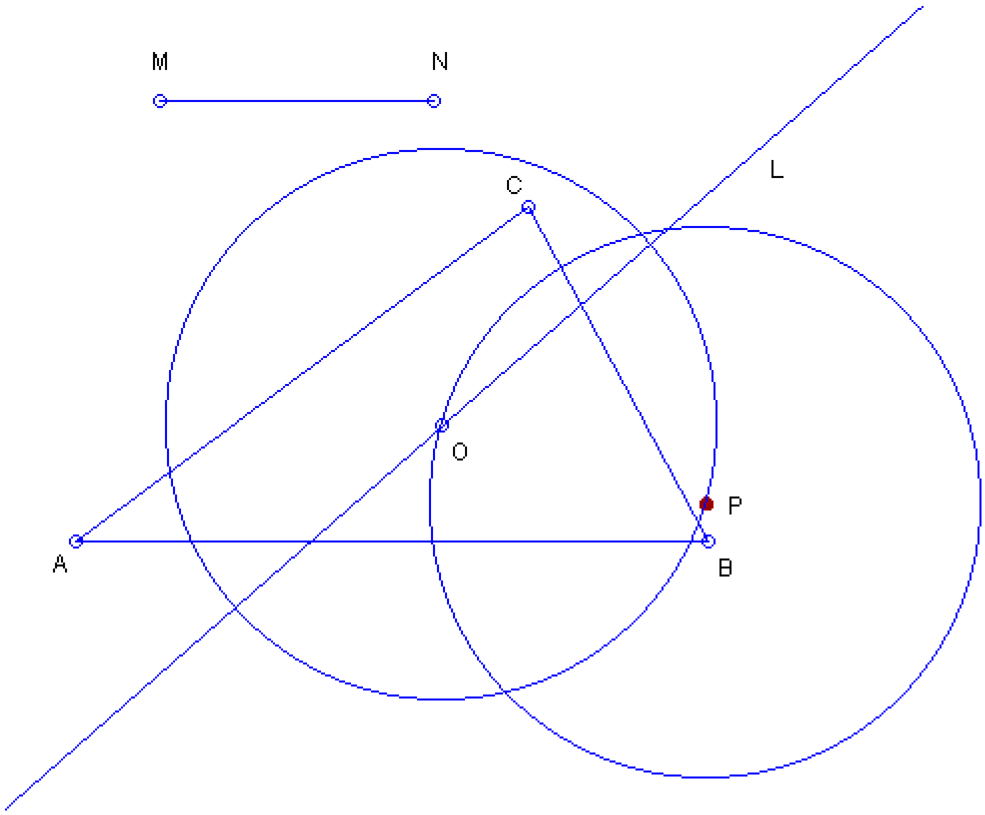
Note

1. If we select another set of theorems, we could obtain another solution.

Now we can construct the Outer Gallatly-Kiepert Circle as follows:

1. Construct the Reflection of the Symmedian Point in the Kiepert Center. Label the point by P. We use here Theorem 1.
2. Construct the Line through the Centroid and the Centroid of the Orthic Triangle. Label the line by L. We use here Theorem 2.
3. Construct the Bevan Circle of the Pedal Triangle of the Center of the Brocard Circle. We use here Theorem 3. Construct a line segment MN congruent to the radius of the constructed circle.
4. Construct a circle with center P and radius MN. Construct the intersection point of circle (P) and line L. Label it by O.
5. Construct a circle with center O and radius MN. The constructed circle is the Outer Gallatly-Kiepert Circle.

See the Figure:



P - Reflection of the Symmedian Point in the Kiepert Center;
L - Line through the Centroid and the Centroid of the Orthic Triangle;
MN - a line segment congruent to the radius of the Bevan Circle of the Pedal Triangle of the Center of the Brocard Circle;
(P) - circle with center P and radius MN;
O - intersection point of circle (P) and line L;
(O) - circle with center O and radius MN = Outer Gallatly-Kiepert Circle.

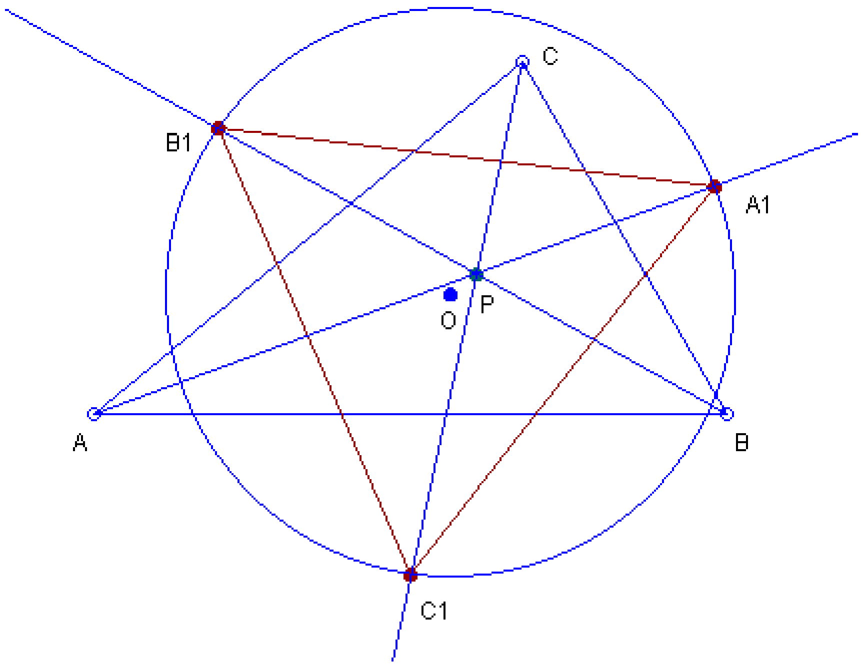
Construction of the Outer Gallatly-Kiepert Triangle

Deko Dekov [3] gave 8 theorems describing triangles perspective to the Outer Gallatly-Kiepert Triangle, and the perspectors. The first theorem states:

The Outer Gallatly-Kiepert Triangle and Triangle ABC are perspective with perspector the Isogonal Conjugate of the Brocard Midpoint.

We use the above theorem and the construction of the Outer Gallatly-Kiepert Circle given above, to construct the Gallatly-Kiepert Triangle.

See the Figure:



P - Outer Gallatly-Kipepert Point = Isogonal Conjugate of the Brocard Midpoint;
(O) - Outer Gallatly-Kiepert Circle;
A_1 - intersection point of line PA and circle (O);
B_1 - intersection point of line PB and circle (O);
C_1 - intersection point of line PC and circle (O);
$A_1B_1C_1$ - Outer Gallatly-Kiepert Triangle.

By using the other 7 theorems concerning perspectives of the Gallatly-Kiepert Triangle and Triangle, give in [3], we obtain in an analogous way 7 additional solutions. We leave these solutions to the reader.

References

1. D. Dekov, Computer-Generated Encyclopedia of Euclidean Geometry, First Edition, 2006, http://www.dekovsoft.com/
2. D. Dekov, Gallatly-Kiepert Triangles, in this journal, 2008.
3. D. Dekov, Construction of the Outer Gallatly-Kiepert Triangle, in this journal, 2008.

Publication Date: 8 January 2008

Hristo Milev, Stara Zagora, Bulgaria