

Problem 4.

The Feuerbach Point is the Perspector of the Euler Triangle and the Triangle of the Incenters of the Corner Triangles of the Orthocenter

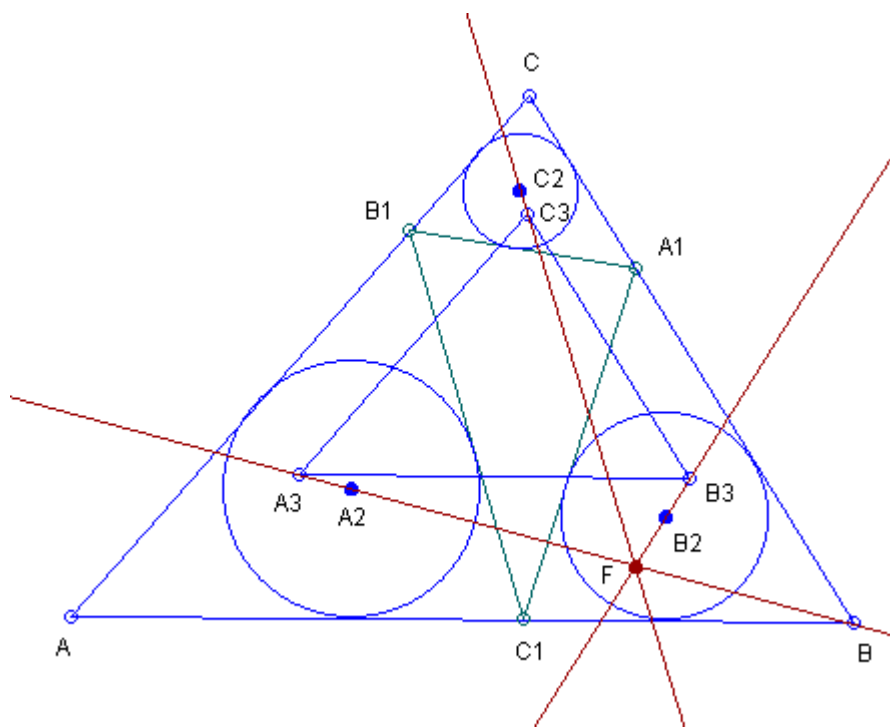
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Prove the following computer-generated theorem:

THEOREM. The Feuerbach Point is the Perspector of the Euler Triangle and the Triangle of the Incenters of the Corner Triangles of the Orthocenter.

The reader may find the definitions in [1 - 3].

Denote by $A_1B_1C_1$ the Orthic triangle of a given triangle ABC . Denote by $A_2B_2C_2$ the Triangle of the Incenters of the Corner Triangles of the Orthocenter. Denote by $A_3B_3C_3$ the Euler Triangle. The theorem states that lines A_2A_3 , B_2B_3 , and C_2C_3 concur at a point, and the point of concurrence is the Feuerbach Point. See the Figure:



$A_1B_1C_1$ - Orthic triangle;

$A_2B_2C_2$ - Triangle of the Incenters of the Corner Triangles of the Orthocenter;

$A_3B_3C_3$ - Euler triangle;
 A_2 is the Incenter of triangle AB_1C_1 ;
 B_2 is the Incenter of triangle BC_1A_1 ;
 C_2 is the Incenter of triangle CA_1B_1 ;
Lines A_2A_3 , B_2B_3 , and C_2C_3 concur at the Feuerbach Point F.

References

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3. D. Dekov, Corner Triangles, in this journal, 2007.
4. Eric W. Weisstein, MathWorld - A Wolfram Web Resource.
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