Problem 23
The Feuerbach Point is the Perspector of the Euler Triangle and the Triangle of the Internal Centers of Similitude of the Incircles and the Circumcircles of the Corner Triangles of the Orthocenter

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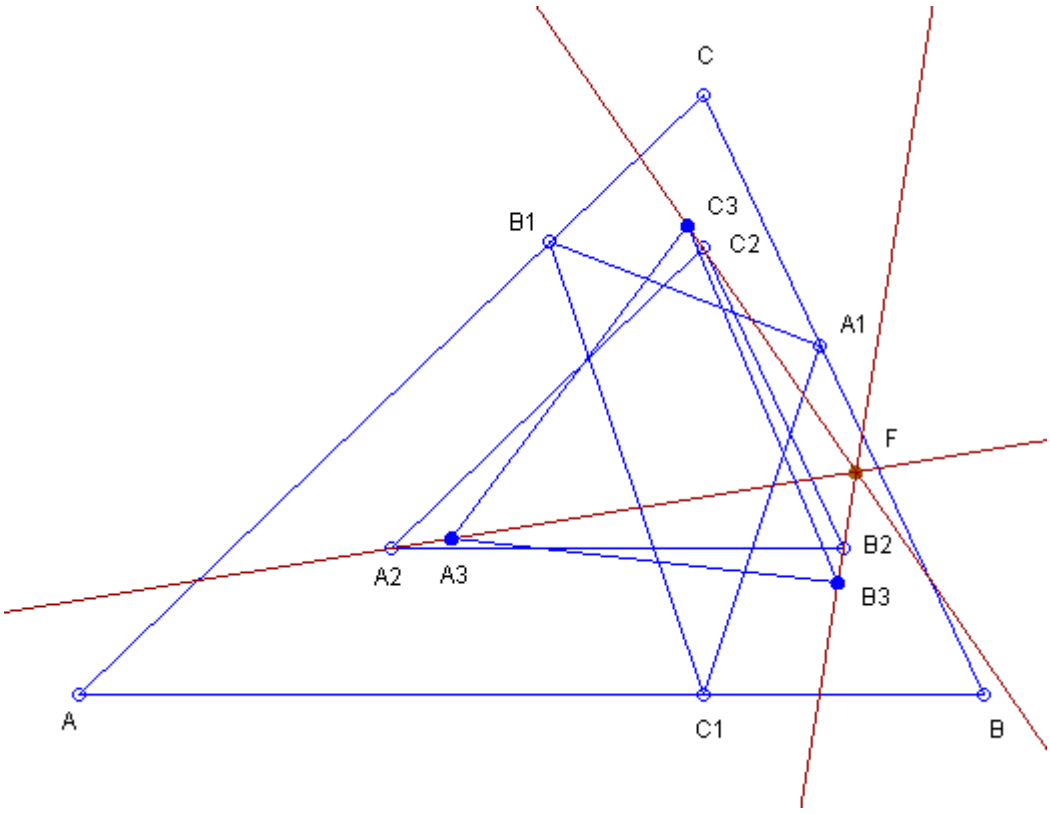
Prove the following computer-generated theorem:

THEOREM. The Feuerbach Point is the Perspector of the Euler Triangle and the Triangle of the Internal Centers of Similitude of the Incircles and the Circumcircles of the Corner Triangles of the Orthocenter.

The reader may find the definitions in [1-4].

See the Figure:



$A_1B_1C_1$ - Orthic Triangle = Cevian Triangle of the Orthocenter;
$A_2B_2C_2$ - Euler Triangle;
A_3 - Internal Center of Similitude of the Incircle and the Circumcircle of triangle AB_1C_1;
B_3 - Internal Center of Similitude of the Incircle and the Circumcircle of triangle BC_1A_1;
C_3 - Internal Center of Similitude of the Incircle and the Circumcircle of triangle CA_1B_1;
$A_3B_3C_3$ - Triangle of the Internal Centers of Similitude of the Incircles and the Circumcircles of the Corner Triangles of the Orthocenter;
The lines A_2A_3, B_2B_3, and C_2C_3 concur at the Feuerbach Point F, that is, the Feuerbach Point is the Perspector of the Euler Triangle and the Triangle of the Internal Centers of Similitude of the Incircles and the Circumcircles of the Corner Triangles of the Orthocenter.

References

1. Quim Castellsaguer, The Triangles Web, http://www.xtec.es/~qcastell/ttw/ttweng/portada.html
2. D. Dekov, Computer-Generated Encyclopedia of Euclidean Geometry, First Edition, 2006, available at the Web: http://www.dekovsoft.com/e1/.
3. D. Dekov, papers in this journal, 2006, 2007, 2008.
4. Eric W. Weisstein, MathWorld - A Wolfram Web Resource. http://mathworld.wolfram.com/