Computer-Generated Mathematics: Eleven Circles passing through the Parry Point

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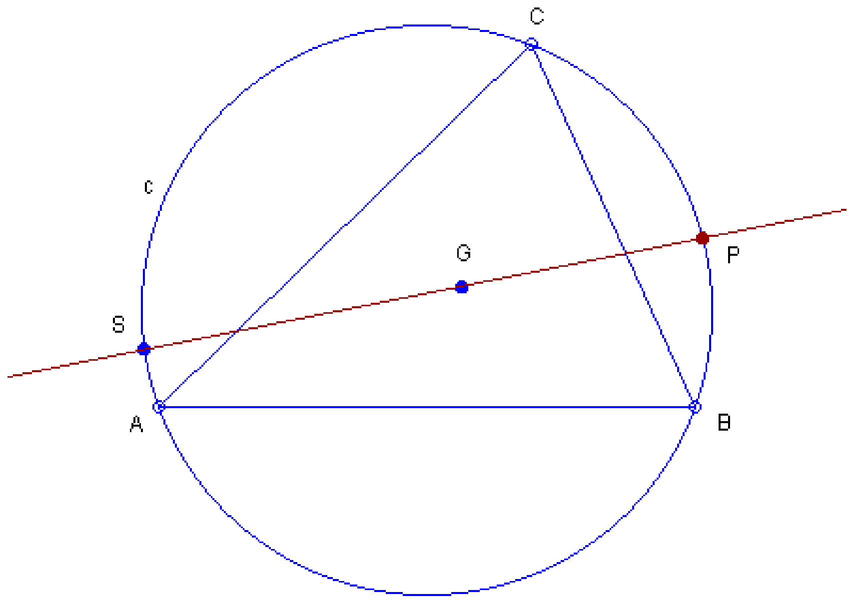
Abstract. By using the computer program "Machine for Questions and Answers", we find eleven circles passing through the Parry point of a given triangle.

"Within ten years a digital computer will discover and prove an important mathematical theorem." (Simon and Newell, 1958).

This is the famous prediction by Simon and Newell [1]. Now is 2008, 50 years later. The first computer program able easily to discover new deep mathematical theorems - The *Machine for Questions and Answers* (The *Machine*) [2,3] has been created by the author of this article, in 2006, that is, 48 years after the prediction. The Machine has discovered a few thousands new mathematical theorems. In 2006, the Machine has produced the first computer-generated encyclopedia [2].

In this paper we illustrate the use of the Machine for discovering new theorems in Euclidean Geometry. We use the Machine to find circles passing through the Parry point.

The reader may find the definitions in [2 - 4]. Given a triangle, the *Parry point* is the point of intersection of the circumcircle and the line passing through the Centroid and the Steiner point. See the Figure:



c - Circumcircle;
G - Centroid;
S - Steiner point;
P - Parry point = point of intersection of the circumcircle and the line passing through the Centroid and the Steiner point.

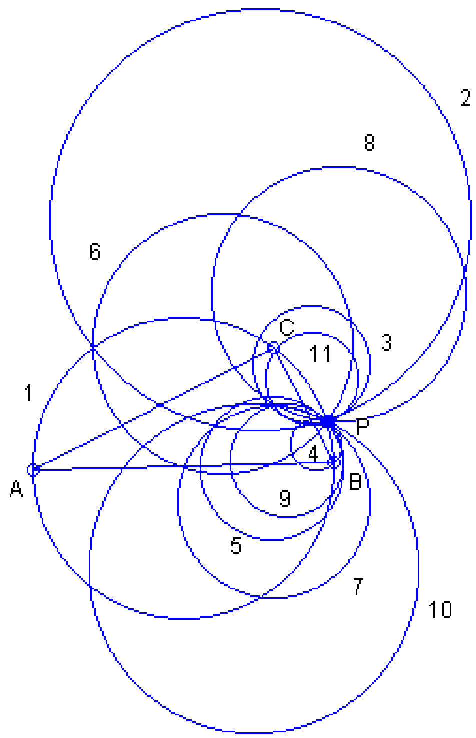
Given an object (circle, point, triangle, line, etc.) the Machine discovers theorems about the object. We ask the Machine to give examples of circles passing through the Parry point of a given triangle. We receive the following theorem:

THEOREM 1. The Parry Point lies on the following circles:

1. Circumcircle.
2. Parry Circle.
3. Parry Circle of the Pedal Triangle of the Outer Fermat Point.
4. Parry Circle of the Pedal Triangle of the Inner Fermat Point.
5. Parry Circle of the Fourth Brocard Triangle.
6. Circle with diameter connecting the Center of the Parry Circle and the Circumcenter.
7. Circle through the Centroid, the Circumcenter and the Symmedian Point.
8. Circle through the Symmedian Point, the Outer Fermat Point and the Second Isodynamic Point.
9. Circle through the Symmedian Point, the Inner Fermat Point and the First Isodynamic Point.
10. Circle through the Nine-Point Center, the Symmedian Point and the Exeter Point.
11. Circle through the Symmedian Point, the Kiepert Center and the Schoute Center.

Parry point lies on the circumcircle, from the definition of the Parry point. It is well-known that the Parry point lies on the Parry circle - see [4, Parry Circle]. We invite the reader to prove the rest of the theorem. Note that the Fourth Brocard triangle is known also as the D-triangle [4, D-Triangle], the Outer (Inner) Fermat point is known also as the First (Second)

Fermat point. Here we will illustrate the theorem. See the Figure:

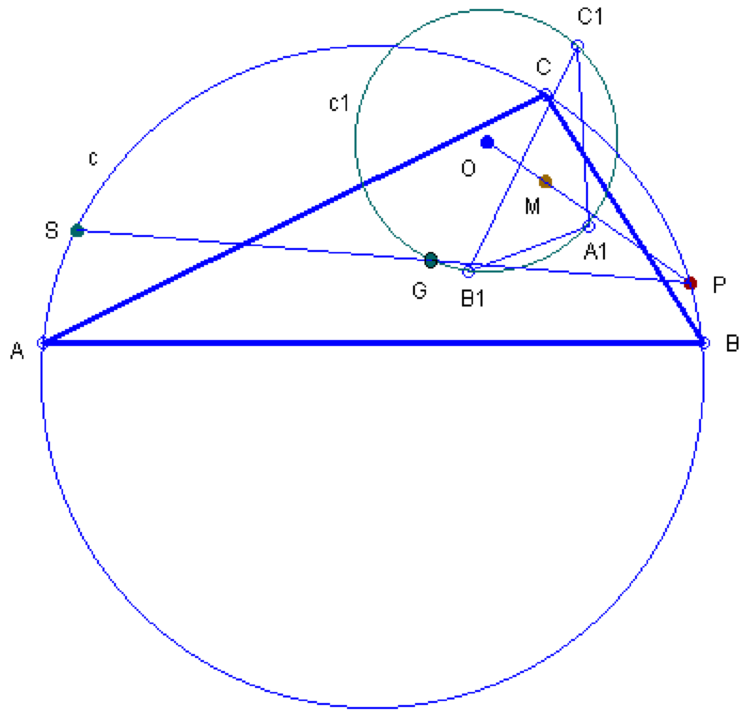


P - Parry Point;
1 - Circumcircle;
2 - Parry Circle;
3 - Parry Circle of the Pedal Triangle of the Outer Fermat Point;
4 - Parry Circle of the Pedal Triangle of the Inner Fermat Point;
5 - Parry Circle of the Fourth Brocard Triangle;
6 - Circle with diameter connecting the Center of the Parry Circle and the Circumcenter;
7 - Circle through the Centroid, the Circumcenter and the Symmedian Point;
8 - Circle through the Symmedian Point, the Outer Fermat Point and the Second Isodynamic Point;
9 - Circle through the Symmedian Point, the Inner Fermat Point and the First Isodynamic Point;
10 - Circle through the Nine-Point Center, the Symmedian Point and the Exeter Point;
11 - Circle through the Symmedian Point, the Kiepert Center and the Schoute Center;

I would like to note also the following theorem, discovered by the Machine:

THEOREM 2. The Parry Point is the Far-Out Point of the Fourth Brocard Triangle.

We invite the reader to prove the theorem. Recall that the Far-Out Point of the Fourth Brocard Triangle is the Inverse point of the Centroid of the Fourth Brocard Triangle in the Fourth Brocard Triangle. Here we will illustrate the theorem. See the Figure:



P - Parry Point;
G - Centroid;
S - Steiner Point;
c - Circumcircle;
$A_1B_1C_1$ - Fourth Brocard Triangle;
c1 - Circumcircle of the Fourth Brocard Triangle;
O - Circumcenter of the Fourth Brocard Triangle;
M - Centroid of the Fourth Brocard Triangle;
The Parry Point P is the Far-Out Point of the Fourth Brocard Triangle.

The figures in this note are produced by using the computer program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download at the Web. It is free and open source. Many thanks to Rene Grothmann for his wonderful program.

References

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