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Computer-Generated Mathematics: The Feuerbach Point

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Abstract. We illustrate the use of the computer program "Machine for Questions and Answers" for discovery of new mathematical theorems.

Keywords: computer-generated mathematics, Euclidean geometry

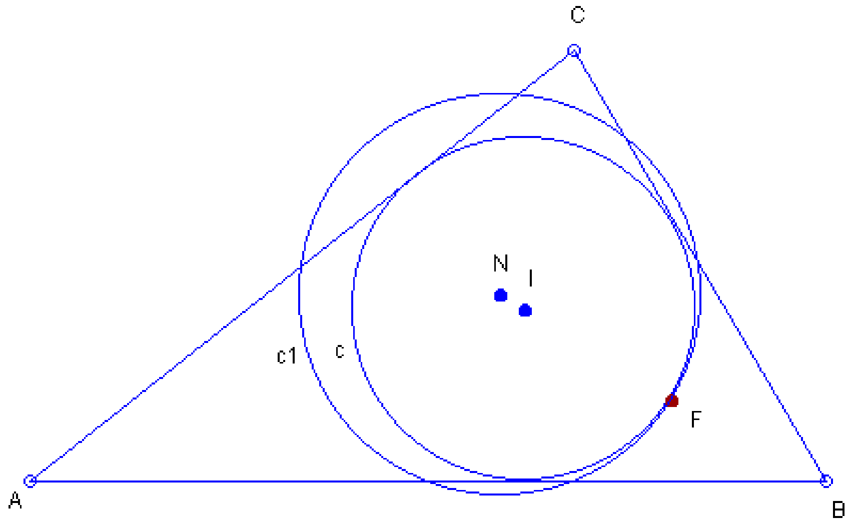
"Within ten years a digital computer will discover and prove an important mathematical theorem." (Simon and Newell, 1958).

This is the famous prediction by Simon and Newell [1]. Now is 2008, 50 years later. The first computer program able easily to discover new deep mathematical theorems - The Machine for Questions and Answers (The Machine) [2,3] has been created by the author of this article, in 2006, that is, 48 years after the prediction. The Machine has produced more than 90% of the possible new mathematical computer-generated theorems, since the prediction by Simon and Newell. In 2006, the Machine has produced the first computer-generated encyclopedia [2].

Given an object (point, triangle, circle, line, etc.), the Machine produces theorems related to the properties of the object. The theorems produced by the Machine are either known theorems, or possible new theorems. A "possible new" theorem means that the theorem is either known theorem, but the source is not available for the author of the Machine, or the theorem is a new theorem.

In 2008, in the Journal of Computer-Generated Euclidean Geometry were published 35 possible new theorems about the Feuerbach point, discovered by the Machine. These theorems are presented as problems for school students and teachers. Possibly, 20 to 30 of these theorems are new theorems. Below we present three additional possible new theorems about the Feuerbach point, discovered by the Machine. We invite the reader to select the new theorems and to prove them.

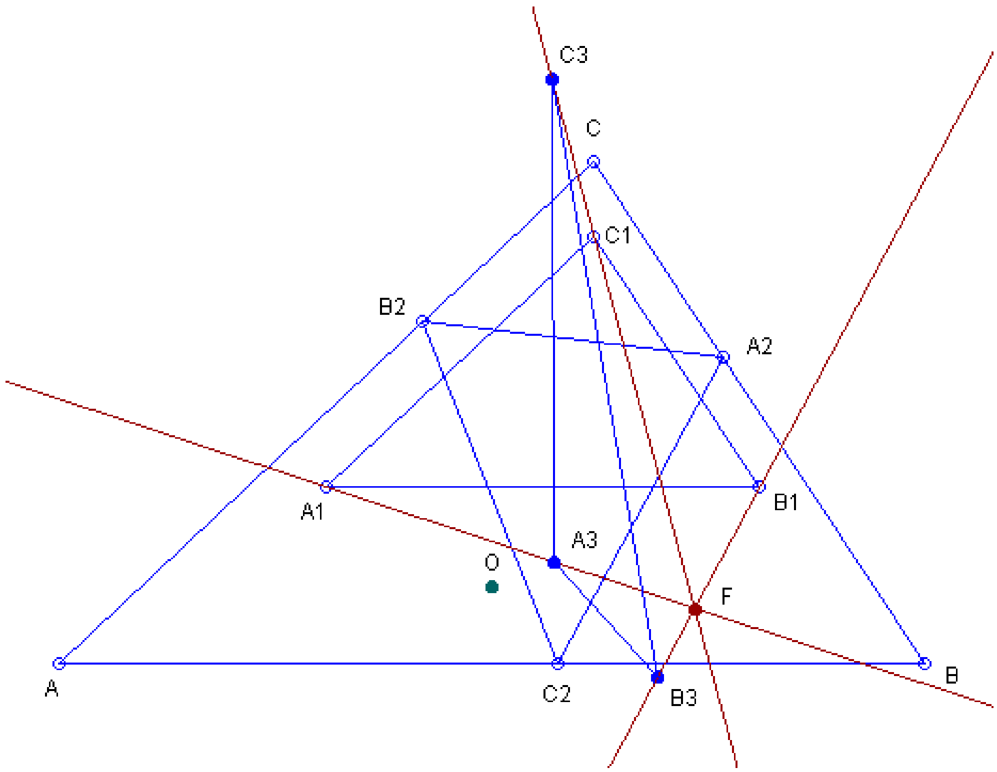
The reader may find the definitions used in this paper, in [2-4]. Recall that the *Feuerbach Point* of a triangle is the point of tangency of the Incircle and Nine-Point circle of the triangle. ([4, Feuerbach Point]). See the Figure:



triangle ABC - an arbitrary triangle;
circle c - Incircle;
I - Incenter = Center of the Incircle;
circle c1 - Nine-Point Circle = Circumcircle of the Medial Triangle;
N - Nine-Point Center = Center of the Nine-Point Circle;
F - Feuerbach Point = Tangency point of the Incircle and Nine-Point Circle.

THEOREM 1. The Feuerbach Point is the Perspector of the Euler Triangle and the Triangle of reflections of the Circumcenter in the sides of the Intouch Triangle.

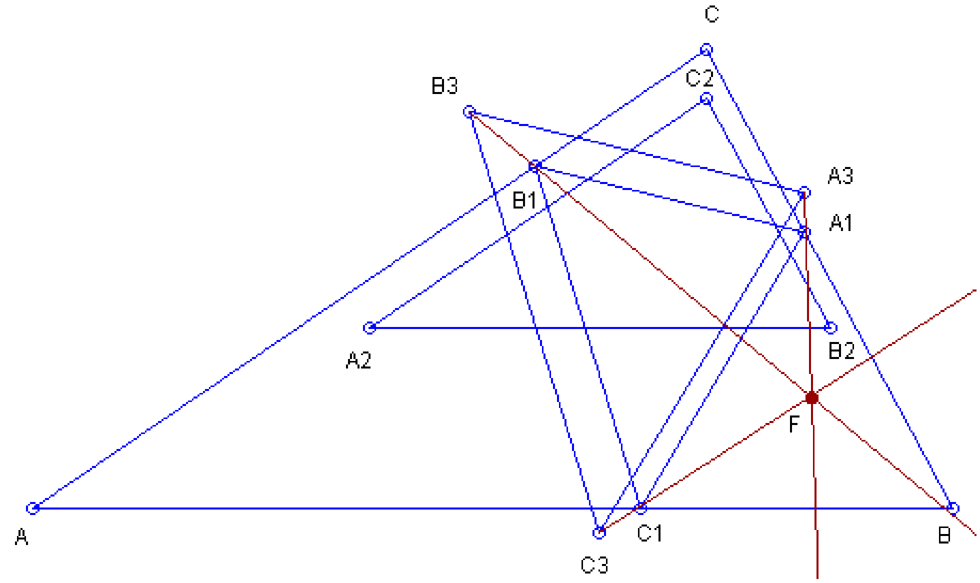
See the Figure:



$A_1B_1C_1$ - Euler Triangle;
$A_2B_2C_2$ - Intouch Triangle = Cevian Triangle of the Gergonne point;
O - Circumcenter;
A_3 - reflection of the Circumcenter O in the sideline B_2C_2;
B_3 - reflection of the Circumcenter O in the sideline C_2A_2;
C_3 - reflection of the Circumcenter O in the sideline A_2B_2;
$A_3B_3C_3$ - Triangle of reflections of the Circumcenter in the sides of the Intouch Triangle;
Lines A_1A_3, B_1B_3 and C_1C_3 concur in the Feuerbach Point F, that is, The Feuerbach Point is the Perspector of the Euler Triangle and the Triangle of reflections of the Circumcenter in the sides of the Intouch Triangle.

THEOREM 2. The Feuerbach Point is the Homothetic Center of the Intouch Triangle and the Circum-Incentral Triangle of the Euler Triangle.

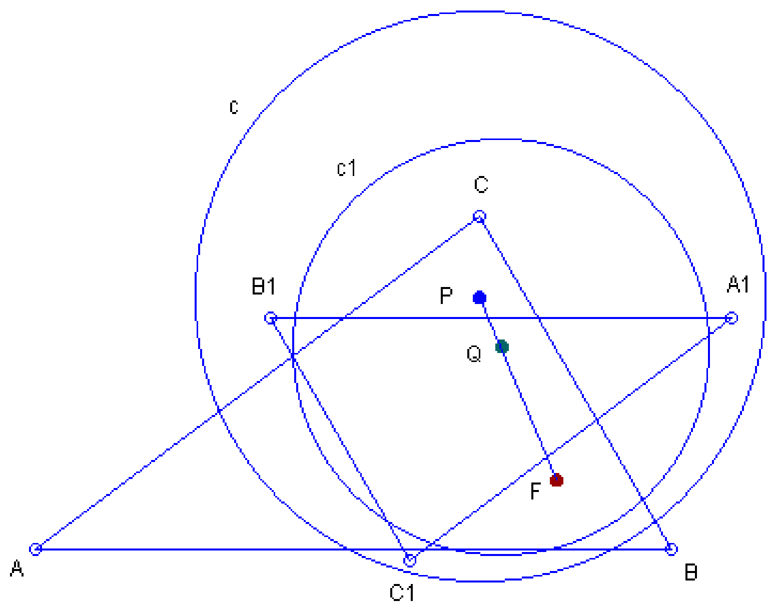
See the Figure:



$A_1B_1C_1$ - Intouch Triangle;
$A_2B_2C_2$ - Euler Triangle;
$A_3B_3C_3$ - Circum-Incentral Triangle of the Euler Triangle;
The Feuerbach Point F is the Homothetic Center of the Intouch Triangle and the Circum-Incentral Triangle of the Euler Triangle

THEOREM 3. The Feuerbach Point is the External Center of Similitude of the First Droz-Farny Circle and the Second Droz-Farny Circle of the Outer Yff Triangle.

See the Figure:



c - First Droz-Farny Circle;
$A_1B_1C_1$ - Outer Yff Triangle;
c1 - Second Droz-Farny Circle of the Outer Yff Triangle;
The Feuerbach Point F is the External Center of Similitude of the First Droz-Farny Circle and the Second Droz-Farny Circle of the Outer Yff Triangle.

Thanks

The figures in this note are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download at the Web. It is free and open source. Many thanks to Rene Grothmann for his wonderful program.

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