Computer-Generated Mathematics: The Kiepert-Parry Point

Deko Dekov

Abstract. We illustrate the use of the computer program "Machine for Questions and Answers" (The Machine) for discovering of new theorems in Euclidean Geometry. The paper contains more than 40 new theorems about the Kiepert-Parry Point, discovered by the Machine.

Keywords: computer-generated mathematics, Euclidean geometry

"Within ten years a digital computer will discover and prove an important mathematical theorem." (Simon and Newell, 1958).

This is the famous prediction by Simon and Newell [1]. Now is 2008, 50 years later. The first computer program able easily to discover new deep mathematical theorems - The *Machine for Questions and Answers* (The *Machine*) [2,3] has been created by the author of this article, in 2006, that is, 48 years after the prediction. The Machine has discovered a few thousands new mathematical theorems, that is, more than 90% of the new mathematical computer-generated theorems since the prediction by Simon and Newell. In 2006, the Machine has produced the first computer-generated encyclopedia [2].

Given an object (point, triangle, circle, line, etc.), the Machine produces theorems related to the properties of the object. The theorems produced by the Machine are either known theorems, or possible new theorems. A *possible new* theorem means that the theorem is either known theorem, but the source is not available for the author of the Machine, or the theorem is a new theorem. I expect that approximately 75 to 90% of the possible new theorems are new theorems. Although the Machine works completely independent from the human thinking, the theorems produced are surprisingly similar to the theorems produced by the people.

The advantages in using the Machine are as follows. (1) It is not necessary we to be inventive and even it is not necessary we to think. It is enough we to click with the mouse in order to obtain the theorems. (2) The Machine produces complete knowledge. If there exists a theorem related to the object, the Machine discovers the theorem. (3) The people make errors, but the computers do not make errors.

In this paper we illustrate the use of the Machine. We present lists with theorems about the Kiepert-Parry Point, discovered by the Machine. The lists include a few well known theorems, as well as possible new theorems. I expect that new theorems are approximately

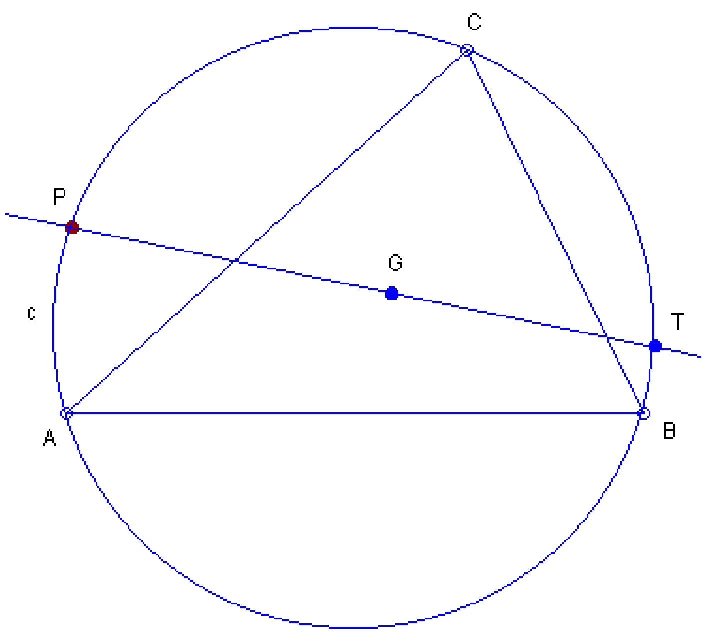
40 to 50 of theorems included into the lists. Hence, the paper contains more than 40 new theorems about the Kiepert-Parry Point, discovered by the Machine. The reader is invited to select the new theorems and to prove them. There are a few additional lists of theorems about the Kiepert-Parry Point, produced by the Machine, which are not included in this paper.

Pedagogical use of the Machine

The Machine could be useful for students and teachers mainly in these directions: (1) The Machine could produce an encyclopedia of Euclidean geometry suitable for school students and teachers. (2) The Machine will give to the school students and teachers the possibility to discover new theorems. (3) The interactive use of the Machine will give to the school students and teachers the possibility to investigate in depth selected problems. (4) The Machine will give to the teachers the possibility easily to produce problems and theorems for textbooks, for use in the classroom, for home works, etc. (5) The Machine will give to the school students and teachers the possibility better to understand the abilities of computers to discover new theorems.

The Kiepert-Parry Point

The reader may find the definitions in [2-4]. Recall the definition of the Kiepert-Parry Point. Given a triangle, the *Kiepert-Parry point* is the point of intersection of the circumcircle and the line passing through the Centroid and the Tarry point. See the Figure:



c - Circumcircle;
G - Centroid;
T - Tarry point;
P - Kiepert-Parry point = point of intersection of the circumcircle and the line passing

through the Centroid and the Tarry point.

The Kiepert-Parry point is also known as the *Focus of the Kiepert Parabola*. See [4, Kiepert Parabola].

Roles of the Kiepert-Parry Point

The list below contains a few theorems about the Roles of the Kiepert-Parry Point, discovered by the Machine.

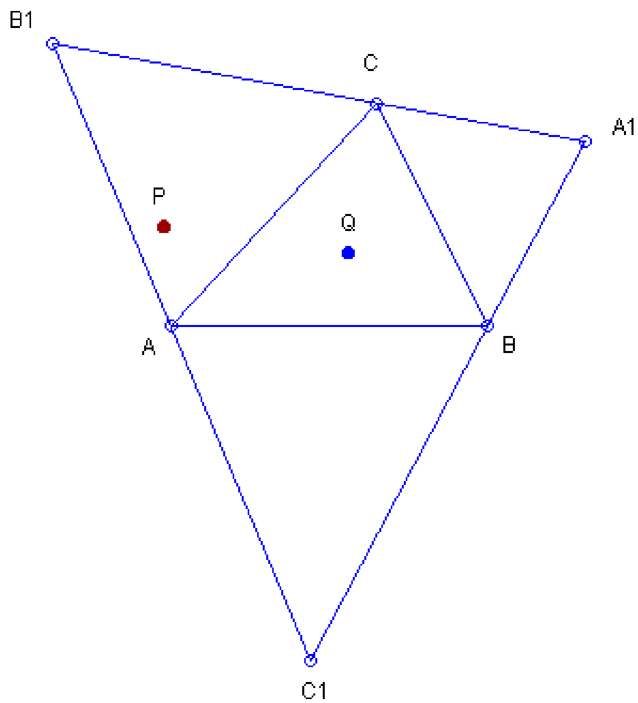
1. The Kiepert-Parry Point is the Kiepert Center of the Antipedal Triangle of the First Isodynamic Point.
2. The Kiepert-Parry Point is the Parry Point of the Circumcevian Triangle of the Symmedian Point.
3. The Kiepert-Parry Point is the Tarry Point of the Circumcevian Triangle of the Outer Fermat Point.
4. The Kiepert-Parry Point is the Tarry Point of the Circumcevian Triangle of the Inner Fermat Point.
5. The Kiepert-Parry Point is the Far-Out Point of the First Brocard Triangle.
6. The Kiepert-Parry Point is the Far-Out Point of the Desmic Mate the Second Brocard Triangle.
7. The Kiepert-Parry Point is the Far-Out Point of the Desmic Mate the Third Brocard Triangle.
8. The Kiepert-Parry Point is the Homothetic Center of the Circum-Orthic Triangle and the Euler Triangle of the Tangential Triangle.
9. The Kiepert-Parry Point is the Perspector of the Johnson Triangle and the Euler Triangle of the Tangential Triangle.
10. The Kiepert-Parry Point is the Homothetic Center of the Tangential Triangle and the Anticomplementary Triangle of the Orthic Triangle of the Anticomplementary Triangle.
11. The Kiepert-Parry Point is the Perspector of the Circum-Incentral Triangle and the Triangle of the Kosnita Points of the Anticevian Corner Triangles of the Incenter.
12. The Kiepert-Parry Point is the Perspector of the Circum-Medial Triangle and the Triangle of the Symmedian Points of the Anticevian Corner Triangles of the Centroid.
13. The Kiepert-Parry Point is the Perspector of the Circum-Medial Triangle and the Triangle of the Centroids of the Anticevian Corner Triangles of the Symmedian Point.
14. The Kiepert-Parry Point is the Perspector of the Circum-Medial Triangle and the Triangle of the reflections of the Symmedian Point in the vertices of the Medial Triangle.
15. The Kiepert-Parry Point is the Perspector of the Circum-Orthic Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Orthocenter.
16. The Kiepert-Parry Point is the Perspector of the Circum-Orthic Triangle and the Triangle of the Circumcenters of the Anticevian Corner Triangles of the Centroid.
17. The Kiepert-Parry Point is the Perspector of the Circum-Orthic Triangle and the Triangle of the Centroids of the Anticevian Corner Triangles of the Circumcenter.
18. The Kiepert-Parry Point is the Perspector of the Circum-Orthic Triangle and the Triangle of the Orthocenters of the Anticevian Corner Triangles of the Symmedian Point.

19. The Kiepert-Parry Point is the Perspector of the Circum-Orthic Triangle and the Triangle of the reflections of the Circumcenter in the vertices of the Medial Triangle.
20. The Kiepert-Parry Point is the Perspector of the Johnson Triangle and the Triangle of the Centroids of the Anticevian Corner Triangles of the Circumcenter.

We illustrate a few of the above theorems. We invite the reader to select the new theorems and to prove them.

Theorem 1. The Kiepert-Parry Point is the Kiepert Center of the Antipedal Triangle of the First Isodynamic Point.

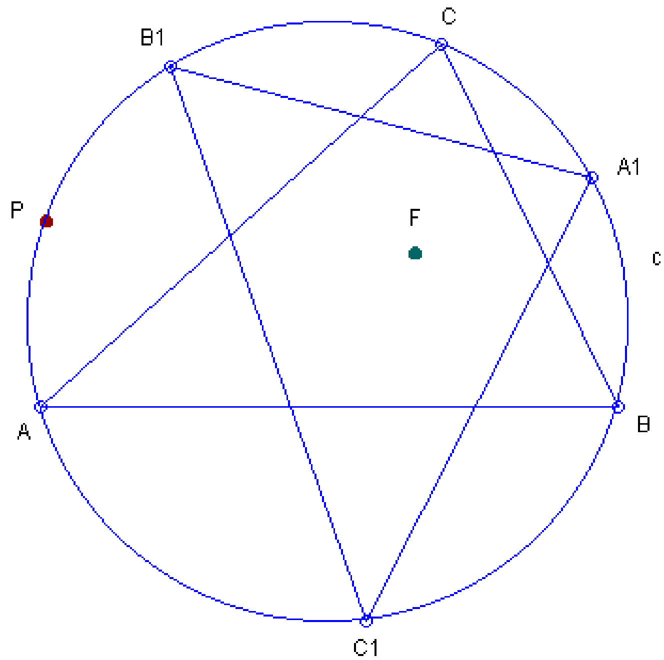
See the Figure:



P - Kiepert-Parry Point;
Q - First Isodynamic Point;
$A_1B_1C_1$ - Antipedal Triangle of the First Isodynamic Point;
The Kiepert-Parry Point P is the Kiepert Center of the Antipedal Triangle of the First Isodynamic Point.

Theorem 3. The Kiepert-Parry Point is the Tarry Point of the Circumcevian Triangle of the Outer Fermat Point.

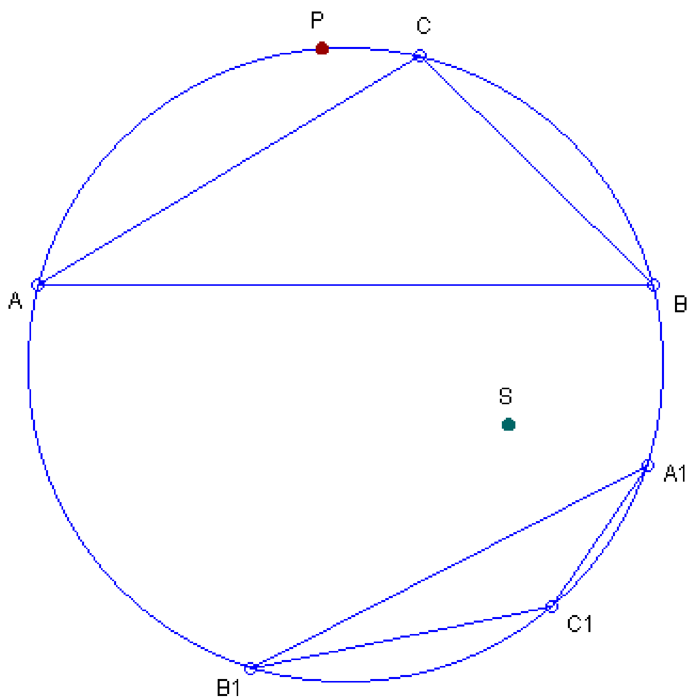
See the Figure:



P - Kiepert-Parry Point;
F - Outer Fermat Point;
$A_1B_1C_1$ - Circumcevian Triangle of the Outer Fermat Point;
The Kiepert-Parry Point is the Tarry Point of the Circumcevian Triangle of the Outer Fermat Point.

Theorem 4. The Kiepert-Parry Point is the Tarry Point of the Circumcevian Triangle of the Inner Fermat Point.

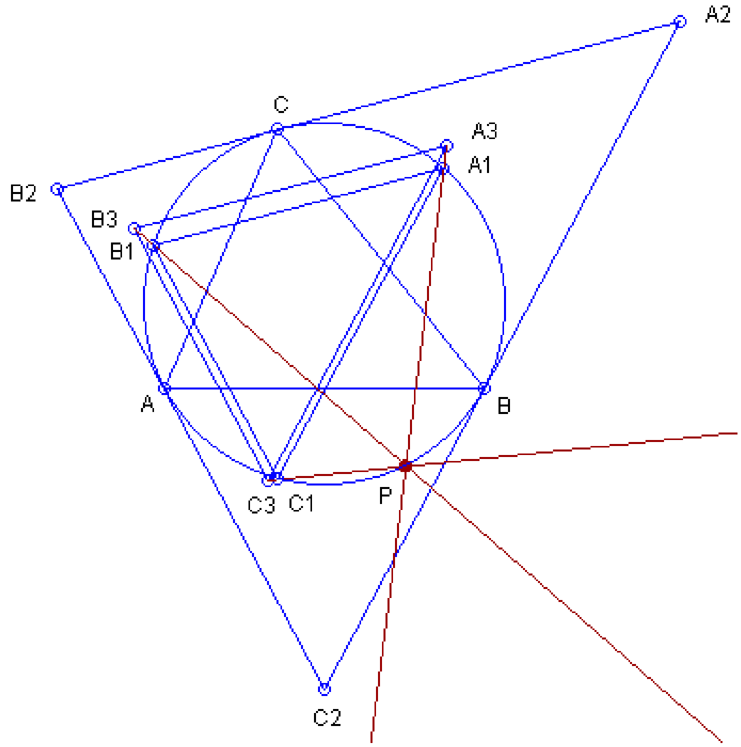
See the Figure:



P - Kiepert-Parry Point;
S - Inner Fermat Point;
$A_1B_1C_1$ - Circumcevian Triangle of the Inner Fermat Point;
The Kiepert-Parry Point is the Tarry Point of the Circumcevian Triangle of the Inner Fermat Point.

Theorem 8. The Kiepert-Parry Point is the Homothetic Center of the Circum-Orthic Triangle and the Euler Triangle of the Tangential Triangle.

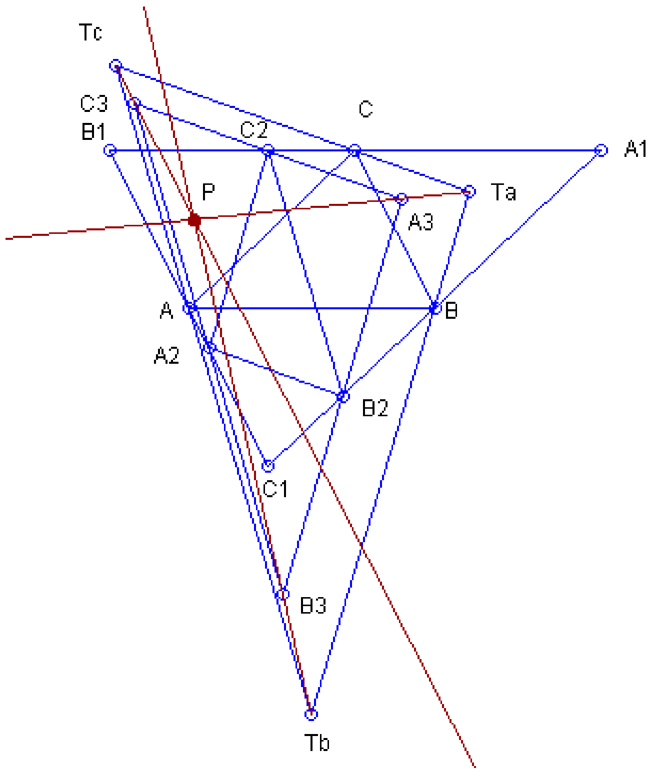
See the Figure:



P - Kiepert-Parry Point;
$A_1B_1C_1$ - Circum-Orthic Triangle;
$A_2B_2C_2$ - Tangential Triangle;
$A_3B_3C_3$ - Euler Triangle of the Tangential Triangle;
The Kiepert-Parry Point P is the Homothetic Center of the Circum-Orthic Triangle and the Euler Triangle of the Tangential Triangle.

Theorem 10. The Kiepert-Parry Point is the Homothetic Center of the Tangential Triangle and the Anticomplementary Triangle of the Orthic Triangle of the Anticomplementary Triangle.

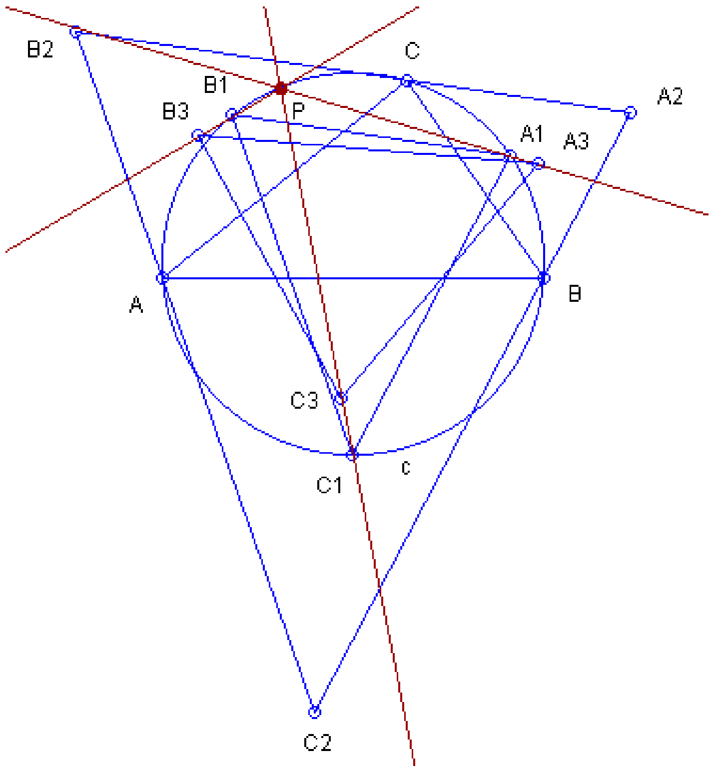
See the Figure:



P - Kiepert-Parry Point;
$T_aT_bT_c$ - Tangential Triangle;
$A_1B_1C_1$ - Anticomplementary Triangle;
$A_2B_2C_2$ - Orthic Triangle of the Anticomplementary Triangle;
$A_3B_3C_3$ - Anticomplementary Triangle of the Orthic Triangle of the Anticomplementary Triangle;
The Kiepert-Parry Point is the Homothetic Center of the Tangential Triangle and the Anticomplementary Triangle of the Orthic Triangle of the Anticomplementary Triangle.

Theorem 11. The Kiepert-Parry Point is the Perspector of the Circum-Incentral Triangle and the Triangle of the Kosnita Points of the Anticevian Corner Triangles of the Incenter.

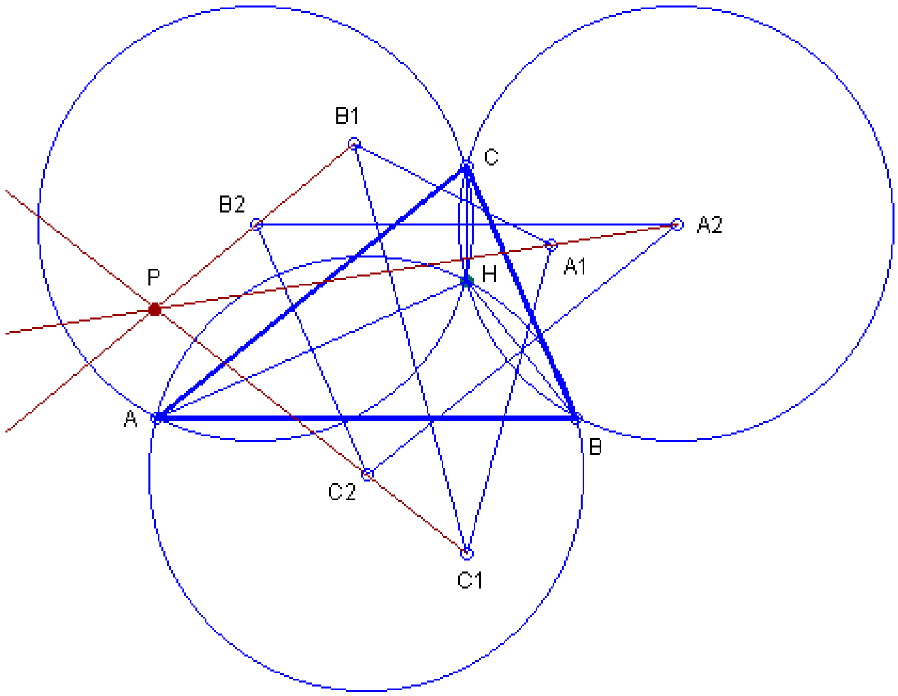
See the Figure:



P - Kiepert-Parry Point;
$A_1B_1C_1$ - Circum-Incentral Triangle;
$A_2B_2C_2$ - Anticevian Triangle of the Incenter = Anticomplementary Triangle;
A_3 - Kosnita Point of triangle BCA_2;
B_3 - Kosnita Point of triangle CAB_2;
C_3 - Kosnita Point of triangle ABC_2;
$A_3B_3C_3$ - Triangle of the Kosnita Points of the Anticevian Corner Triangles of the Incenter;
Lines A_1A_3, B_1B_3 and C_1C_3 concur in point P, that is, the Kiepert-Parry Point P is the Perspector of the Circum-Incentral Triangle and the Triangle of the Kosnita Points of the Anticevian Corner Triangles of the Incenter.

Theorem 15. The Kiepert-Parry Point is the Perspector of the Circum-Orthic Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Orthocenter.

See the Figure:



P - Kiepert-Parry Point;
H - Orthocenter;
$A_1B_1C_1$ - Circum-Orthic Triangle;
$A_2B_2C_2$ - Triangle of the Circumcenters of the Triangulation Triangles of the Orthocenter;
The Kiepert-Parry Point is the Perspector of the Circum-Orthic Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Orthocenter.

Circles passing through the Kiepert-Parry Point

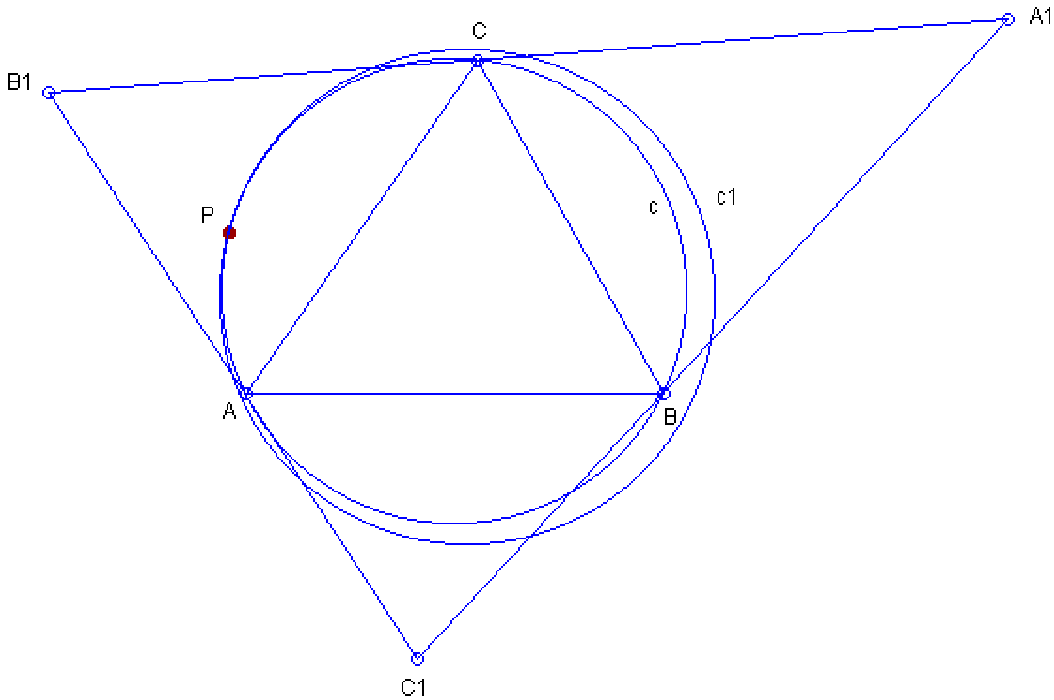
The list below contains a few theorems about the Kiepert-Parry Point, discovered by the Machine.

1. The Kiepert-Parry Point lies on the Nine-Point Circle of the Anticevian Triangle of the Circumcenter.
2. The Kiepert-Parry Point lies on the Nine-Point Circle of the Tangential Triangle.
3. The Kiepert-Parry Point lies on the Nine-Point Circle of the Anticevian Triangle of the Malfatti-Moses Point.
4. The Kiepert-Parry Point lies on the Nine-Point Circle of the Antipedal Triangle of the Symmedian Point.
5. The Kiepert-Parry Point lies on the Nine-Point Circle of the Antipedal Triangle of the First Isodynamic Point.
6. The Kiepert-Parry Point lies on the Moses Circle of the Antipedal Triangle of the First Isodynamic Point.
7. The Kiepert-Parry Point lies on the Nine-Point Circle of the Antipedal Triangle of the Second Isodynamic Point.
8. The Kiepert-Parry Point lies on the Moses Circle of the Antipedal Triangle of the Second Isodynamic Point.

9. The Kiepert-Parry Point lies on the Nine-Point Circle of the Antipedal Triangle of the Kosnita Point.
10. The Kiepert-Parry Point lies on the Nine-Point Circle of the Antipedal Triangle of the Prasolov Point.
11. The Kiepert-Parry Point lies on the Nine-Point Circle of the Antipedal Triangle of the Evans Perspector.
12. The Kiepert-Parry Point lies on the Parry Circle of the First Brocard Triangle.
13. The Kiepert-Parry Point lies on the Circle centered at the Schoute Center through the Far-Out Point.
14. The Kiepert-Parry Point lies on the Circle centered at the First Beltrami Point through the Far-Out Point.
15. The Kiepert-Parry Point lies on the Circle centered at the Second Beltrami Point through the Far-Out Point.
16. The Kiepert-Parry Point lies on the Circle with diameter from the Center of the Parry Circle to the Circumcenter.
17. The Kiepert-Parry Point lies on the Circle through the Brocard Midpoint, the Centroid and the Third Power Point.
18. The Kiepert-Parry Point lies on the Circle through the External Center of Similitude of the Incircle and the Circumcircle, the Internal Center of Similitude of the Incircle and the Circumcircle and the Schiffler Point.

It well known that the Kiepert-Parry point lies on the circumcircle and on the Parry circle. The Machine discovers 18 additional circles passing through the Kiepert-Parry point. Hence, now we know 20 different circles passing through the Kiepert-Parry Point. We illustrate a few the above theorems. We invite the reader to select the new theorems and to prove them.

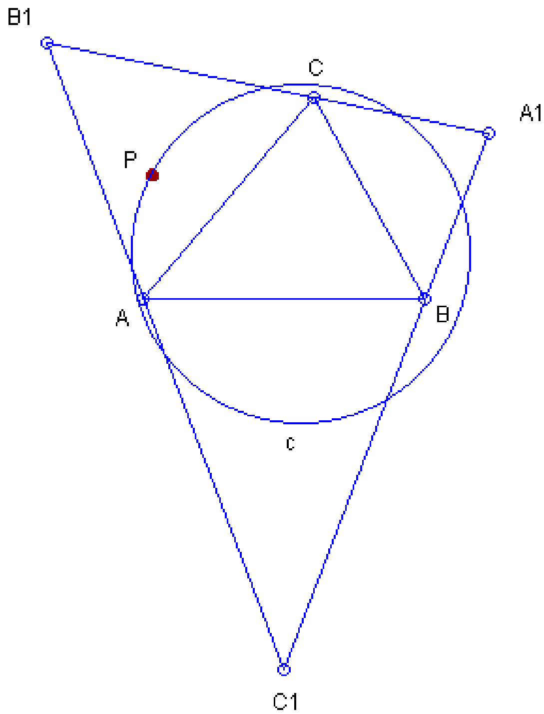
Theorem 1. The Kiepert-Parry Point lies on the Nine-Point Circle of the Anticevian Triangle of the Circumcenter.



P - Kiepert-Parry Point;
c - Circumcircle;
$A_1B_1C_1$ - Anticevian Triangle of the Circumcenter;
c1 - Nine-Point Circle of triangle $A_1B_1C_1$;
The Kiepert-Parry Point P lies on the Nine-Point Circle of the Anticevian Triangle of the Circumcenter.

Theorem 2. The Kiepert-Parry Point lies on the Nine-Point Circle of the Tangential Triangle.

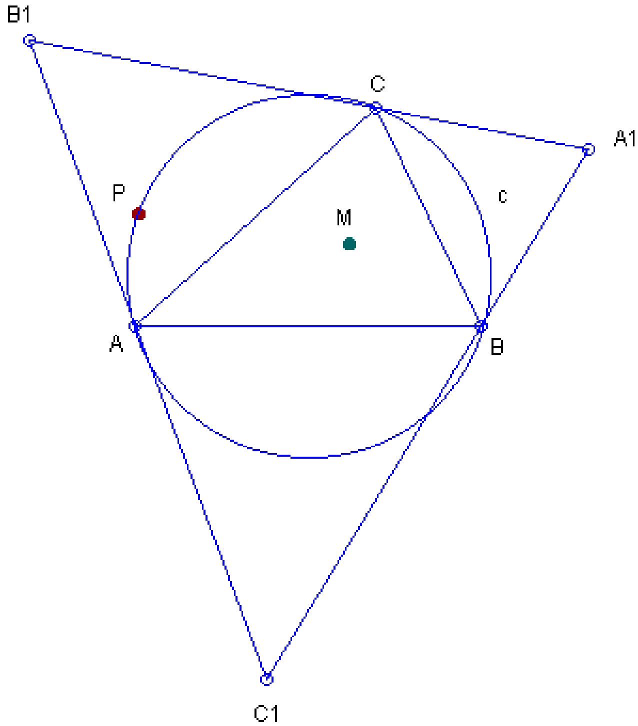
See the Figure:



P - Kiepert-Parry Point;
$A_1B_1C_1$ - Tangential Triangle;
c - Nine-Point Circle of the Tangential Triangle;
The Kiepert-Parry Point lies on the Nine-Point Circle of the Tangential Triangle.

Theorem 3. The Kiepert-Parry Point lies on the Nine-Point Circle of the Anticevian Triangle of the Malfatti-Moses Point.

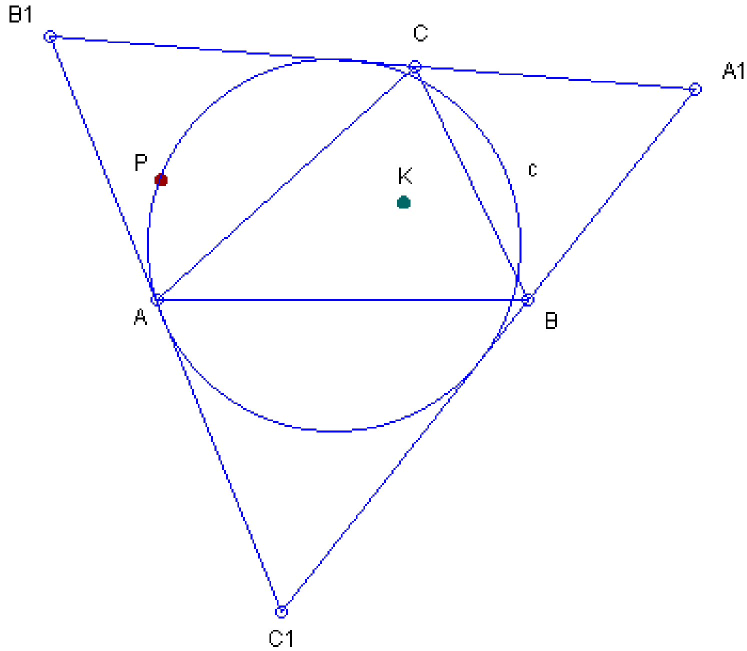
See the Figure:



P - Kiepert-Parry Point;
M - Malfatti-Moses Point;
$A_1B_1C_1$ - Anticevian Triangle of the Malfatti-Moses Point;
c - Nine-Point Circle of the Anticevian Triangle of the Malfatti-Moses Point;
The Kiepert-Parry Point lies on the Nine-Point Circle of the Anticevian Triangle of the Malfatti-Moses Point.

Theorem 4. The Kiepert-Parry Point lies on the Nine-Point Circle of the Antipedal Triangle of the Symmedian Point.

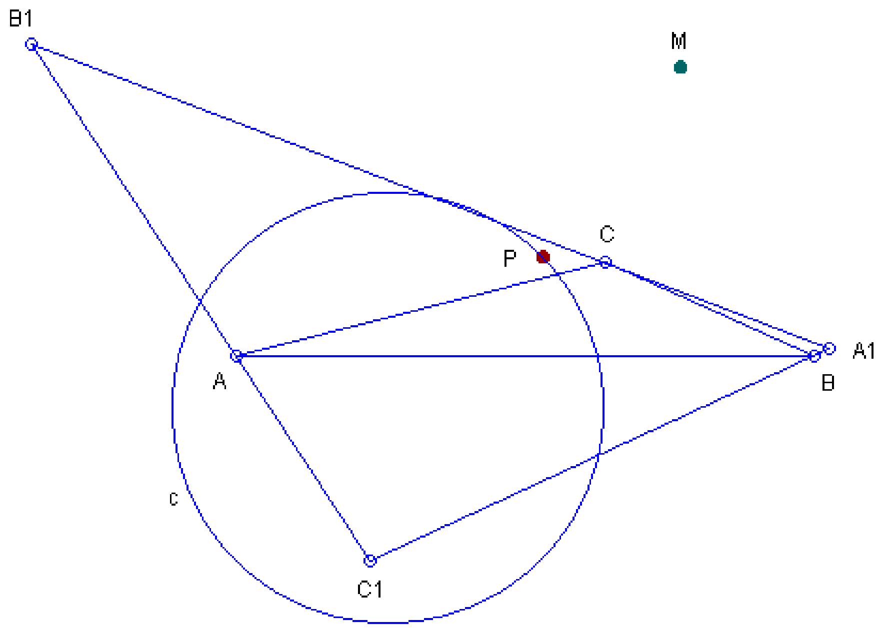
See the Figure:



P - Kiepert-Parry Point;
K - Symmedian Point;
$A_1B_1C_1$ - Antipedal Triangle of the Symmedian Point;
c - Nine-Point Circle of the Antipedal Triangle of the Symmedian Point;
The Kiepert-Parry Point lies on the Nine-Point Circle of the Antipedal Triangle of the Symmedian Point.

Theorem 7. The Kiepert-Parry Point lies on the Nine-Point Circle of the Antipedal Triangle of the Second Isodynamic Point.

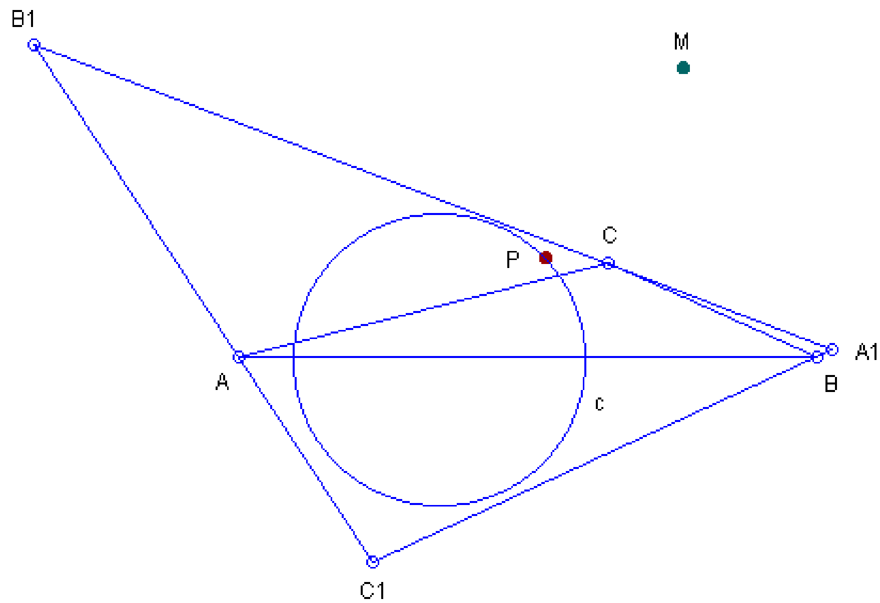
See the Figure:



P - Kiepert-Parry Point;
M - Second Isodynamic Point;
$A_1B_1C_1$ - Antipedal Triangle of the Second Isodynamic Point;
c - Nine-Point Circle of the Antipedal Triangle of the Second Isodynamic Point;
The Kiepert-Parry Point lies on the Nine-Point Circle of the Antipedal Triangle of the Second Isodynamic Point.

Theorem 8. The Kiepert-Parry Point lies on the Moses Circle of the Antipedal Triangle of the Second Isodynamic Point.

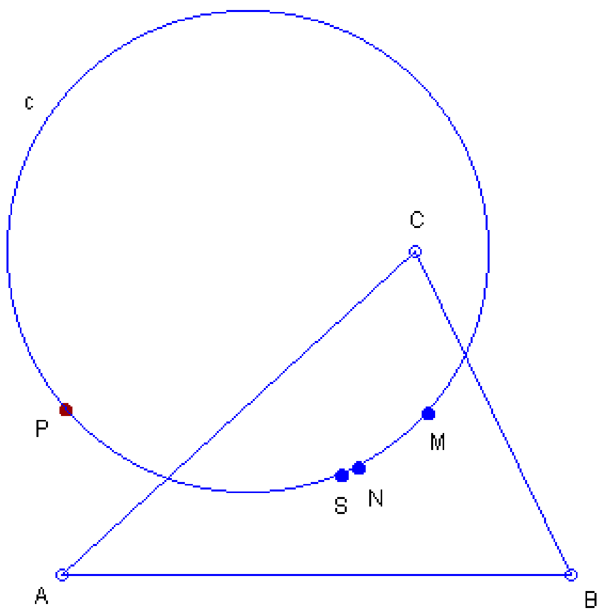
See the Figure:



P - Kiepert-Parry Point;
M - Second Isodynamic Point;
$A_1B_1C_1$ - Antipedal Triangle of the Second Isodynamic Point;
c - Moses Circle of the Antipedal Triangle of the Second Isodynamic Point;
The Kiepert-Parry Point lies on the Moses Circle of the Antipedal Triangle of the Second Isodynamic Point.

Theorem 18. The Kiepert-Parry Point lies on the Circle through the External Center of Similitude of the Incircle and the Circumcircle, the Internal Center of Similitude of the Incircle and the Circumcircle and the Schiffler Point.

See the Figure:



P - Kiepert-Parry Point;
M - External Center of Similitude of the Incircle and the Circumcircle;
N - Internal Center of Similitude of the Incircle and the Circumcircle;
S - Schiffler Point;
c - Circle through M, N and S;
The Kiepert-Parry Point lies on the Circle through the External Center of Similitude of the Incircle and the Circumcircle, the Internal Center of Similitude of the Incircle and the Circumcircle and the Schiffler Point.

Theorems about Kiepert-Parry Points of triangles

Here we give discovered by the Machine theorems about the Kiepert-Parry Points of different triangles.

1. The Kiepert-Parry Point of the Medial Triangle of the Fuhrmann Triangle is the Spieker Center.
2. The Kiepert-Parry Point of the Anticomplementary Triangle of the Fuhrmann Triangle is the Bevan Point.
3. The Kiepert-Parry Point of the Circum-Incentral Triangle of the Euler Triangle is the First Feuerbach Point.
4. The Kiepert-Parry Point of the Fuhrmann Triangle of the Incentral Triangle is the Orthocenter of the Incentral Triangle.
5. The Kiepert-Parry Point of the Fuhrmann Triangle of the Medial Triangle is the Circumcenter.
6. The Kiepert-Parry Point of the Fuhrmann Triangle of the Orthic Triangle is the Orthocenter of the Orthic Triangle.
7. The Kiepert-Parry Point of the Fuhrmann Triangle of the Symmedial Triangle is the Orthocenter of the Symmedial Triangle.
8. The Kiepert-Parry Point of the Fuhrmann Triangle of the Intouch Triangle is the

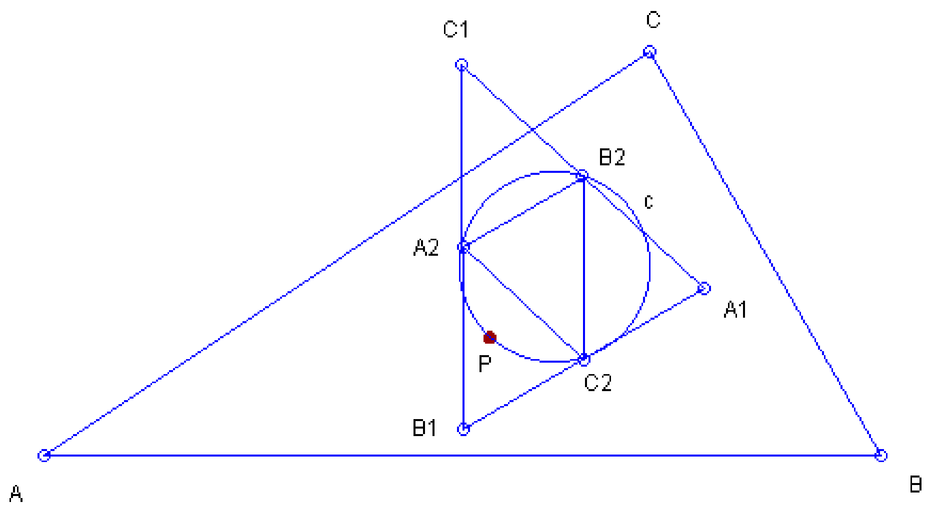
Orthocenter of the Intouch Triangle.

9. The Kiepert-Parry Point of the Fuhrmann Triangle of the Extouch Triangle is the Orthocenter of the Extouch Triangle.
10. The Kiepert-Parry Point of the Fuhrmann Triangle of the Excentral Triangle is the Incenter.
11. The Kiepert-Parry Point of the Fuhrmann Triangle of the Anticomplementary Triangle is the de Longchamps Point.
12. The Kiepert-Parry Point of the Fuhrmann Triangle of the Tangential Triangle is the Orthocenter of the Tangential Triangle.
13. The Kiepert-Parry Point of the Fuhrmann Triangle of the Circum-Incentral Triangle is the Incenter.
14. The Kiepert-Parry Point of the Fuhrmann Triangle of the Outer Vecten Triangle is the Outer Vecten Point.
15. The Kiepert-Parry Point of the Fuhrmann Triangle of the Inner Vecten Triangle is the Inner Vecten Point.
16. The Kiepert-Parry Point of the Fuhrmann Triangle of the Euler Triangle is the Orthocenter.
17. The Kiepert-Parry Point of the Euler Triangle of the Fuhrmann Triangle is the Midpoint of the Incenter and the Orthocenter.
18. The Kiepert-Parry Point of the Fuhrmann Triangle of the Fuhrmann Triangle is the Incenter.
19. The Kiepert-Parry Point of the Johnson Triangle of the Fuhrmann Triangle is the Circumcenter.
20. The Kiepert-Parry Point of the Fuhrmann Triangle of the Mid-Arc Triangle is the Incenter of the Intouch Triangle.
21. The Kiepert-Parry Point of the Fuhrmann Triangle of the Hexyl Triangle is the Bevan Point.
22. The Kiepert-Parry Point of the Fuhrmann Triangle of the Johnson Triangle is the Circumcenter.
23. The Kiepert-Parry Point of the Fuhrmann Triangle of the Inner Yff Triangle is the Center of the Inner Johnson-Yff Circle.
24. The Kiepert-Parry Point of the Fuhrmann Triangle of the Outer Yff Triangle is the Center of the Outer Johnson-Yff Circle.

It is known that the Kiepert-Parry Point of the Intouch Triangle is the First Feuerbach Point. Also, it is known that the Kiepert-Parry Point of the Fuhrmann Triangle is the Orthocenter. [4, Fuhrmann Triangle]. We illustrate a few of the above theorems. We invite the reader to select the new theorems and to prove them.

Theorem 1. The Kiepert-Parry Point of the Medial Triangle of the Fuhrmann Triangle is the Spieker Center.

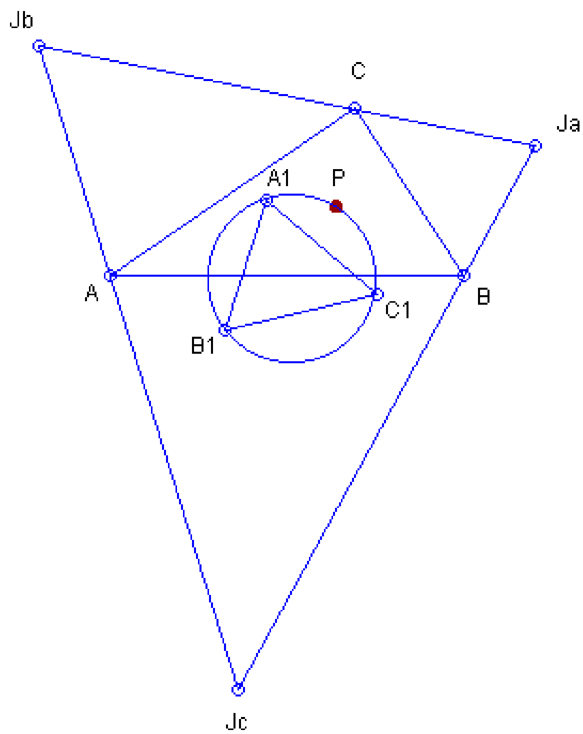
See the Figure:



P - Spieker Center;
$A_1B_1C_1$ - Fuhrmann Triangle;
$A_2B_2C_2$ - Medial Triangle of the Fuhrmann Triangle;
The Spieker Center coincides with the Kiepert-Parry Point of the Medial Triangle of the Fuhrmann Triangle.

Theorem 10 The Kiepert-Parry Point of the Fuhrmann Triangle of the Excentral Triangle is the Incenter.

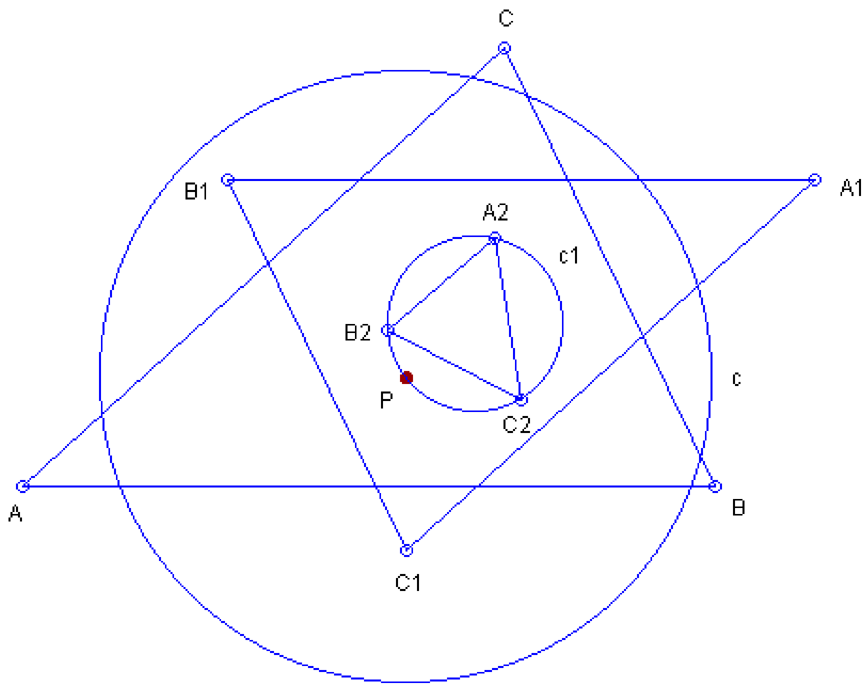
See the Figure:



P - Incenter;
JaJbJc - Excentral Triangle;
$A_1B_1C_1$ - Fuhrmann Triangle of the Excentral Triangle;
The Incenter coincides with the Kiepert-Parry Point of the Fuhrmann Triangle of the Excentral Triangle.

Theorem 24 The Kiepert-Parry Point of the Fuhrmann Triangle of the Outer Yff Triangle is the Center of the Outer Johnson-Yff Circle.

See the Figure:



c - Outer Johnson-Yff Circle;
P - Center of the Outer Johnson-Yff Circle;
$A_1B_1C_1$ - Outer Yff Triangle;
$A_2B_2C_2$ - Fuhrmann Triangle of the Outer Yff Triangle;
The Center of the Outer Johnson-Yff Circle coincides with the Kiepert-Parry Point of the Fuhrmann Triangle of the Outer Yff Triangle

Thanks

The figures in this note are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download at the Web. It is free and open source. Many thanks to Rene Grothmann for his wonderful program.

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Dr. Deko Dekov
Stara Zagora
Bulgaria
ddekov@dekovsoft.com.

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