

Computer-Generated Mathematics: The Pohoata Point

Deko Dekov

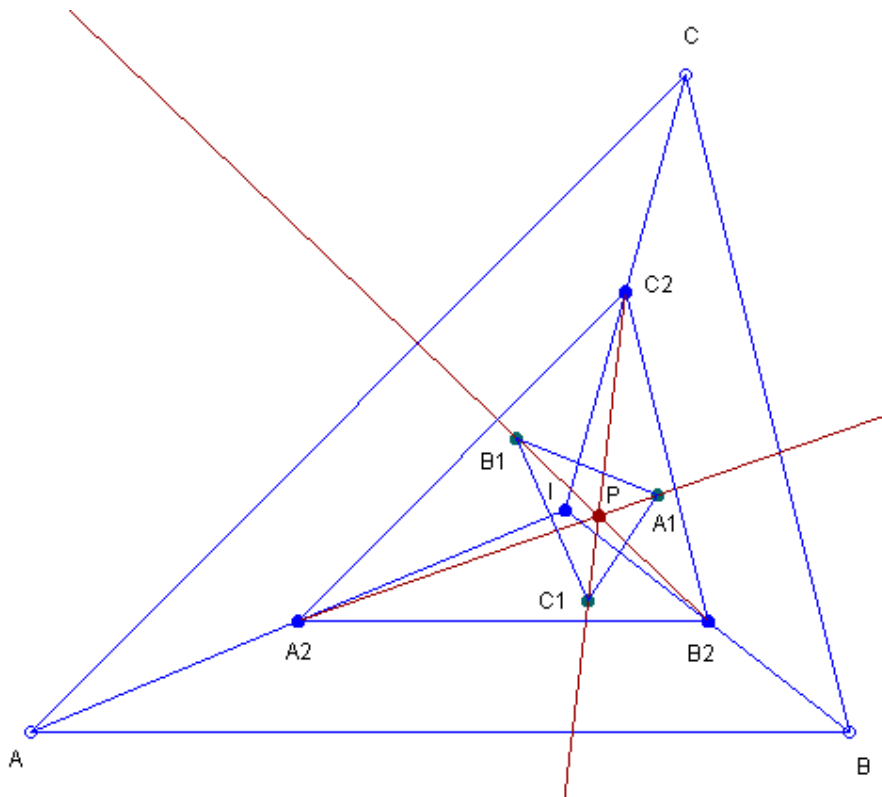
Abstract. By using the computer program "Machine for Questions and Answers", we find properties of the Pohoata Point.

Keywords: computer-generated mathematics, Euclidean geometry

We use the terminology in accordance with [1-6]. For the triangulation triangles, see [4]. For the Euler triangles, see [5].

The Pohoata Point is named in honor of Cosmin Pohoata, a Romanian Mathematician. The *Pohoata Point* is the Perspector of the Triangle of the Symmedian Points of the Triangulation Triangles of the Incenter and the Euler Triangle of the Incenter.

See the Figure:



I - Incenter;
A₁ - Symmedian point of triangle BCI;
B₁ - Symmedian point of triangle CAI;
C₁ - Symmedian point of triangle ABI;
A₁B₁C₁ - Triangle of the Symmedian Points of the Triangulation Triangles of the Incenter;
A₂ - Midpoint of segment AI;
B₂ - Midpoint of segment BI;
C₂ - Midpoint of segment CI;
A₁B₂C₂ - Euler Triangle of the Incenter;
Lines A₁A₂, B₁B₂ and C₁C₂ concur in a point (labeled by P), that is, triangles A₁B₁C₁ and A₁B₂C₂ are perspective. The perspector P, is the Pohoata Point.

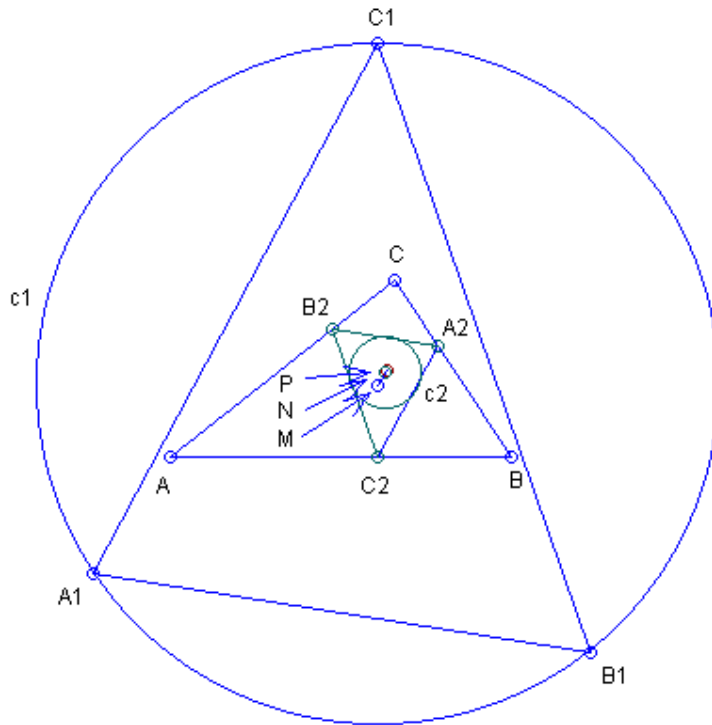
In 2006, the author of this paper created a computer program named the *Machine for Questions and Answers* (The *Machine*). The Machine is designed to discover mathematical theorems. Since then, The Machine has discovered a few thousands new mathematical theorems [2,3]. In 2006, the Machine has produced the first computer-generated encyclopedia [2].

Given an object (point, triangle, circle, line, etc.), the Machine produces theorems related to the properties of the object. The theorems produced by the Machine are either known theorems, or possible new theorems. A *possible new* theorem means that the theorem is either known theorem, but the source is not available for the author of the Machine, or the theorem is a new theorem.

In this paper we illustrate the use of the Machine. We present below five possible new theorems about the Pohoata Point, discovered by the Machine. The reader is invited to select the new theorems and to prove them.

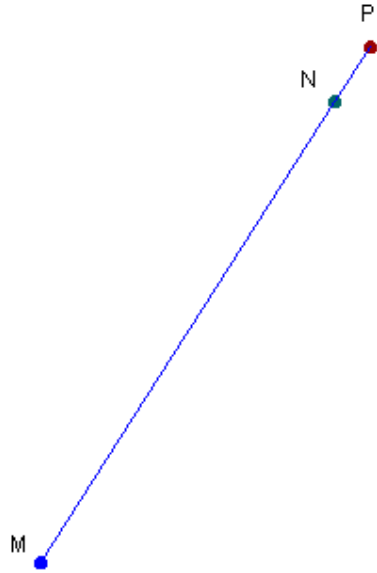
Theorem 1. The Pohoata Point is the External Center of Similitude of the Hexyl Circle and the Nine-Point Circle of the Intouch Triangle.

See the Figure:



- P - Pohoata Point;
- $A_1B_1C_1$ - Hexyl Triangle;
- c_1 - Hexyl Circle = Circumcircle of the Hexyl Triangle;
- M - Center of the Hexyl Circle = Circumcenter of the Hexyl Triangle;
- $A_2B_2C_2$ - Intouch Triangle;
- c_2 - Nine-Point Circle of the Intouch Triangle;
- N - Center of the Nine-Point Circle of the Intouch Triangle;
- P - Pohoata Point = External Center of Similitude of circles c_1 and c_2 .

The Figure below gives more clear picture:



M - Center of the Hexyl Circle = Circumcenter of the Hexyl Triangle;

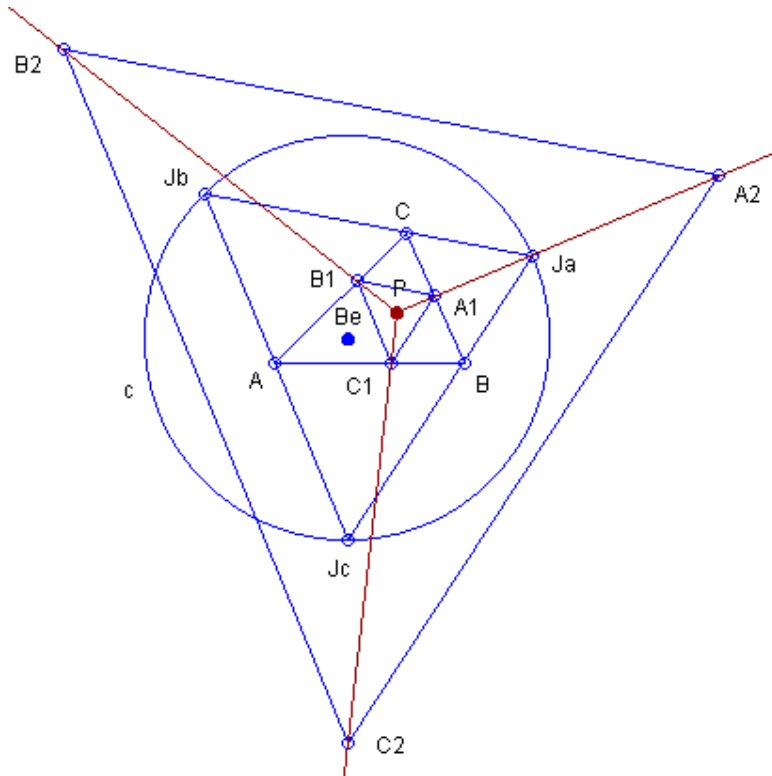
N - Center of the Nine-Point Circle of the Intouch Triangle;

The Pohoata Point P divides externally line segment MN in ratio radius of circle c_1 : radius of circle c_2 , that is, P lies on line MN, P is outside segment MN, and

$$\frac{\text{radius of circle } c_1}{\text{radius of circle } c_2} = \frac{MP}{NP}$$

Theorem 2. The Pohoata Point is the Homothetic Center of the Intouch Triangle and the Triangle of the reflections of the Bevan Point in the vertices of the Excentral Triangle.

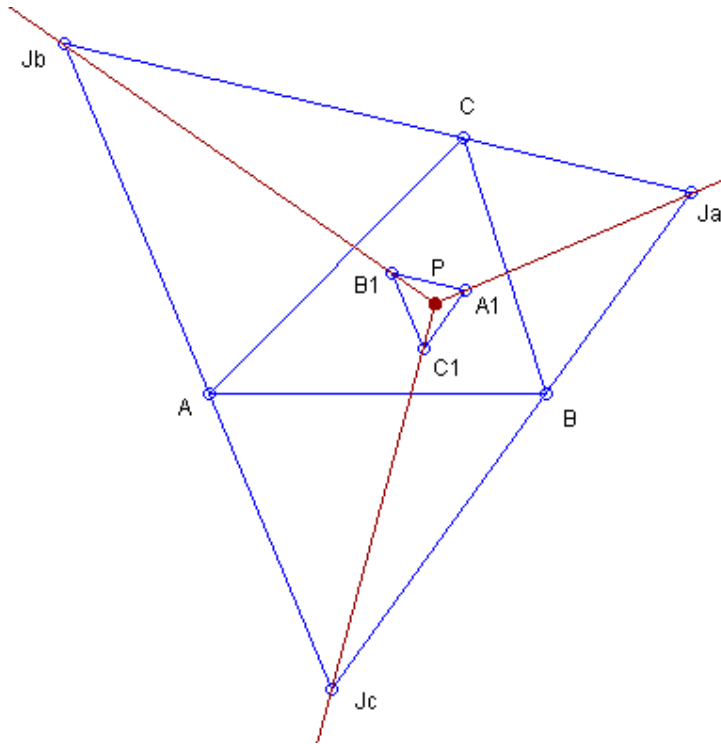
See the Figure:



P - Pohoata Point;
 $A_1B_1C_1$ - Intouch Triangle;
 J_a, J_b, J_c - Excenters;
 $J_aJ_bJ_c$ - Excentral Triangle;
 c - Circumcircle of the Excentral Triangle;
 Be - Bevan Point = Center of the Circumcircle of the Excentral Triangle = Ciircumcenter of the Excentral Triangle;
 A_2 - reflection of point Be in point J_a ;
 B_2 - reflection of point Be in point J_b ;
 C_2 - reflection of point Be in point J_c ;
 $A_2B_2C_2$ - Triangle of the reflections of the Bevan Point in the vertices of the Excentral Triangle;
 Triangles $A_1B_1C_1$ and $A_2B_2C_2$ are homothetic and the Pohoata Point P is the center of the homothety.

Theorem 3. The Pohoata Point is the Homothetic Center of the Excentral Triangle and the Intouch Triangle of the Orthic Triangle of the Circum-Incentral Triangle.

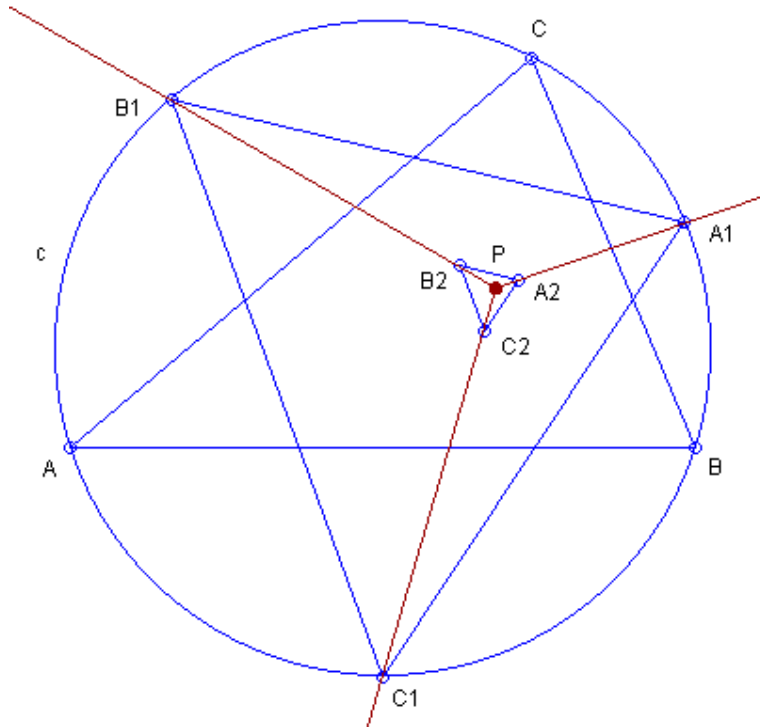
See the Figure:



P - Pohoata Point;
 $J_aJ_bJ_c$ - Excentral Triangle;
 $A_1B_1C_1$ - Intouch Triangle of the Orthic Triangle of the Circum-Incentral Triangle;
 Triangles $J_aJ_bJ_c$ and $A_1B_1C_1$ are homothetic and the Pohoata Point P is the center of the homothety.

Theorem 4. The Pohoata Point is the Homothetic Center of the Circum-Incentral Triangle and the Medial Triangle of the Medial Triangle of the Intouch Triangle.

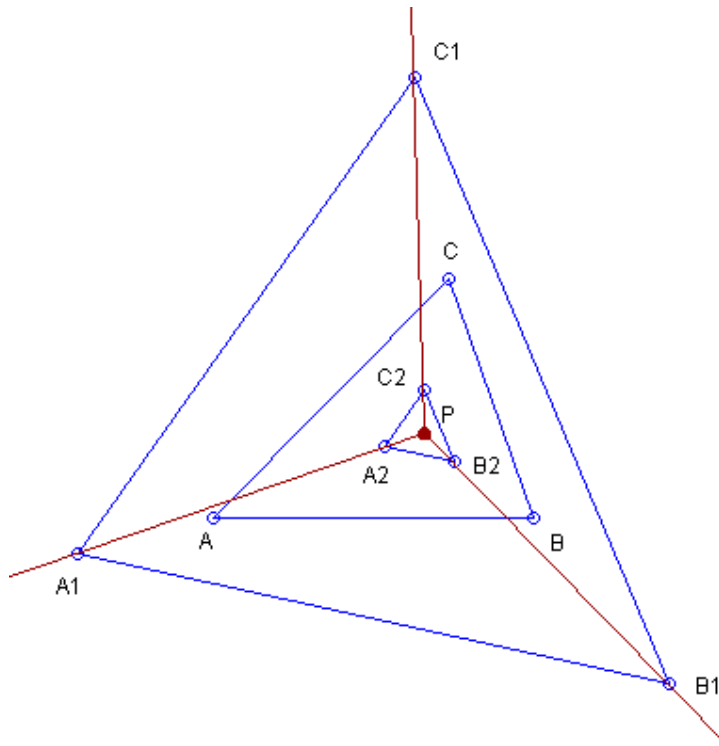
See the Figure:



P - Pohoata Point;
 c - Circumcircle;
 $A_1B_1C_1$ - Circum-Incidental Triangle;
 $A_2B_2C_2$ - Medial Triangle of the Medial Triangle of the Intouch Triangle;
 Triangles $A_1B_1C_1$ and $A_2B_2C_2$ are homothetic and the Pohoata Point P is the center of the homothety.

Theorem 5. The Pohoata Point is the Homothetic Center of the Hexyl Triangle and the Medial Triangle of the Intouch Triangle.

See the Figure:



P - Pohoata Point;
 $A_1B_1C_1$ - Hexyl Triangle;
 $A_2B_2C_2$ - Medial Triangle of the Intouch Triangle;
 Triangles $A_1B_1C_1$ and $A_2B_2C_2$ are homothetic and the Pohoata Point P is the center of the homothety.

References

1. Quim Castellsaguer, The Triangles Web,
<http://www.xtec.es/~qcastell/ttw/ttweng/portada.html>
2. D. Dekov, Computer-Generated Encyclopedia of Euclidean Geometry, First Edition, 2006, <http://www.dekovsoft.com/e1/>
3. D. Dekov, papers in the Journal of Computer-Generated Euclidean Geometry, 2006, 2007, 2008 <http://www.dekovsoft.com/j/>
4. D. Dekov, Triangulation Triangles, Journal of Computer-Generated Euclidean Geometry, **2**, 2007. <http://www.dekovsoft.com/j/>
5. D. Dekov, Euler Triangles, Journal of Computer-Generated Euclidean Geometry, **2**, 2007. <http://www.dekovsoft.com/j/>
6. Eric W. Weisstein, MathWorld - A Wolfram Web Resource.
<http://mathworld.wolfram.com/>

Dr.Deko Dekov

Zahari Knjazheski 81
6000 Stara Zagora
Bulgaria
ddekov@dekovsoft.com.

Preprint: 23 April 2008
Publication Date: 10 March 2009