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Computer-Generated Mathematics: The Schroder Point

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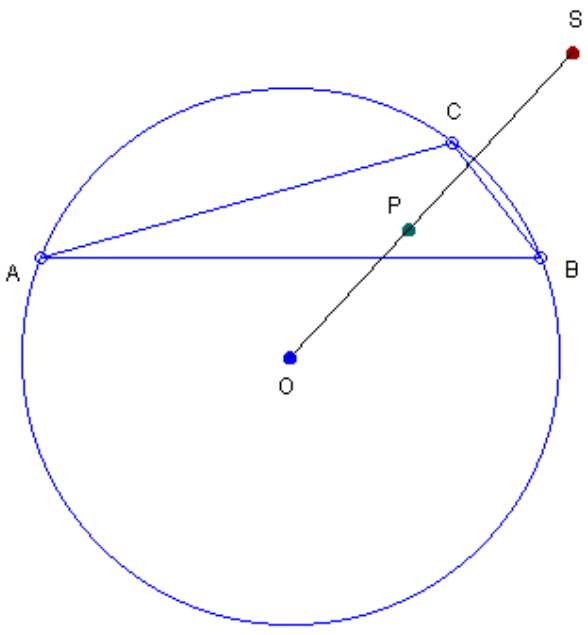
Abstract. By using the computer program "Machine for Questions and Answers", we find properties of the Schroder Point.

Keywords: computer-generated mathematics, Euclidean geometry

Recently Darij Grinberg [4] has collected known results about the Schroder points of a triangle. For the Schroder point see also [1, Schroder Point].. We use the terminology in accordance with [1-5].

The *Schroder Point* is the inverse image of the Internal Center of Similitude of the Incircle and the Circumcircle with respect to the Circumcircle.

See the Figure:



S - Schroder Point;
O - Circumcenter;

P - Internal Center of Similitude of the Incircle and the Circumcircle.

In 2006, the author of this paper created a computer program named the *Machine for Questions and Answers* (The *Machine*). The Machine is designed to discover mathematical theorems. Since then, The Machine has discovered a few thousands new mathematical theorems [2,3]. In 2006, the Machine has produced the first computer-generated encyclopedia [2].

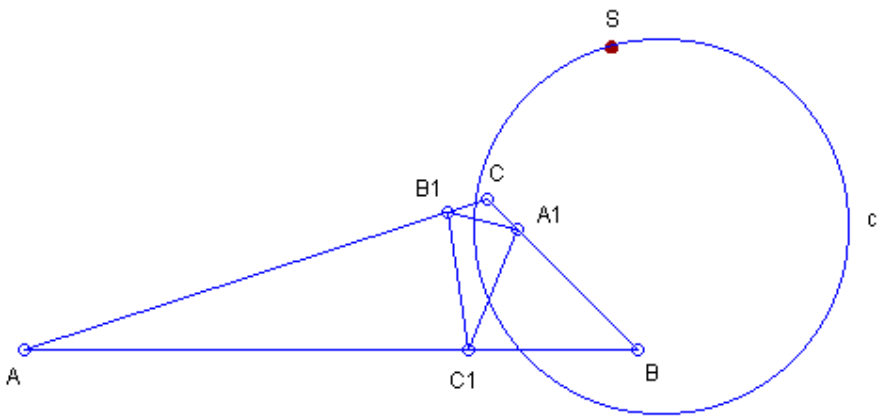
Given an object (point, triangle, circle, line, etc.), the Machine produces theorems related to the properties of the object. The theorems produced by the Machine are either known theorems, or possible new theorems. A *possible new* theorem means that the theorem is either known theorem, but the source is not available for the author of the Machine, or the theorem is a new theorem.

In this paper we illustrate the use of the Machine. We present below a few possible new theorems about the Schroder Point, discovered by the Machine.

The Machine finds circles passing through a given point. Below we give a few theorems about circles passing through the Schroder Point, discovered by the Machine. We illustrate the theorems and invite the reader to select the possible new theorems and to prove them.

Theorem 1. The Schroder Point lies on the Parry Circle of the Intouch Triangle.

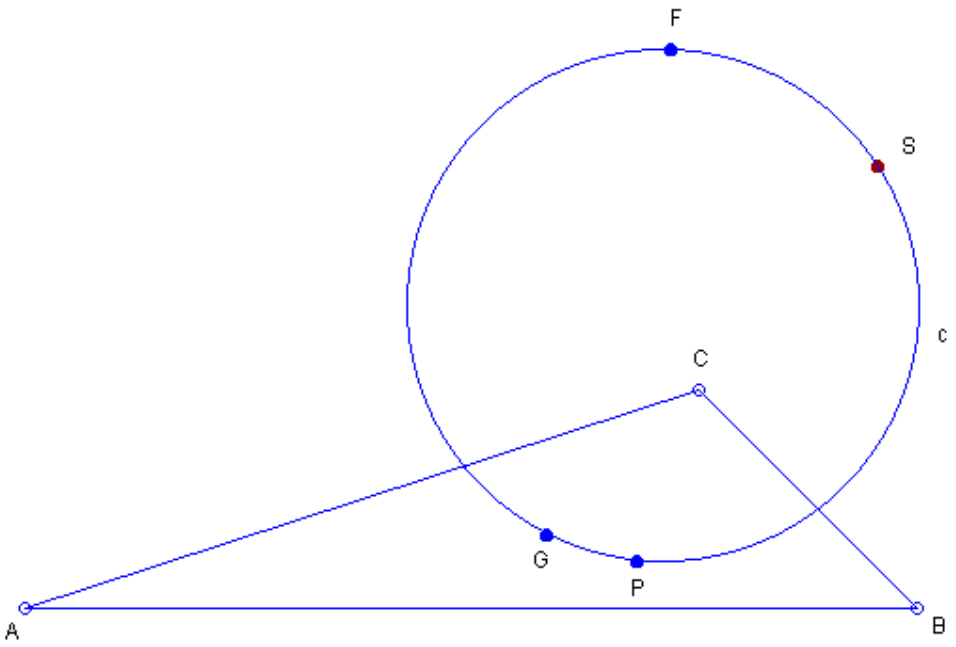
See the Figure:



S - Schroder Point;
$A_1B_1C_1$ - Intouch Triangle;
c - Parry Circle of the Intouch Triangle;
The Schroder Point lies on circle c.

Theorem 2. The Schroder Point lies on the Circle through the Centroid, the Far-Out Point and the Internal Center of Similitude of the Incircle and the Circumcircle.

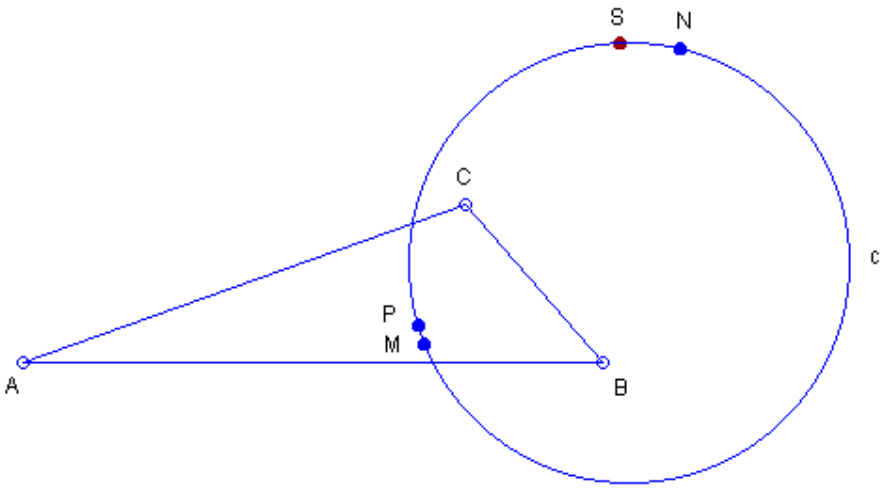
See the Figure:



S - Schroder Point;
G - Centroid;
F - Far-Out Point;
P - Internal Center of Similitude of the Incircle and the Circumcircle;
c - Circle through the Centroid, the Far-Out Point and the Internal Center of Similitude of the Incircle and the Circumcircle;
The Schroder Point lies on circle c.

Theorem 3. The Schroder Point lies on the Circle through the First Isodynamic Point, the Second Isodynamic Point and the Internal Center of Similitude of the Incircle and the Circumcircle.

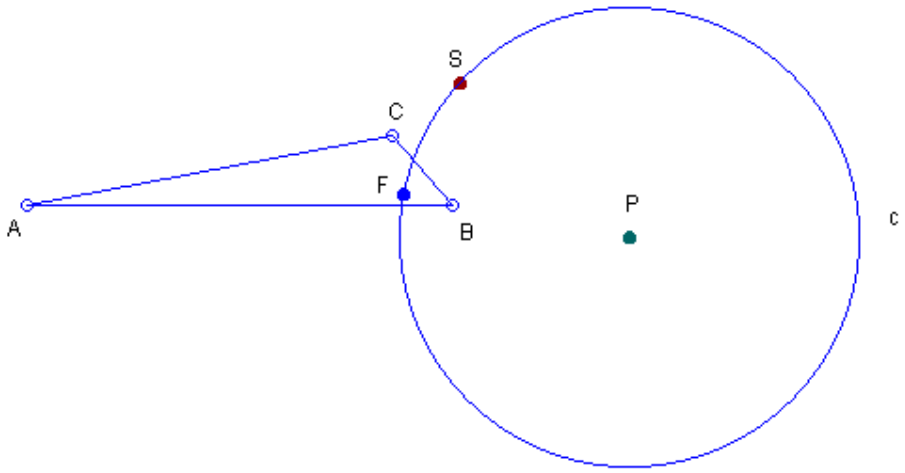
See the Figure:



S - Schroder Point;
M - First Isodynamic Point;
N - Second Isodynamic Point;
P - Internal Center of Similitude of the Incircle and the Circumcircle;
c - Circle through the First Isodynamic Point, the Second Isodynamic Point and the Internal Center of Similitude of the Incircle and the Circumcircle;
The Schroder Point lies on circle c.

Theorem 4. The Schroder Point lies on the Circle centered at the Center of the Stevanovic Circle through the First Feuerbach Point.

See the Figure:

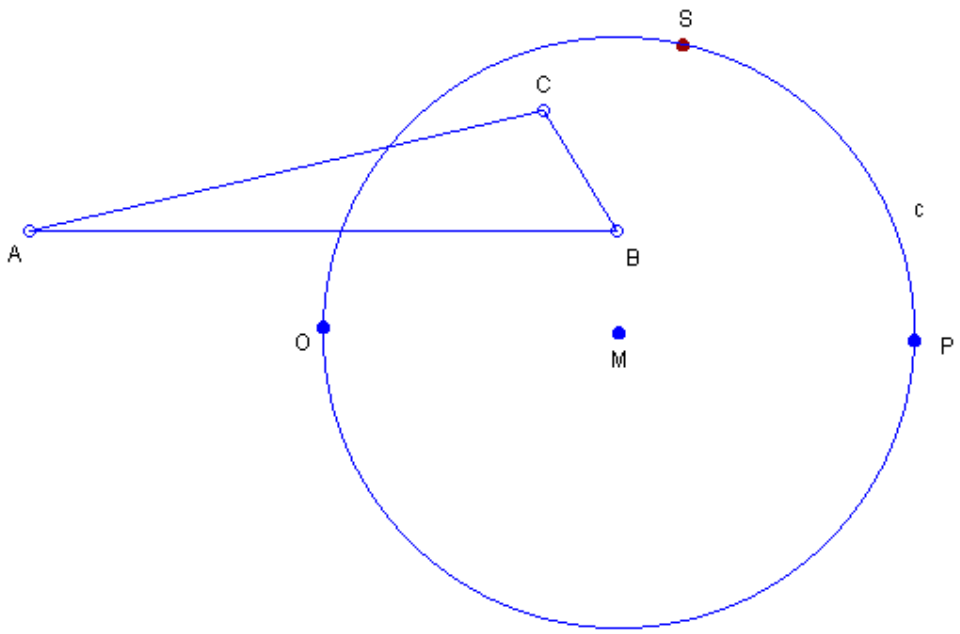


S - Schroder Point;
P - Center of the Stevanovic Circle;
F - First Feuerbach Point;

c - Circle centered at the Center of the Stevanovic Circle through the First Feuerbach Point;
The Schroder Point lies on circle c.

Theorem 5. The Schroder Point lies on the Circle with diameter connecting the Center of the Stevanovic Circle and the Circumcenter.

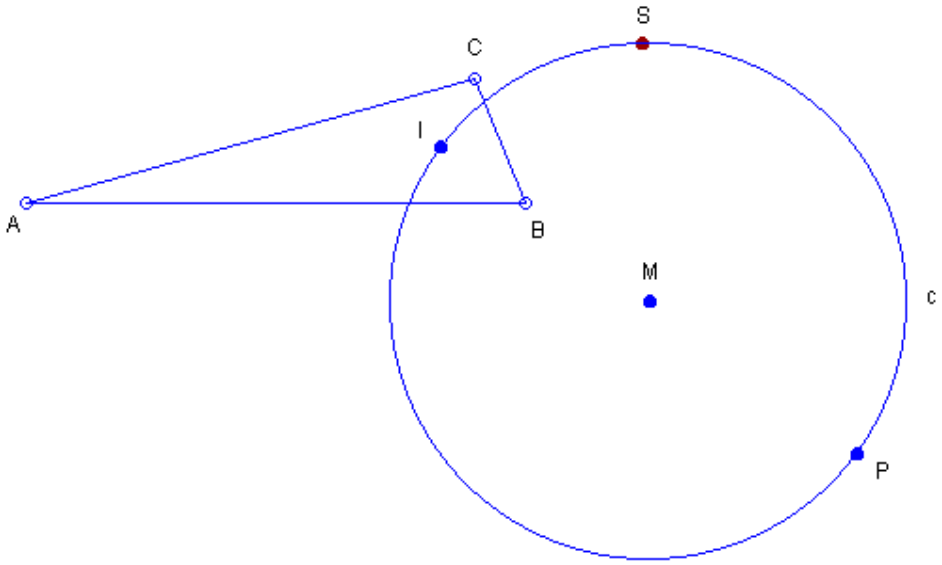
See the Figure:



S - Schroder Point;
P - Center of the Stevanovic Circle;
O - Circumcenter;
c - Circle with diameter connecting the Center of the Stevanovic Circle and the Circumcenter;
M - Midpoint of the Center of the Stevanovic Circle and the Circumcenter = Center of circle c;
The Schroder Point lies on circle c.

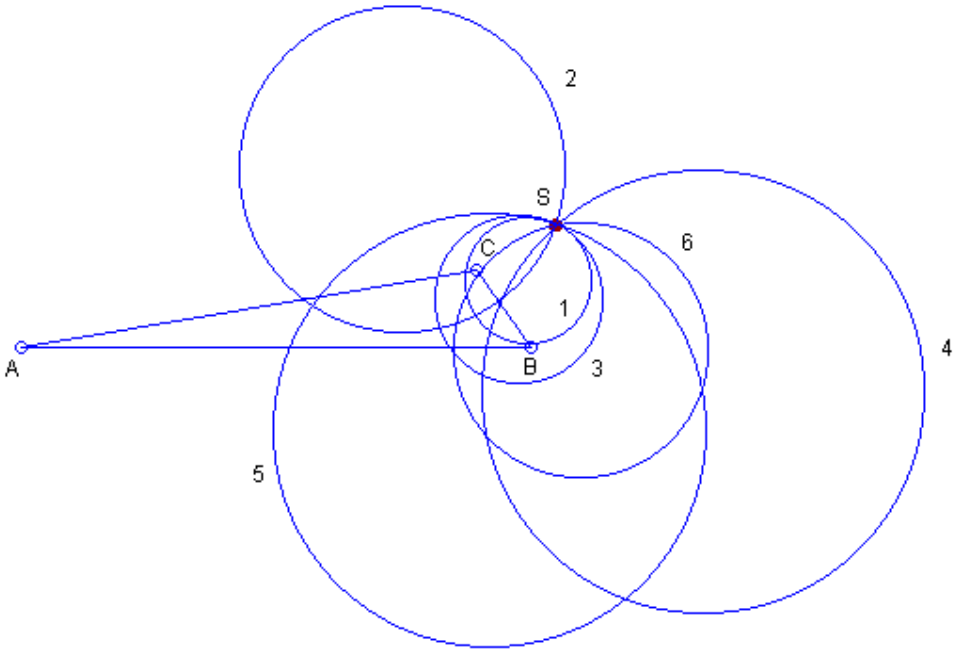
Theorem 6. The Schroder Point lies on the Circle with diameter connecting the Center of the Stevanovic Circle and the Incenter.

See the Figure:



S - Schroder Point;
P - Center of the Stevanovic Circle;
I - Incenter;
c - Circle with diameter connecting the Center of the Stevanovic Circle and the Incenter;
M - Midpoint of the Center of the Stevanovic Circle and the Incenter = Center of circle c;
The Schroder Point lies on circle c.

The Figure below represents the six circles which appear in theorems 1 to 6:



S - Schroder Point;
circle 1 - Parry Circle of the Intouch Triangle;
circle 2 - Circle through the Centroid, the Far-Out Point and the Internal Center of Similitude of the Incircle and the Circumcircle;
circle 3 - Circle through the First Isodynamic Point, the Second Isodynamic Point and the Internal Center of Similitude of the Incircle and the Circumcircle;
circle 4 - Circle centered at the Center of the Stevanovic Circle through the First Feuerbach Point;
circle 5 - Circle with diameter connecting the Center of the Stevanovic Circle and the Circumcenter;
circle 6 - Circle with diameter connecting the Center of the Stevanovic Circle and the Incenter.

Theorem 7. The Schroder Point lies on the Circle with diameter connecting the Center of the Stevanovic Circle and the Moses Point.

Theorem 8. The Schroder Point lies on the Circle with diameter connecting the Center of the Stevanovic Circle and the Bevan Point.

Theorem 9. The Schroder Point lies on the Circle with diameter connecting the Center of the Stevanovic Circle and the Internal Center of Similitude of the Incircle and the Circumcircle.

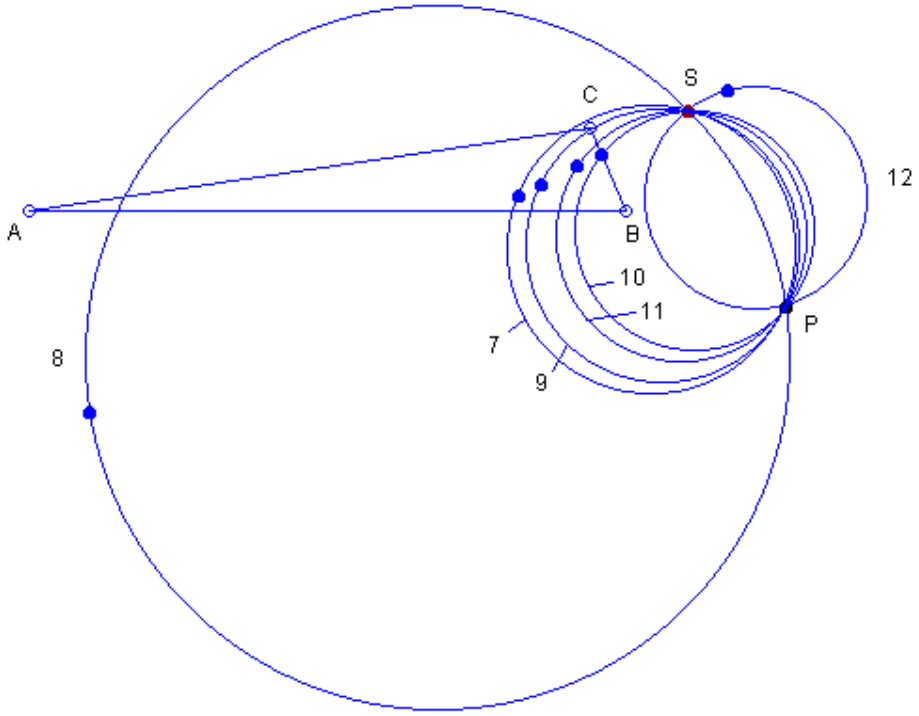
Theorem 10. The Schroder Point lies on the Circle with diameter connecting the Center of the Stevanovic Circle and the External Center of Similitude of the Incircle and the Circumcircle.

Theorem 11. The Schroder Point lies on the Circle with diameter connecting the Center of

the Stevanovic Circle and the Weill Point.

Theorem 12. The Schroder Point lies on the Circle with diameter connecting the Center of the Stevanovic Circle and the Evans Perspector.

The Figure below represents the six circles which appear in theorems 7 to 12:



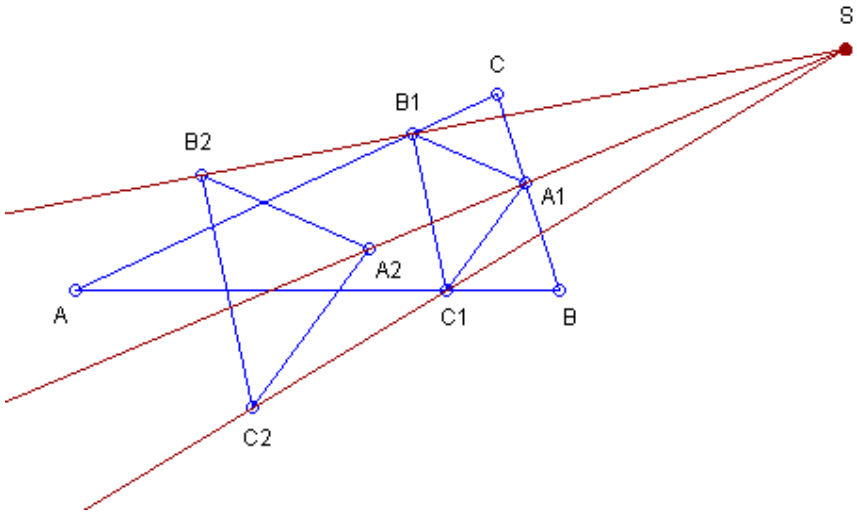
S - Schroder Point;
circle 7 - Circle with diameter connecting the Center of the Stevanovic Circle and the Moses Point;
circle 8 - Circle with diameter connecting the Center of the Stevanovic Circle and the Bevan Point;
circle 9 - Circle with diameter connecting the Center of the Stevanovic Circle and the Internal Center of Similitude of the Incircle and the Circumcircle;
circle 10 - Circle with diameter connecting the Center of the Stevanovic Circle and the External Center of Similitude of the Incircle and the Circumcircle;
circle 11 - Circle with diameter connecting the Center of the Stevanovic Circle and the Weill Point;
circle 12 - Circle with diameter connecting the Center of the Stevanovic Circle and the Evans Perspector.

Below we give two theorems about the Schroder Point as Homothetic Center, discovered by the Machine. We illustrate the theorems and invite the reader to select the possible new theorems and to prove them.

Theorem 13. The Schroder Point is the Homothetic Center of the Intouch Triangle and the

Medial Triangle of the Medial Triangle of the Excentral Triangle.

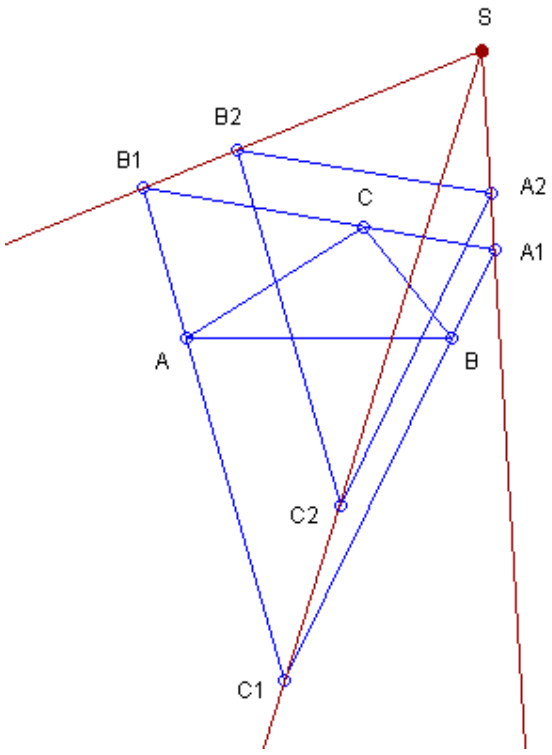
See the Figure:



S - Schroder Point;
$A_1B_1C_1$ - Intouch Triangle;
$A_2B_2C_2$ - Medial Triangle of the Medial Triangle of the Excentral Triangle;
The Schroder Point is the Homothetic Center of triangles $A_1B_1C_1$ and $A_2B_2C_2$.

Theorem 14. The Schroder Point is the Homothetic Center of the Excentral Triangle and the Anticomplementary Triangle of the Anticomplementary Triangle of the Intouch Triangle.

See the Figure:



S - Schroder Point;
$A_1B_1C_1$ - Excentral Triangle;
$A_2B_2C_2$ - Anticomplementary Triangle of the Anticomplementary Triangle of the Intouch Triangle;
The Schroder Point is the Homothetic Center of triangles $A_1B_1C_1$ and $A_2B_2C_2$.

Thanks

The figures in this paper are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download at the Web. It is free and open source. Many thanks to Rene Grothmann for his wonderful program.

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