

Computer-Generated Mathematics: The Bevan-Schroder Point

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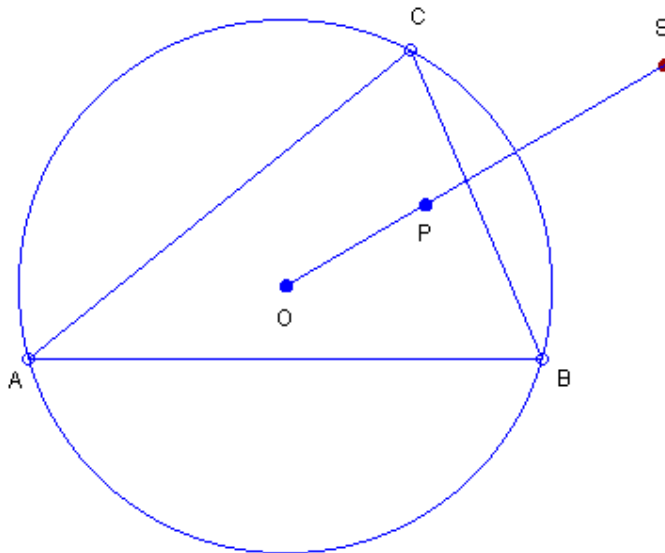
Abstract. By using the computer program "Machine for Questions and Answers", we find properties of the Bevan-Schroder Point.

Keywords: computer-generated mathematics, Euclidean geometry

Recently Darij Grinberg [4] has collected known results about the Bevan-Schroder point of a triangle. For the Bevan-Schroder point see also [1, Bevan-Schroder Point].. We use the terminology in accordance with [1-5].

The *Bevan-Schroder Point* is the inverse image of the External Center of Similitude of the Incircle and the Circumcircle with respect to the Circumcircle.

See the Figure:



S - Bevan-Schroder Point;
O - Circumcenter;

P - External Center of Similitude of the Incircle and the Circumcircle.

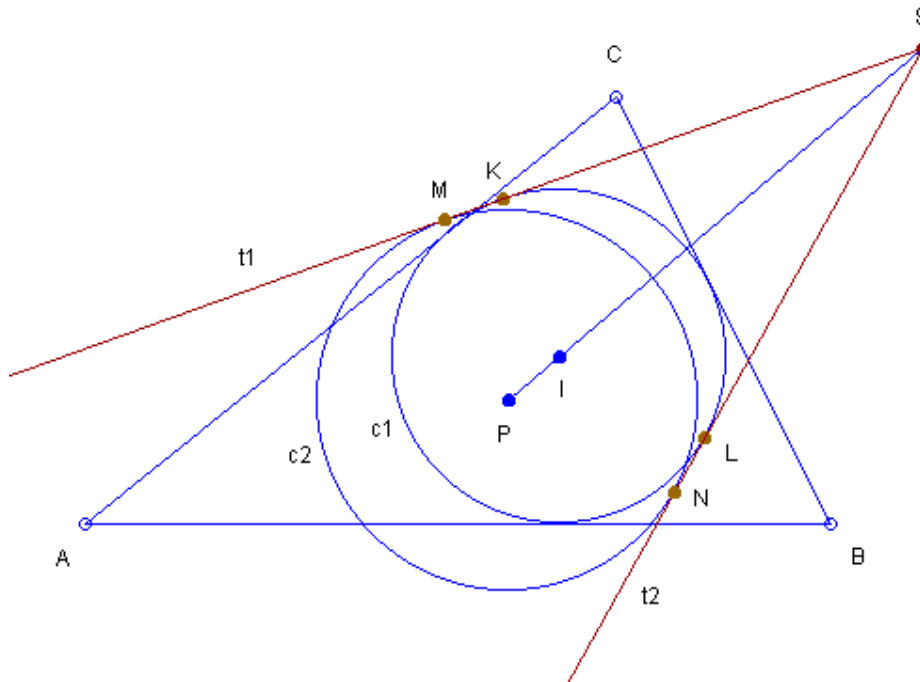
In 2006, the author of this paper created a computer program named the *Machine for Questions and Answers* (The *Machine*). The Machine is designed to discover mathematical theorems. Since then, The Machine has discovered a few thousands new mathematical theorems [2,3]. In 2006, the Machine has produced the first computer-generated encyclopedia [2].

Given an object (point, triangle, circle, line, etc.), the Machine produces theorems related to the properties of the object. The theorems produced by the Machine are either known theorems, or possible new theorems. A *possible new* theorem means that the theorem is either known theorem, but the source is not available for the author of the Machine, or the theorem is a new theorem.

In this paper we illustrate the use of the Machine. We present below a few possible new theorems about the Bevan-Schroder Point, discovered by the Machine. We illustrate the theorems and invite the reader to select the possible new theorems and to prove them.

Theorem 1. The Bevan-Schroder Point is the External Center of Similitude of the Incircle and the Nine-Point Circle of the Circum-Incentral Triangle.

See the Figure:



- S - Bevan-Schroder Point;
- c1 - Incircle;
- I - Incenter;
- c2 - Nine-Point Circle of the Circum-Incentral Triangle;
- P - Nine-Point Center of the Circum-Incentral Triangle = Center of the Nine-Point Circle of

the Circum-Incentral Triangle;

t_1, t_2 - tangents from point S to circle c_1 ;

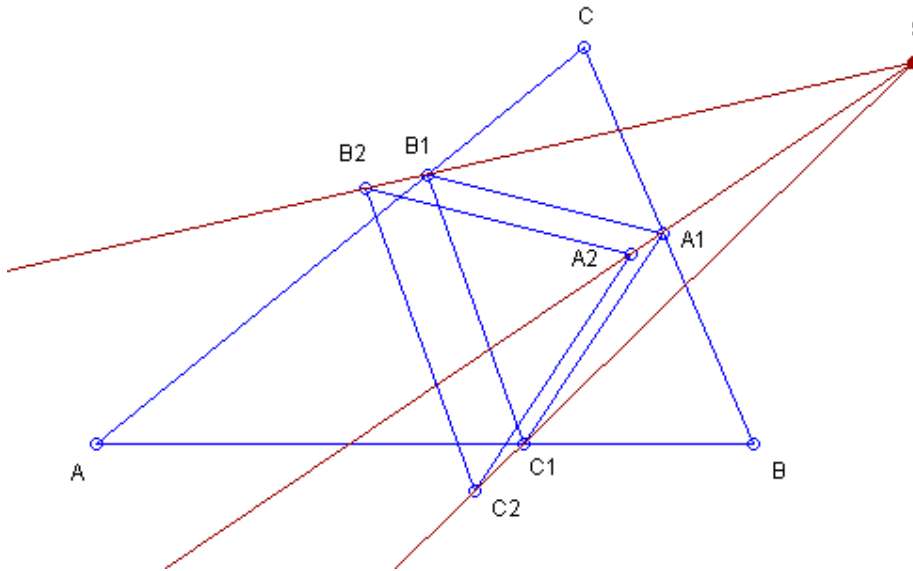
K, L - tangent points of circle c_1 and lines t_1 and t_2 , respectively;

The Bevan-Schroder Point is the External Center of Similitude of circles c_1 and c_2 . Lines t_1 and t_2 are tangent to circle c_2 . The Bevan-Schroder Point S divides externally line segment PI in ratio radius of circle c_2 : radius of circle c_1 , that is, S lies on line PI , S is outside segment PI , and

$$\frac{IS}{PS} = \frac{\text{radius of circle } c_1}{\text{radius of circle } c_2}$$

Theorem 2. The Bevan-Schroder Point is the Homothetic Center of the Intouch Triangle and the Euler Triangle of the Circum-Incentral Triangle.

See the Figure:



S - Bevan-Schroder Point;

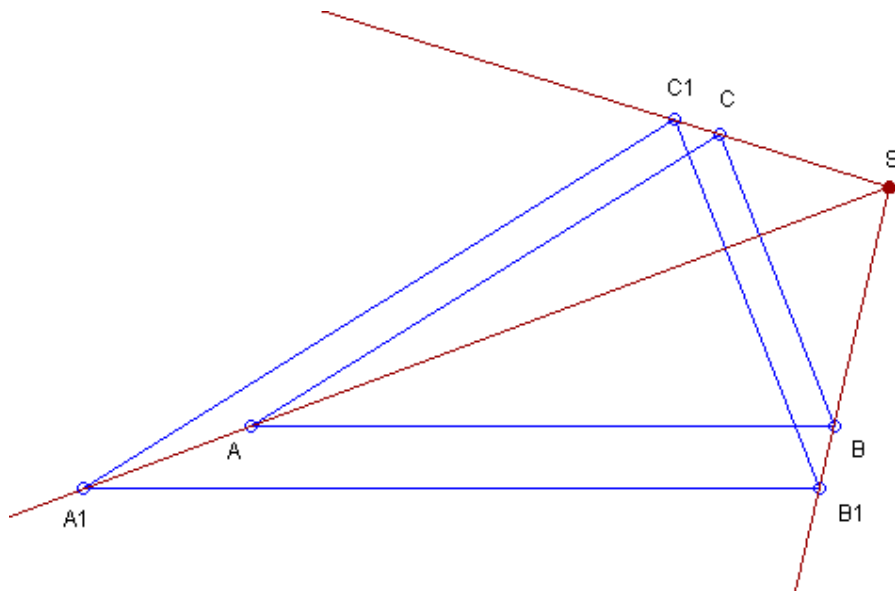
$A_1B_1C_1$ - Intouch Triangle;

$A_2B_2C_2$ - Euler Triangle of the Circum-Incentral Triangle;

The Bevan-Schroder Point is the Homothetic Center of triangles $A_1B_1C_1$ and $A_2B_2C_2$.

Theorem 3. The Bevan-Schroder Point is the Homothetic Center of Triangle ABC and the Tangential Triangle of the Euler Triangle of the Circum-Incentral Triangle.

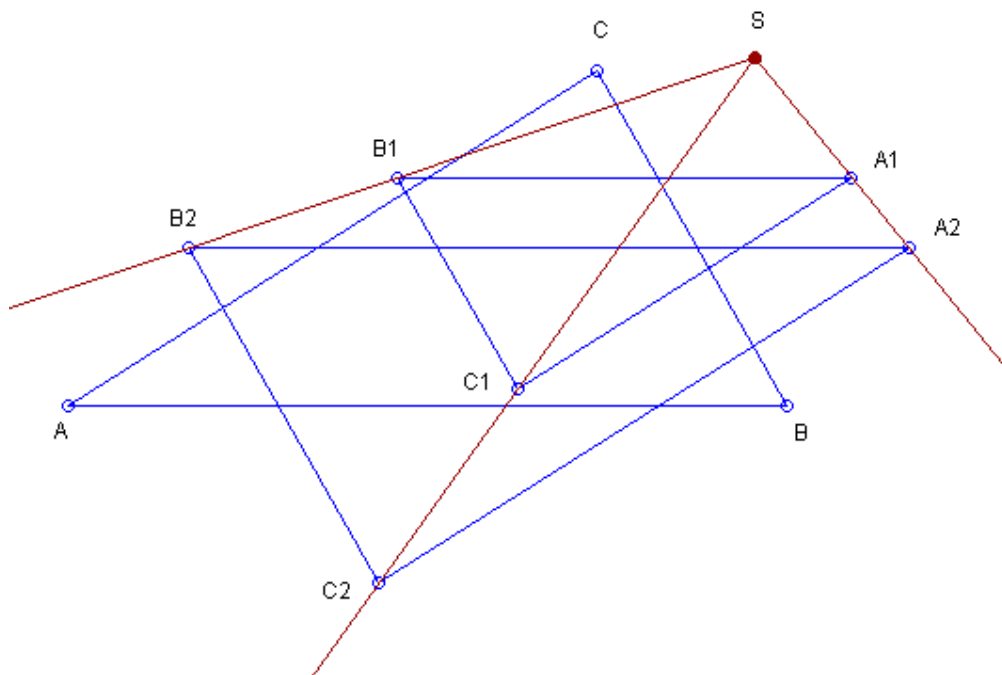
See the Figure:



S - Bevan-Schroder Point;
 $A_1B_1C_1$ - Tangential Triangle of the Euler Triangle of the Circum-Incentral Triangle;
 The Bevan-Schroder Point is the Homothetic Center of triangles ABC and $A_1B_1C_1$.

Theorem 4. The Bevan-Schroder Point is the Homothetic Center of the Outer Yff Triangle and the Triangle of reflections of the Center of the Fuhrmann Circle in the vertices of the Medial Triangle.

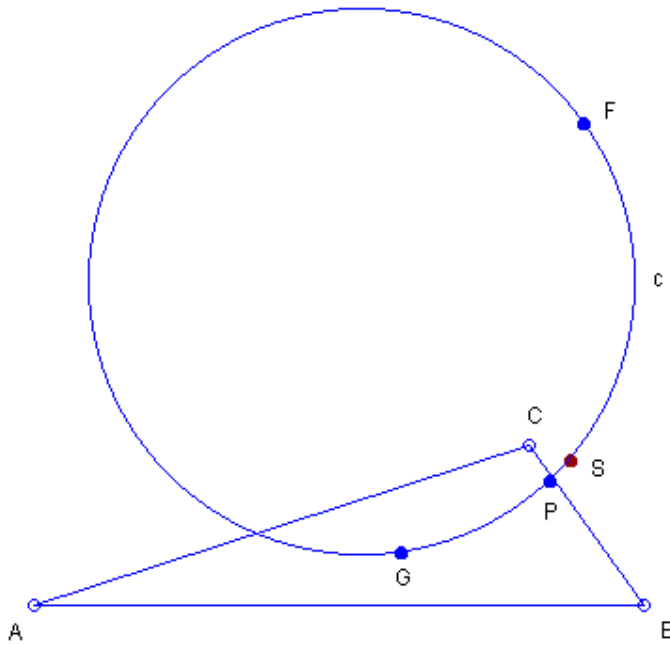
See the Figure:



S - Bevan-Schroder Point;
 $A_1B_1C_1$ - Outer Yff Triangle;
 $A_2B_2C_2$ - Triangle of reflections of the Center of the Fuhrmann Circle in the vertices of the Medial Triangle;
 The Bevan-Schroder Point is the Homothetic Center of triangles $A_1B_1C_1$ and $A_2B_2C_2$.

Theorem 5. The Bevan-Schroder Point lies on the Circle through the Centroid, the Far-Out Point and the External Center of Similitude of the Incircle and the Circumcircle.

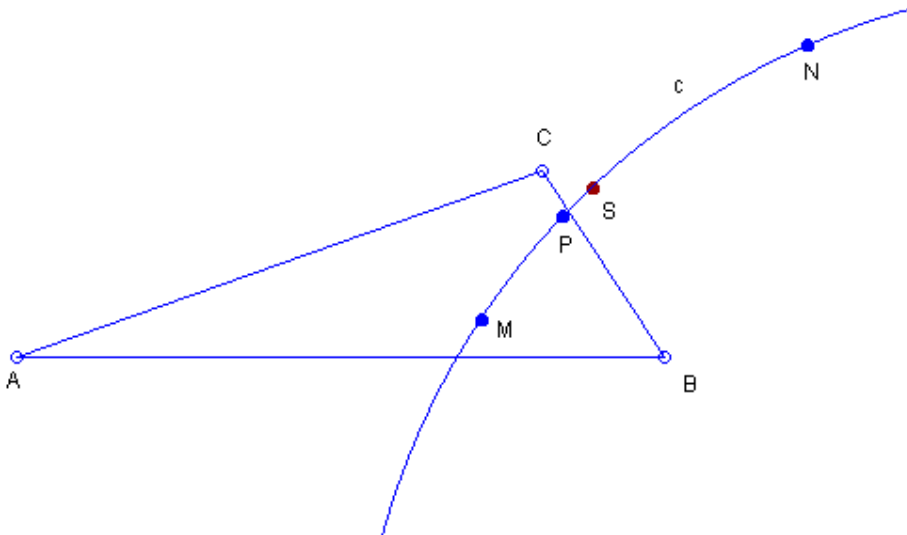
See the Figure:



S - Bevan-Schroder Point;
 G - Centroid;
 F - Far-Out Point;
 P - External Center of Similitude of the Incircle and the Circumcircle;
 c - Circle defined by points G, F and P;
 The Bevan-Schroder Point lies on circle c.

Theorem 6. The Bevan-Schroder Point lies on the Circle through the First Isodynamic Point, the Second Isodynamic Point and the External Center of Similitude of the Incircle and the Circumcircle.

See the Figure:



S - Bevan-Schroder Point;
M - First Isodynamic Point;
N - Second Isodynamic Point;
P - External Center of Similitude of the Incircle and the Circumcircle;
c - Circle defined by points M, N and P;
The Bevan-Schroder Point lies on circle c.

Thanks

The figures in this paper are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download at the Web. It is free and open source. Many thanks to Rene Grothmann for his wonderful program.

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