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Computer-Generated Mathematics: Construction of the Inner Apollonius Circle of the Mixtilinear Incircles

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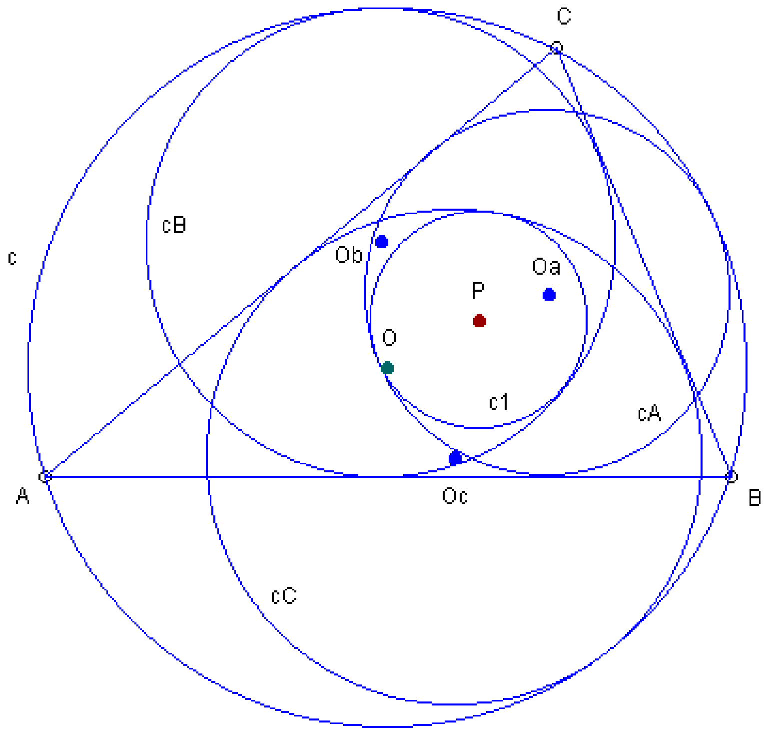
Abstract. We illustrate the use of the computer program "Machine for Questions and Answers" (The Machine) for discovering of new theorems in Euclidean Geometry. By using the Machine we find a few new straightedge-and-compass constructions of the inner Apollonius circle of the mixtilinear incircles.

Keywords: computer-generated mathematics, Euclidean geometry

Given a triangle ABC. The A-Mixtilinear incircle is the circle (Oa) that touches internally the rays AB and AC and the circumcircle. Define the B- and C-Mixtilinear incircles (Ob) and (Oc) analogously ([5,7]). The circle ca = (P,ra) tangent internally to the mixtilinear incircles is the *inner Apollonius circle of the mixtilinear incircles*. See the Figure:



In 2006, the author of this paper created a computer program named the *Machine for Questions and Answers* (The *Machine*). The Machine is designed to discover mathematical theorems. Since then, The Machine has discovered a few thousands new mathematical theorems [1-4]. In 2006, the Machine has produced the first computer-generated encyclopedia [1].

Given an object (point, triangle, circle, line, etc.), the Machine produces theorems related to the properties of the object. The theorems produced by the Machine are either known theorems, or possible new theorems. A *possible new* theorem means that the theorem is either known theorem, but the source is not available for the author of the Machine, or the theorem is a new theorem.

In this paper we illustrate the use of the Machine. We present below a few possible new theorems about the Inner Apollonius circle of the mixtilinear incircles, discovered by the Machine. We illustrate the theorems and invite the reader to select the possible new theorems and to prove them.

There are many ways to construct the inner Apollonius circle of the mixtilinear incircles, ca ,by using a straightedge and compass only. We can construct the center of the circle ca as the point dividing segment from the circumcenter to the incenter in ratio 4R : r, where R is the circumradius and r is the inradius.([5, Theorem 11.2],also a theorem produced by the Machine), and radius (see [5, Theorem 11.2]):

$$ra = \frac{3Rr}{4R+r}$$

The theorems below give us the possibility to construct the radius ra in alternative ways.

The Machine produces the following theorems:

Theorem 1. The External Center of Similitude of the Inner Apollonius Circle of the Mixtillinear Incircles and the Incircle is the Internal Center of Similitude of the Incircle and the Circumcircle.

Theorem 2. The Internal Center of Similitude of the Inner Apollonius Circle of the Mixtillinear Incircles and the Circumcircle is the Internal Center of Similitude of the Incircle and the Circumcircle.

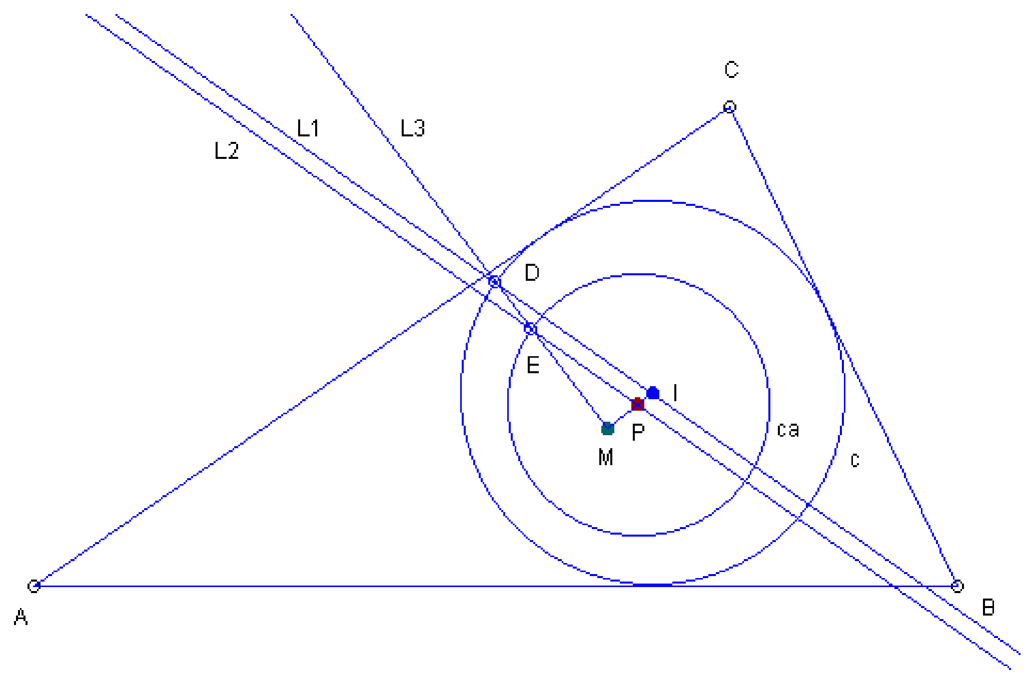
Theorem 3. The Inverse of the Incenter in the Inner Apollonius Circle of the Mixtillinear Incircles is the Inverse of the Incenter in the Circumcircle.

Theorem 4. The Inner Apollonius Circle of the Mixtillinear Incircles is orthogonal to the Circle through the Incenter, the Centroid and the Inverse of the Centroid in the Circumcircle.

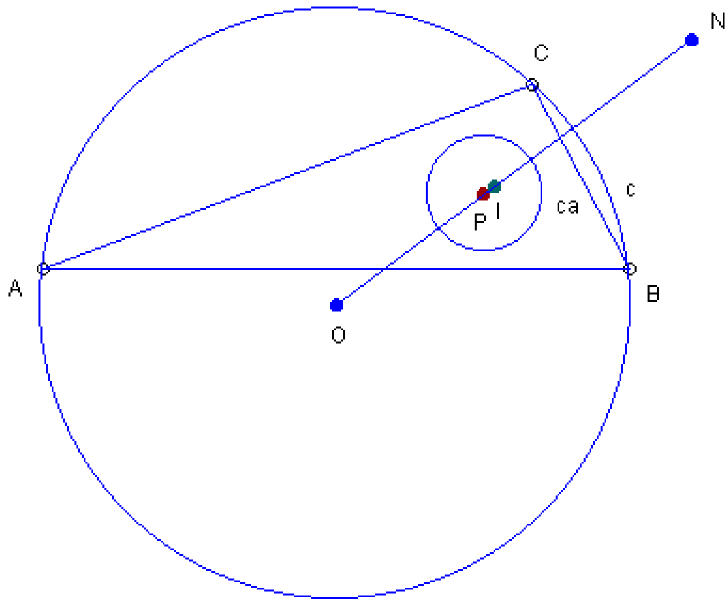
Theorem 5. The Inner Apollonius Circle of the Mixtillinear Incircles is orthogonal to the Circle through the Incenter, the First Isodynamic Point and the Second Isodynamic Point.

We could use the above theorems to construct the circle ca as follows.

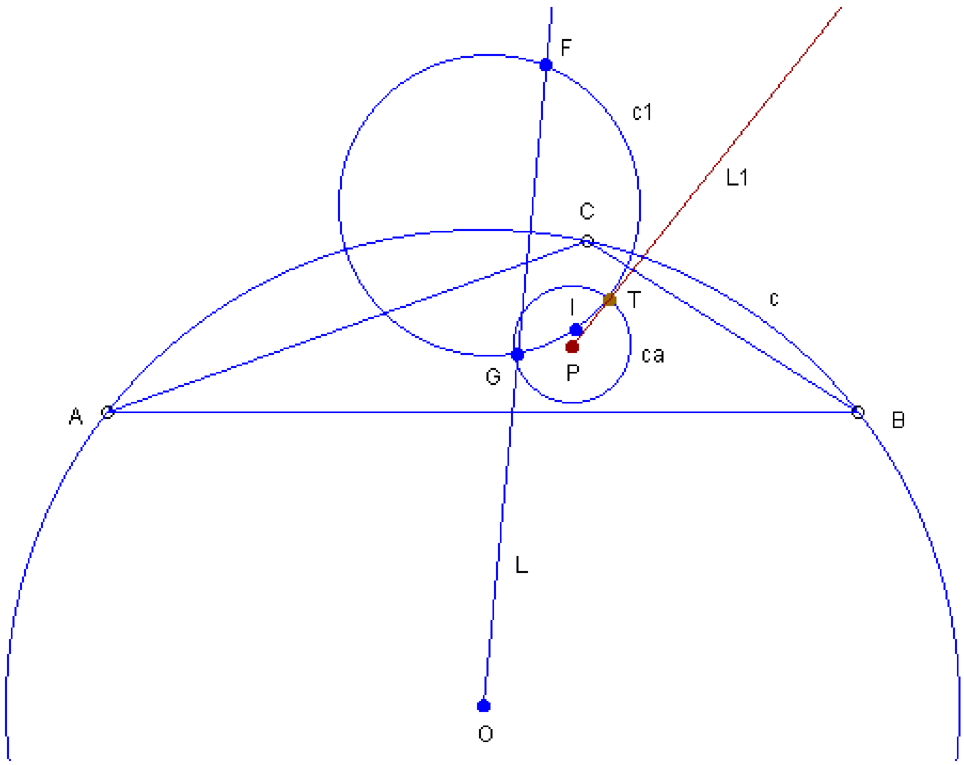
To apply theorem 1, construct the incenter I, the incircle c, and the internal center of similitude of the incircle and the circumcircle M. By theorem 1, point M divides externally the line segment IP in ratio r : ra, where r is the inradius and ra is the radius of circle ca. Hence, we can construct easily the radius ra as follows: Construct a line L1 through point I. Label D the intersection point of L1 and circle c. Construct the line L2 through P parallel to line L1. Construct the line through points M and D. Label E the intersection point of lines MD and L2. Construct circle ca as the circle with center P and radius PE. See the Figure:



We can use theorem 2 to obtain a similar construction. To apply theorem 3, constrict point N, the inverse point of the incenter I with respect to the circumcircle c. Construct circle ca as the circle centered at P with radius $ra = \sqrt{PI.PN}$ See the Figure:



To apply theorem 4, construct circle c1 passing through the incenter I, the centroid G and the inverse of the centroid in the circumcircle, F. Construct a tangent line from point P to circle c1 and label by T the tangency point. Construct circle ca as the circle centered at P with radius PT. See the Figure:



We can use theorem 5 to obtain a similar construction. We give below a few additional theorems produced by the Machine. The reader may find the definitions in the Weisstein's encyclopedia [6].

Theorem 6. The Radical Center of the Mixtillinear Incircles lies on the Line through the Midpoint of the Incenter and the Centroid and the Midpoint of the Incenter and the Mittenpunkt.

Theorem 7. The Inner Apollonius Circle of the Mixtillinear Incircles is orthogonal to the Circle through the Incenter, the First Isodynamic Point and the Second Isodynamic Point.

Theorem 8. The Inner Apollonius Circle of the Mixtillinear Incircles is orthogonal to the Circle through the Incenter, the First Brocard Point and the Inverse of the First Brocard Point in the Circumcircle.

Theorem 6 gives us an additional way to construct the center P of circle ca, as the point of intersection of the line passing through the incenter and circumcenter, and the line passing through the midpoint of the incenter and the centroid and the midpoint of the incenter and the mittenpunkt. Theorems 6 to 8 give us additional ways to construct circle ca.

Thanks

The figures in this note are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download at the Web. It is free and open source. Many thanks to Rene Grothmann for his wonderful program.

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