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Computer-Generated Mathematics: The Stanilov Triangle

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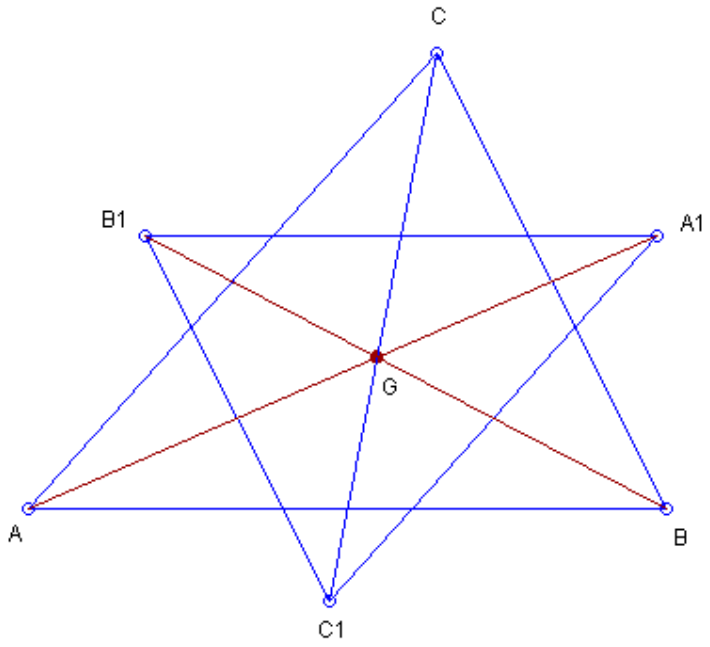
Abstract. By using the computer program "Machine for Questions and Answers", we find examples of triangles perspective with the Stanilov Triangle.

Grozyo Stanilov [4,5] defines the Stanilov triangle (the term *Stanilov triangle* is introduced in this paper in honor of Grozyo Stanilov) by using statements from differential geometry. He proves that the Stanilov triangle is the homothetic image of triangle ABC under the homothety with center the centroid and ratio equal to -4/5 ([5], theorem 2). We use this result to give an alternative definition of the Stanilov triangle, as follows. The *Stanilov triangle* is the homothetic image of triangle ABC under homothety with center the centroid of triangle ABC and ratio equal to -4/5. See the Figure:



In the Figure, point G is the Gentroid of triangle ABC. Point A_1 lies on line AG and the length of segment GA_1 is equal to 4/5 of the length of segment AG. Similarly cosntruct points B_1 and C_1. Triangle $A_1B_1C_1$ is the Stanilov Triangle.

We use the definitions in accordance with [1,2,3,6].

The Machine for Questions and Answers produces examples of triangles perspective with the Stanilov Triangle. We select a few examples given below.

The Stanilov Triangle is

1. homothetic to Triangle ABC.
2. homothetic to the Circumcevian Triangle of the Circumcenter.
3. homothetic to the Euler Triangle.
4. perspective with the First Brocard Triangle.
5. perspective with the Fourth Brocard Triangle.
6. perspective with the Neuberg Triangle.
7. perspective with the Reflected Neuberg Triangle.
8. homothetic to the Johnson Triangle.
9. homothetic to the Inner Yff Triangle.
10. homothetic to the Outer Yff Triangle.
11. homothetic to the Outer Grebe Triangle.
12. homothetic to the Inner Grebe Triangle.
13. perspective with the Outer Fermat Triangle.
14. perspective with the Inner Fermat Triangle.
15. homothetic to the Medial Triangle of the Medial Triangle.
16. perspective with the Cevian Triangle of the Circumcenter of the Intouch Triangle.
17. homothetic to the Orthic Triangle of the Intouch Triangle.
18. perspective with the Cevian Triangle of the de Longchamps Point of the Intouch Triangle.
19. homothetic to the Tangential Triangle of the Excentral Triangle.
20. homothetic to the Anticomplementary Triangle of the Anticomplementary Triangle.
21. perspective with the Pedal Triangle of the Skordev Point of the Anticomplementary Triangle.
22. homothetic to the Antipedal Triangle of the Circumcenter of the Excentral Triangle.
23. homothetic to the Antipedal Triangle of the Orthocenter of the Anticomplementary Triangle.
24. homothetic to the Circum-Orthic Triangle of the Excentral Triangle.
25. perspective with the Circum-Medial Triangle of the Anticomplementary Triangle.
26. homothetic to the Circumcevian Triangle of the Circumcenter of the Anticomplementary Triangle.
27. homothetic to the Intangents Triangle of the Excentral Triangle.
28. homothetic to the Extangents Triangle of the Excentral Triangle.
29. homothetic to the Orthic Triangle of the Circum-Incentral Triangle.
30. homothetic to the Tangential Triangle of the Circum-Incentral Triangle.
31. homothetic to the Pedal Triangle of the Orthocenter of the Circum-Incentral Triangle.
32. homothetic to the Antipedal Triangle of the Circumcenter of the Circum-Incentral Triangle.
33. homothetic to the Intangents Triangle of the Circum-Incentral Triangle.
34. homothetic to the Extangents Triangle of the Circum-Incentral Triangle.
35. homothetic to the Medial Triangle of the Euler Triangle.
36. perspective with the Medial Triangle of the First Brocard Triangle.
37. homothetic to the Orthic Triangle of the Yff Central Triangle.

38. perspective with the Medial Triangle of the Neuberg Triangle.
39. perspective with the Medial Triangle of the Reflected Neuberg Triangle.
40. homothetic to the Orthic Triangle of the Hexyl Triangle.
41. homothetic to the Medial Triangle of the Johnson Triangle.
42. homothetic to the Medial Triangle of the Inner Yff Triangle.
43. homothetic to the Medial Triangle of the Outer Yff Triangle.
44. homothetic to the Anticomplementary Triangle of the Euler Triangle.
45. perspective with the Anticomplementary Triangle of the First Brocard Triangle.
46. homothetic to the Tangential Triangle of the Yff Central Triangle.
47. perspective with the Anticomplementary Triangle of the Neuberg Triangle.
48. perspective with the Anticomplementary Triangle of the Reflected Neuberg Triangle.
49. homothetic to the Tangential Triangle of the Hexyl Triangle.
50. homothetic to the Anticomplementary Triangle of the Johnson Triangle.
51. homothetic to the Anticomplementary Triangle of the Inner Yff Triangle.
52. homothetic to the Anticomplementary Triangle of the Outer Yff Triangle.
53. homothetic to the Euler Triangle of the Euler Triangle.
54. homothetic to the Johnson Triangle of the Euler Triangle.
55. homothetic to the Inner Yff Triangle of the Euler Triangle.
56. homothetic to the Outer Yff Triangle of the Euler Triangle.
57. homothetic to the First Brocard Triangle of the First Brocard Triangle.
58. perspective with the Second Brocard Triangle of the First Brocard Triangle.
59. perspective with the Neuberg Triangle of the First Brocard Triangle.
60. perspective with the Reflected Neuberg Triangle of the First Brocard Triangle.
61. homothetic to the Intangents Triangle of the Yff Central Triangle.
62. homothetic to the Malfatti Squares Triangle of the Malfatti Squares Triangle.
63. perspective with the First Brocard Triangle of the Neuberg Triangle.
64. perspective with the Neuberg Triangle of the Neuberg Triangle.
65. perspective with the Reflected Neuberg Triangle of the Neuberg Triangle.
66. perspective with the First Brocard Triangle of the Reflected Neuberg Triangle.
67. perspective with the Neuberg Triangle of the Reflected Neuberg Triangle.
68. perspective with the Reflected Neuberg Triangle of the Reflected Neuberg Triangle.
69. homothetic to the Intangents Triangle of the Hexyl Triangle.
70. homothetic to the Extangents Triangle of the Hexyl Triangle.
71. homothetic to the Euler Triangle of the Johnson Triangle.
72. homothetic to the Inner Yff Triangle of the Johnson Triangle.
73. homothetic to the Outer Yff Triangle of the Johnson Triangle.
74. homothetic to the Euler Triangle of the Inner Yff Triangle.
75. homothetic to the Johnson Triangle of the Inner Yff Triangle.
76. homothetic to the Inner Yff Triangle of the Inner Yff Triangle.
77. homothetic to the Outer Yff Triangle of the Inner Yff Triangle.
78. homothetic to the Euler Triangle of the Outer Yff Triangle.
79. homothetic to the Johnson Triangle of the Outer Yff Triangle.
80. homothetic to the Pedal Triangle of the Circumcenter of the Euler Triangle.
81. perspective with the Pedal Triangle of the Circumcenter of the First Brocard Triangle.
82. homothetic to the Pedal Triangle of the Orthocenter of the Yff Central Triangle.
83. homothetic to the Pedal Triangle of the Symmedian Point of the Malfatti Squares Triangle.
84. perspective with the Pedal Triangle of the Circumcenter of the Neuberg Triangle.

85. perspective with the Pedal Triangle of the Circumcenter of the Reflected Neuberg Triangle.
86. homothetic to the Pedal Triangle of the Orthocenter of the Hexyl Triangle.
87. homothetic to the Pedal Triangle of the Circumcenter of the Johnson Triangle.
88. homothetic to the Pedal Triangle of the Circumcenter of the Inner Yff Triangle.
89. homothetic to the Pedal Triangle of the Circumcenter of the Outer Yff Triangle.
90. perspective with the Antipedal Triangle of the Centroid of the Euler Triangle.
91. homothetic to the Antipedal Triangle of the Orthocenter of the Euler Triangle.
92. perspective with the Antipedal Triangle of the Orthocenter of the First Brocard Triangle.
93. homothetic to the Antipedal Triangle of the Circumcenter of the Yff Central Triangle.
94. homothetic to the Antipedal Triangle of the Centroid of the Malfatti Squares Triangle.
95. perspective with the Antipedal Triangle of the Orthocenter of the Neuberg Triangle.
96. perspective with the Antipedal Triangle of the Orthocenter of the Reflected Neuberg Triangle.
97. homothetic to the Antipedal Triangle of the Circumcenter of the Hexyl Triangle.
98. homothetic to the Antipedal Triangle of the Orthocenter of the Johnson Triangle.
99. homothetic to the Antipedal Triangle of the Orthocenter of the Inner Yff Triangle.
100. homothetic to the Antipedal Triangle of the Orthocenter of the Outer Yff Triangle.
101. homothetic to the Circumcevian Triangle of the Circumcenter of the Euler Triangle.
102. homothetic to the Circum-Orthic Triangle of the Yff Central Triangle.
103. perspective with the Circumcevian Triangle of the Circumcenter of the Malfatti Squares Triangle.
104. homothetic to the Circum-Orthic Triangle of the Hexyl Triangle.
105. homothetic to the Circumcevian Triangle of the Circumcenter of the Johnson Triangle.
106. homothetic to the Circumcevian Triangle of the Circumcenter of the Inner Yff Triangle.
107. homothetic to the Circumcevian Triangle of the Circumcenter of the Outer Yff Triangle.
108. perspective with the Outer Fermat Triangle of the First Brocard Triangle.
109. perspective with the Inner Fermat Triangle of the First Brocard Triangle.
110. perspective with the Outer Fermat Triangle of the Neuberg Triangle.
111. perspective with the Inner Fermat Triangle of the Neuberg Triangle.
112. perspective with the Outer Fermat Triangle of the Reflected Neuberg Triangle.
113. perspective with the Inner Fermat Triangle of the Reflected Neuberg Triangle.
114. homothetic to the Triangle of the Centroids of the Triangulation Triangles of the Incenter.
115. perspective with the Triangle of the Skordev Points of the Triangulation Triangles of the Incenter.
116. homothetic to the Triangle of the Centroids of the Triangulation Triangles of the Centroid.
117. homothetic to the Triangle of the Centroids of the Triangulation Triangles of the Circumcenter.

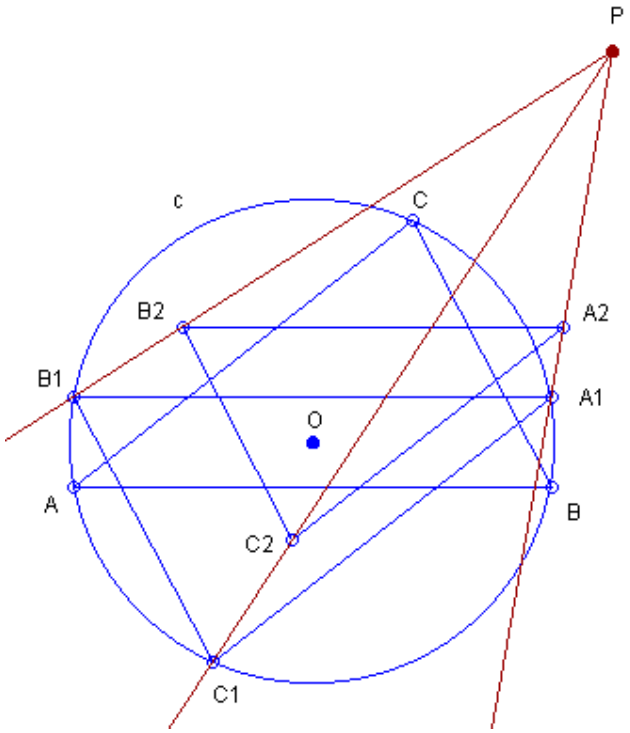
118. homothetic to the Triangle of the First Feuerbach Points of the Triangulation Triangles of the Circumcenter.
119. homothetic to the Triangle of the Kiepert Centers of the Triangulation Triangles of the Circumcenter.
120. perspective with the Triangle of the Skordev Points of the Triangulation Triangles of the Circumcenter.
121. homothetic to the Triangle of the Centroids of the Triangulation Triangles of the Orthocenter.
122. homothetic to the Triangle of the Circumcenters of the Triangulation Triangles of the Orthocenter.
123. homothetic to the Triangle of the de Longchamps Points of the Triangulation Triangles of the Orthocenter.
124. homothetic to the Triangle of the Centers of the Orthocentroidal Circles of the Triangulation Triangles of the Orthocenter.
125. homothetic to the Triangle of the Centers of the Taylor Circles of the Triangulation Triangles of the Orthocenter.
126. homothetic to the Triangle of the Skordev Points of the Triangulation Triangles of the Orthocenter.
127. perspective with the Triangle of the Circumcenters of the Triangulation Triangles of the Outer Fermat Point.
128. homothetic to the Triangle of the Centroids of the Triangulation Triangles of the Inner Fermat Point.
129. perspective with the Triangle of the Circumcenters of the Triangulation Triangles of the Inner Fermat Point.
130. perspective with the Triangle of the de Longchamps Points of the Triangulation Triangles of the Outer Vecten Point.
131. homothetic to the Triangle of the Centroids of the Triangulation Triangles of the Inner Vecten Point.
132. perspective with the Triangle of the de Longchamps Points of the Triangulation Triangles of the Inner Vecten Point.
133. homothetic to the Stevanovic Triangle of the Centroids of the Triangulation triangles of the Incenter.
134. homothetic to the Stevanovic Triangle of the Centroids of the Triangulation triangles of the Centroid.
135. homothetic to the Stevanovic Triangle of the Incenters of the Triangulation triangles of the Circumcenter.
136. homothetic to the Triangle of the Incenters of the Corner Triangles of the Centroid.
137. homothetic to the Triangle of the Orthocenters of the Anticevian Corner Triangles of the Circumcenter.
138. homothetic to the Triangle of the Orthocenters of the Anticevian Corner Triangles of the Orthocenter.
139. homothetic to the Triangle of the Orthocenters of the Anticevian Corner Triangles of the Symmedian Point.
140. homothetic to the Triangle of the First Feuerbach Points of the Anticevian Corner Triangles of the Symmedian Point.
141. homothetic to the Triangle of the Kiepert Centers of the Anticevian Corner Triangles of the Symmedian Point.
142. perspective with the Triangle of the Skordev Points of the Anticevian Corner Triangles of the Symmedian Point.

143. homothetic to the Triangle of reflections of the Nine-Point Center in the sides of the Medial Triangle.
144. homothetic to the Triangle of reflections of the Orthocenter in the sides of the Orthic Triangle.
145. perspective with the Triangle of reflections of the Skordev Point in the sides of the Orthic Triangle.
146. homothetic to the Triangle of reflections of the Incenter in the sides of the Excentral Triangle.
147. perspective with the Triangle of reflections of the Symmedian Point in the sides of the Excentral Triangle.
148. homothetic to the Triangle of reflections of the Orthocenter in the sides of the Anticomplementary Triangle.
149. homothetic to the Triangle of reflections of the de Longchamps Point in the sides of the Anticevian Triangle of the Orthocenter.
150. homothetic to the Triangle of reflections of the Circumcenter in the sides of the Tangential Triangle.
151. perspective with the Triangle of reflections of the Skordev Point in the sides of the Tangential Triangle.
152. homothetic to the Triangle of reflections of the Incenter in the vertices of the Medial Triangle.
153. homothetic to the Triangle of reflections of the Centroid in the vertices of the Medial Triangle.
154. homothetic to the Triangle of reflections of the Second Beltrami Point in the vertices of the Anticomplementary Triangle.
155. perspective with the Triangle of reflections of the Skordev Point in the vertices of the Anticevian Triangle of the Orthocenter.
156. homothetic to the Triangle of reflections of the vertices of the Medial Triangle in the Incenter.
157. homothetic to the Triangle of reflections of the Circumcenter in the sides of Triangle ABC.
158. perspective with the Hatzipolakis Triangle of the Centroid.
159. homothetic to the Desmic Mate the Euler Triangle.
160. perspective with the Desmic Mate the Third Brocard Triangle.
161. homothetic to the Desmic Mate the Inner Yff Triangle.

Theorems 1 and 20 are a part of theorem 2, [5]. We invite the reader to prove the other theorems. We illustrate a few of the above theorems.

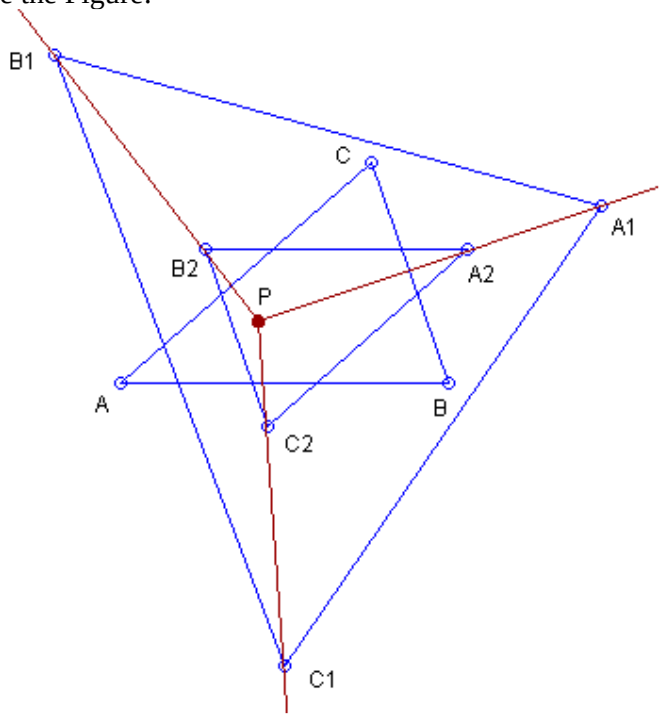
Theorem 2. The Stanilov Triangle is homothetic to the Circumcevian Triangle of the Circumcenter.

See the Figure:



O - Circumcenter;
c - Circumcircle;
$A_1B_1C_1$ - Circumcevian Triangle of the Circumcenter;
$A_2B_2C_2$ - Stanilov Triangle;
Triangles $A_1B_1C_1$ and $A_2B_2C_2$ are homothetic with center of homothety point P.

Theorem 13. The Stanilov Triangle is perspective with the Outer Fermat Triangle.
See the Figure:

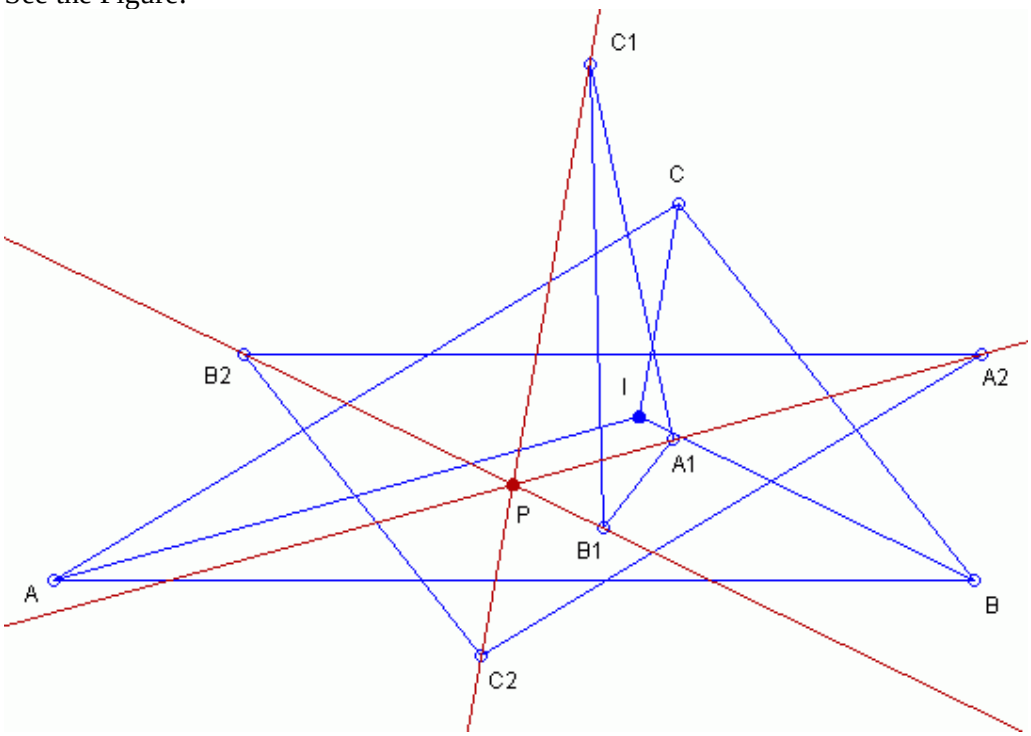


$A_1B_1C_1$ - Outer Fermat Triangle;

$A_2B_2C_2$ - Stanilov Triangle;
Lines A_1A_2, B_1B_2 and C_1C_2 concur in point P, that is, triangles $A_1B_1C_1$ and $A_2B_2C_2$ are perspective with perspector point P.

Theorem 115. The Stanilov Triangle is perspective with the Triangle of the Skordev Points of the Triangulation Triangles of the Incenter.

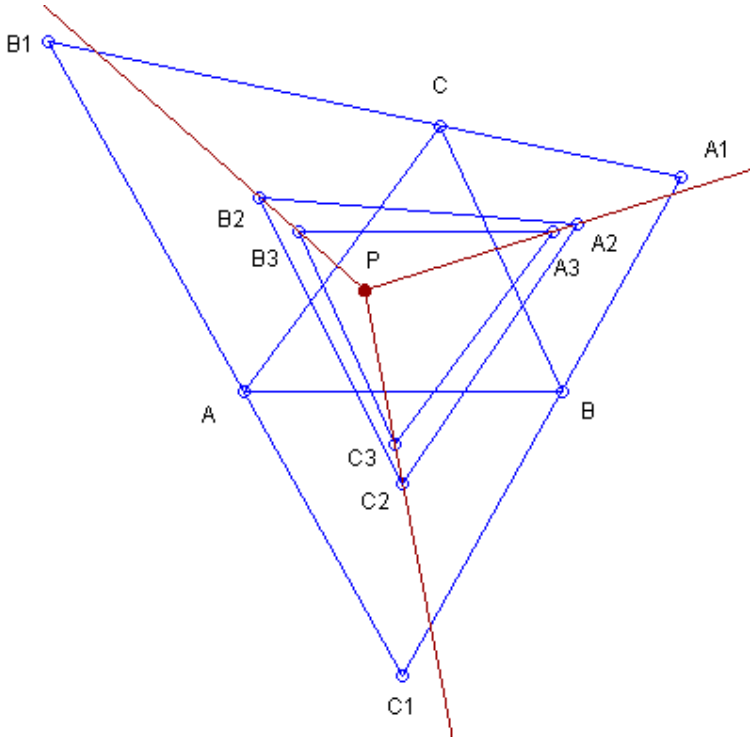
See the Figure:



I - Incenter;
A_1 - Skordev Point of triangle BCI;
B_1 - Skordev Point of triangle CAI;
C_1 - Skordev Point of triangle ABI;
$A_1B_1C_1$ - Triangle of the Skordev Points of the Triangulation Triangles of the Incenter;
$A_2B_2C_2$ - Stanilov Triangle;
Lines A_1A_2, B_1B_2 and C_1C_2 concur in point P, that is, triangles $A_1B_1C_1$ and $A_2B_2C_2$ are perspective with perspector point P.

Theorem 142. The Stanilov Triangle is perspective with the Triangle of the Skordev Points of the Anticevian Corner Triangles of the Symmedian Point.

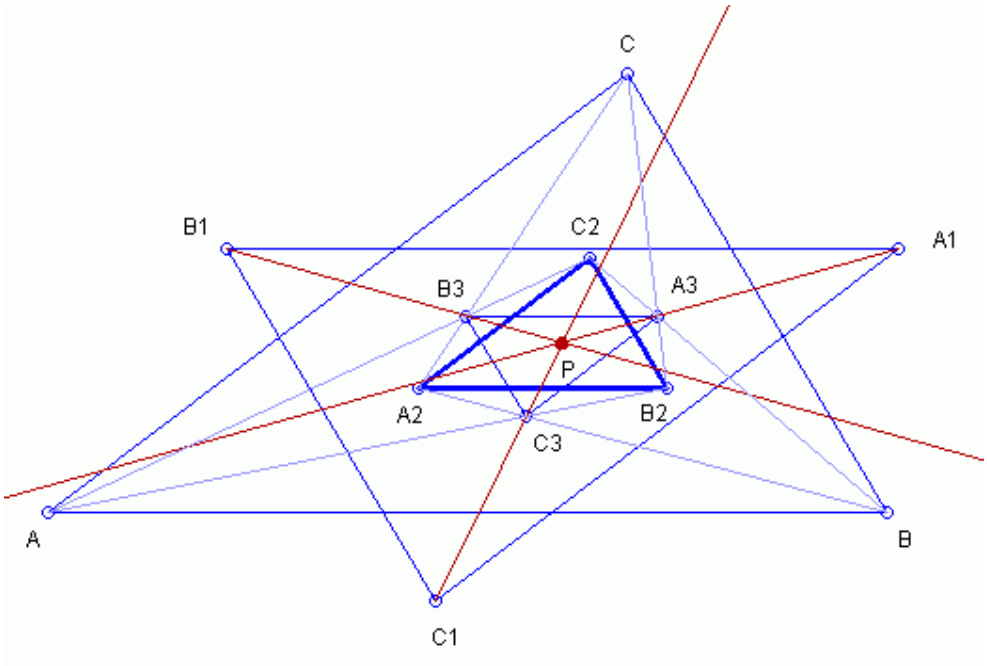
See the Figure:



$A_1B_1C_1$ - Anticevian Triangles of the Symmedian Point = Tangential Triangle;
A_2 - Skordev Point of triangle BCA_1;
B_2 - Skordev Point of triangle CAB_1;
C_2 - Skordev Point of triangle ABC_1;
$A_2B_2C_2$ - Triangle of the Skordev Points of the Anticevian Corner Triangles of the Symmedian Point;
$A_3B_3C_3$ - Stanilov Triangle;
Lines A_2A_3, B_2B_3 and C_2C_3 concur in point P, that is, triangles $A_2B_2C_2$ and $A_3B_3C_3$ are perspective with perspector point P.

Theorem 161. The Stanilov Triangle is homothetic to the Desmic Mate the Inner Yff Triangle.

See the Figure:



$A_1B_1C_1$ - Stanilov Triangle;
$A_2B_2C_2$ - Inner Yff Triangle;
A_3 - intersection point of lines BC_2 and CB_2;
B_3 - intersection point of lines CA_2 and AC_2;
C_3 - intersection point of lines AB_2 and BA_2;
$A_3B_3C_3$ - Desmic Mate the Inner Yff Triangle;
P - homothetic center of triangles $A_1B_1C_1$ and $A_3B_3C_3$.

Thanks

The figures in this note are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download in the Web: Rene Grothmann's C.a.R.. It is free and open source. The reader may verify easily the statements of this paper by using C.a.R. Many thanks to Rene Grothmann for his wonderful program.

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