Computer-Generated Mathematics:
Construction of the Stanilov Triangle

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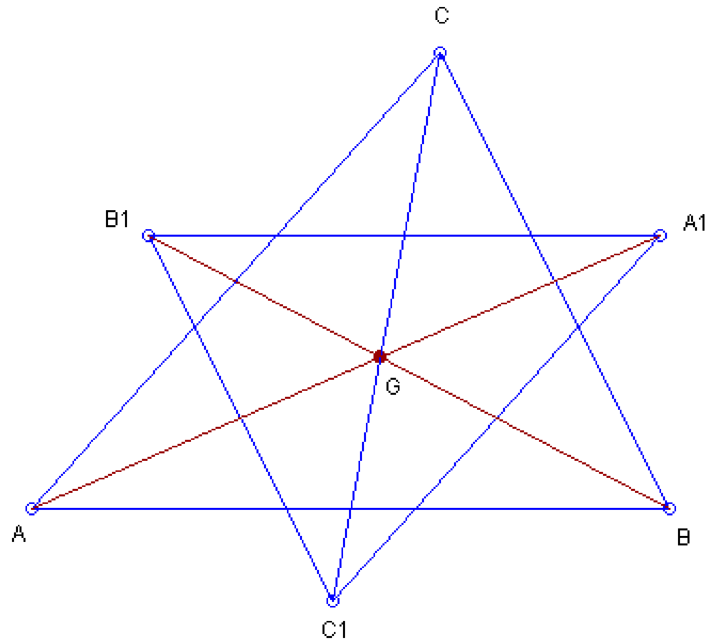
Abstract. By using the computer program "Machine for Questions and Answers", we find 171 different ways to construct the Stanilov Triangle by using a straightedge and compass.

Constructions of geometric objects in the plane by using straightedge and compass only, is an essential part of school education in Geometry. In this paper we illustrate the use of the computer program "Machine for Questions and Answers" (The Machine), created by the author of the paper, for discovering of theorems useful for the straightedge-and-compass constructions. We show 171 ways to construct the Stanilov Triangle by using straightedge and compass. The method used in this paper could be useful in many similar situations. See e,g. applications of the method for construction of the Apollonius circle [4], Malfatti squares triangle [5], Outer Gallatly-Kiepert triangle [6], Intangents triangle [7].

Grozyo Stanilov [9,10] defines the Stanilov triangle by using statements from differential geometry. He proves that the Stanilov triangle is homothetic to triangle ABC under the homothety with center the centroid and ratio equal to -4/5. We use this result to give an alternative definition of the Stanilov triangle, as follows. The *Stanilov triangle* is the homothetic image of triangle ABC under homothety with center the centroid of triangle ABC and ratio equal to -4/5. See the Figure:



In the Figure, point G is the Gentroid of triangle ABC. Point A_1 lies on line AG and the length of segment GA_1 is equal to 4/5 of the length of segment AG. Similarly cosntruct points B_1 and C_1. Triangle $A_1B_1C_1$ is the Stanilov Triangle.

We could construct the Stanilov Triangle by using the above definition. We use the Machine to find 171 additional ways how to construct the Stanilov Triangle. In these ways we do not need to use any ration of any homothety.

We use the following method. We can construct a triangle, if we can construct

- A triangle perspective to the triangle, and the perspector,
- and, a second triangle perspective to the triangle, and the second perspector.

We use the Machine to specify a few theorems given in the paper [8]. We obtain the following theorems (the list below could be extended by the reader by adding additional similar theorems):

Theorem 1. The Stanilov Triangle and the Triangle of reflections of the de Longchamps Point in the vertices of the Anticomplementary Triangle are homothetic with homothetic center the Circumcenter.

Theorem 2. The Stanilov Triangle and the Triangle of reflections of the Nine-Point Center in the vertices of the Medial Triangle are homothetic with homothetic center the Orthocenter.

Theorem 3. The Stanilov Triangle and the Triangle of reflections of the Orthocenter in the vertices of the Anticomplementary Triangle are homothetic with homothetic center the Nine-Point Center.

Theorem 4. The Stanilov Triangle and the Triangle of reflections of the Nagel Point in the vertices of the Anticomplementary Triangle are homothetic with homothetic center the Spieker Center.

Theorem 5. The Stanilov Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Orthocenter are homothetic with homothetic center the de Longchamps Point.

Theorem 6. The Stanilov Triangle and the Triangle of reflections of the Equal Parallelians Point in the vertices of the Anticomplementary Triangle are homothetic with homothetic center the Grinberg Point.

Theorem 7. The Stanilov Triangle and the Triangle of reflections of the Grinberg Point in the vertices of the Medial Triangle are homothetic with homothetic center the Equal Parallelians Point.

Theorem 8. The Stanilov Triangle and the Triangle of reflections of the Skordev Point in the sides of the Orthic Triangle are perspective with perspector the Skordev Point.

Theorem 9. The Stanilov Triangle and the Triangle of reflections of the Mittenpunkt in the vertices of the Medial Triangle are homothetic with homothetic center the Gergonne Point of the Anticomplementary Triangle.

Theorem 10. The Stanilov Triangle and the Triangle of reflections of the Incenter in the vertices of the Medial Triangle are homothetic with homothetic center the Nagel Point of the Anticomplementary Triangle.

Theorem 11. The Stanilov Triangle and the Triangle of reflections of the Symmedian Point in the vertices of the Medial Triangle are homothetic with homothetic center the Perspector of the Orthic Triangle and the Anticomplementary Triangle.

Theorem 12. The Stanilov Triangle and the Triangle of reflections of the Brocard Midpoint in the vertices of the Medial Triangle are homothetic with homothetic center the Perspector of the Symmedial Triangle and the Anticomplementary Triangle.

Theorem 13. The Stanilov Triangle and the Triangle of reflections of the Circumcenter in the vertices of the Anticomplementary Triangle are homothetic with homothetic center the Complement of the Nine-Point Center.

Theorem 14. The Stanilov Triangle and the Triangle of reflections of the Gergonne Point in the vertices of the Anticomplementary Triangle are homothetic with homothetic center the Complement of the Mittenpunkt.

Theorem 15. The Stanilov Triangle and the Triangle of reflections of the Incenter in the vertices of the Anticomplementary Triangle are homothetic with homothetic center the Complement of the Spieker Center.

Theorem 16. The Stanilov Triangle and the Triangle of reflections of the Center of the Orthocentroidal Circle in the vertices of the Anticomplementary Triangle are homothetic with homothetic center the Midpoint of the Centroid and the Nine-Point Center.

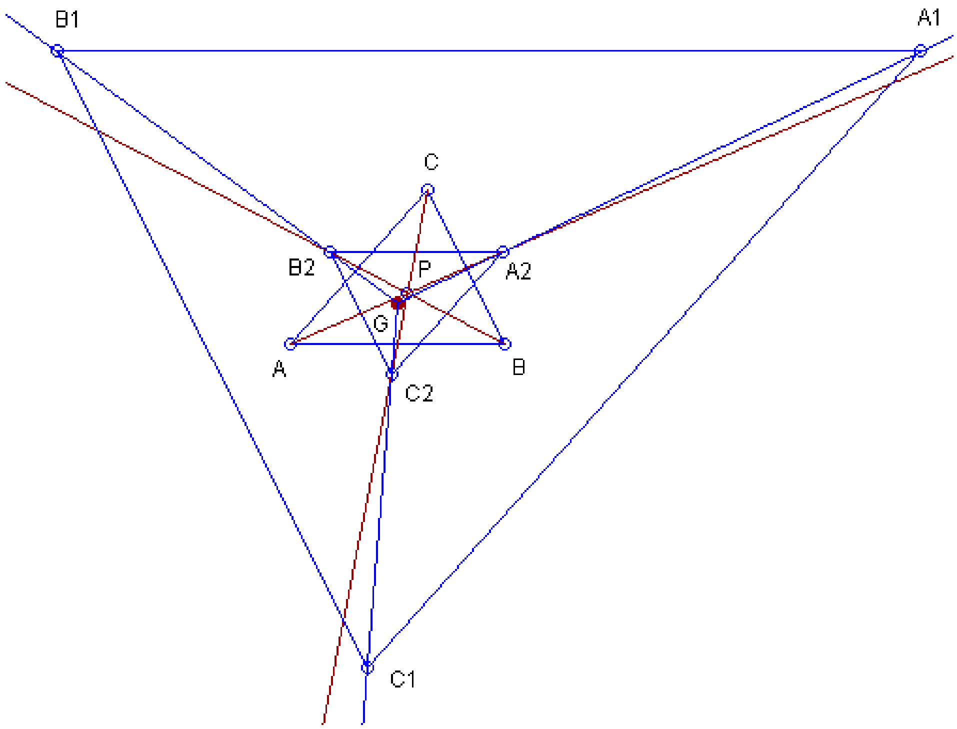
Theorem 17. The Stanilov Triangle and the Triangle of reflections of the Bevan Point in the vertices of the Anticomplementary Triangle are homothetic with homothetic center the Midpoint of the Circumcenter and the Spieker Center.

Theorem 18. The Stanilov Triangle and the Triangle of reflections of the Center of the Fuhrmann Circle in the vertices of the Anticomplementary Triangle are homothetic with homothetic center the Midpoint of the Nine-Point Center and the Spieker Center.

The reader may find the definitions in [1,2,3,11]. We invite the reader to prove the above theorems..
We use the above theorems to obtain 171 ways to construct the Stanilov Triangle: The definition of the Stanilov triangle and the above 18 perspectives give 171 ways (Clearly, if the reader extends the list of the perspectives, he/she will obtain additional ways.)

Solution 1

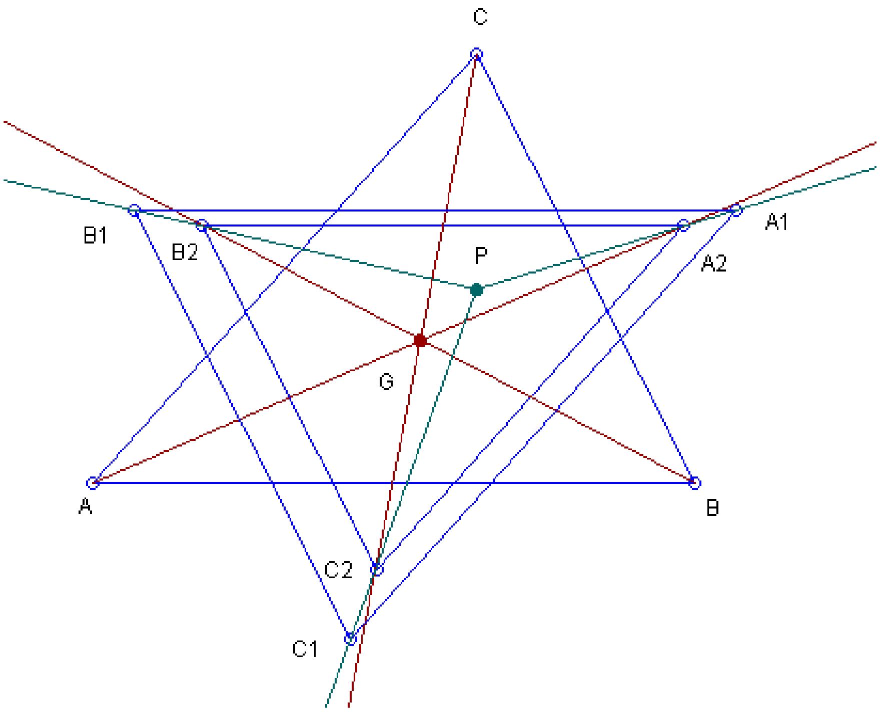
We use the definition and Theorem 1. See the Figure:



G - Centroid;
P - Circumcenter;
$A_1B_1C_1$ - Triangle of reflections of the de Longchamps Point in the vertices of the Anticomplementary Triangle;
A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

Solution 2

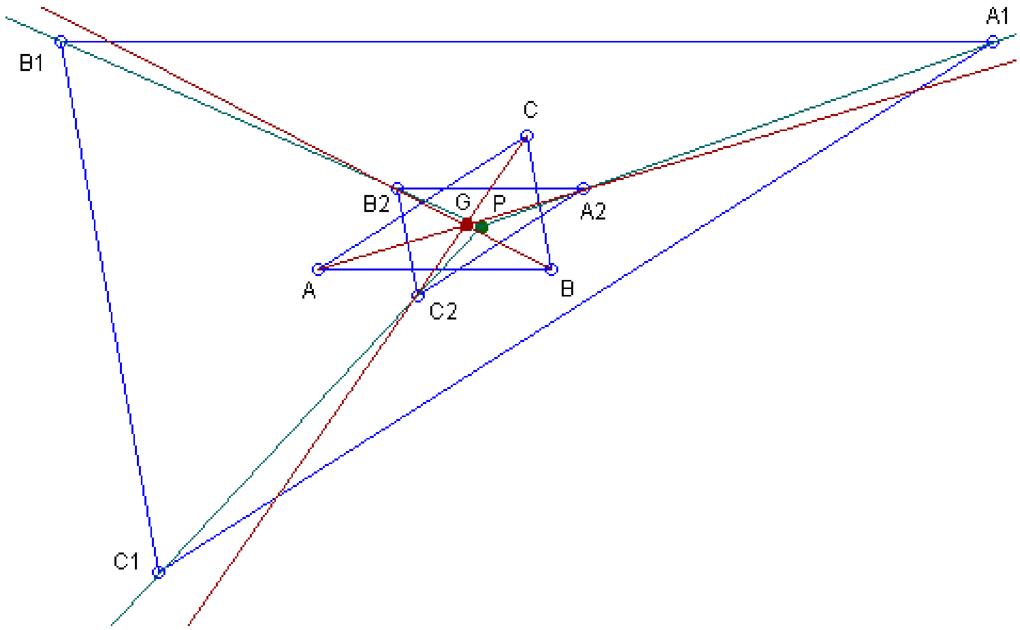
We use the definition and Theorem 2. See the Figure:



G - Centroid;
P - Orthocenter;
$A_1B_1C_1$ - Triangle of reflections of the Nine-Point Center in the vertices of the Medial Triangle;
A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

Solution 3

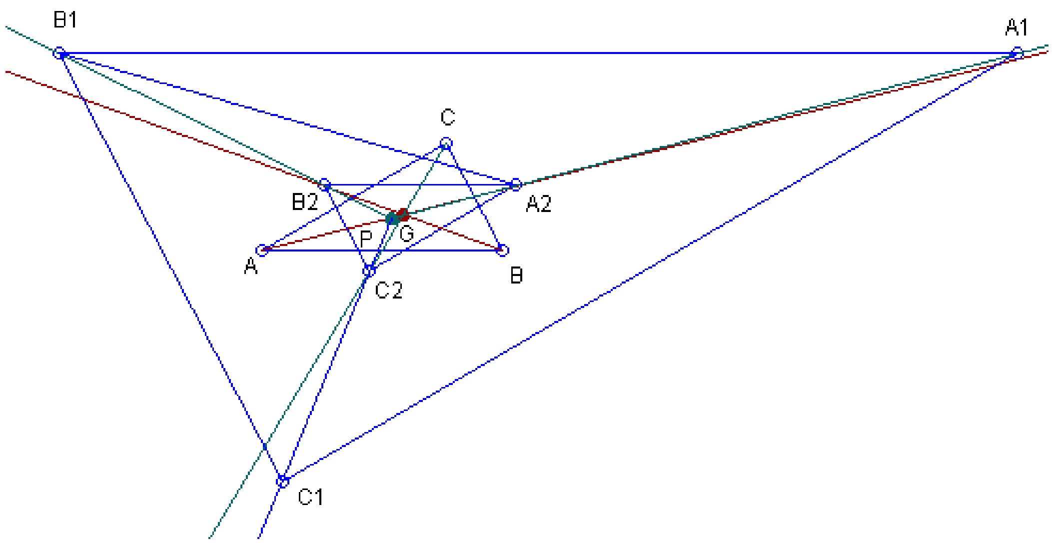
We use the definition and Theorem 3. See the Figure:



G - Centroid;
P - Nine-Point Center;
$A_1B_1C_1$ - Triangle of reflections of the Orthocenter in the vertices of the Anticomplementary Triangle;
A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

Solution 4

We use the definition and Theorem 4. See the Figure:

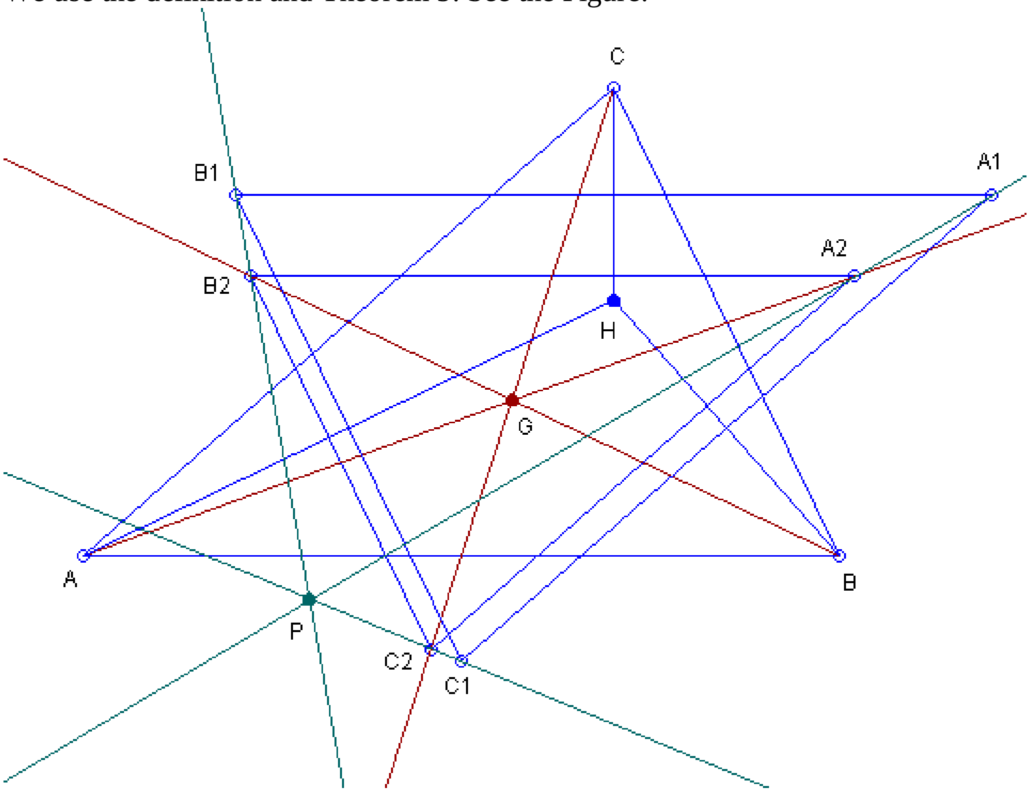


G - Centroid;
P - Spieker Center;
$A_1B_1C_1$ - Triangle of reflections of the Nagel Point in the vertices of the Anticomplementary Triangle;

A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

Solution 5

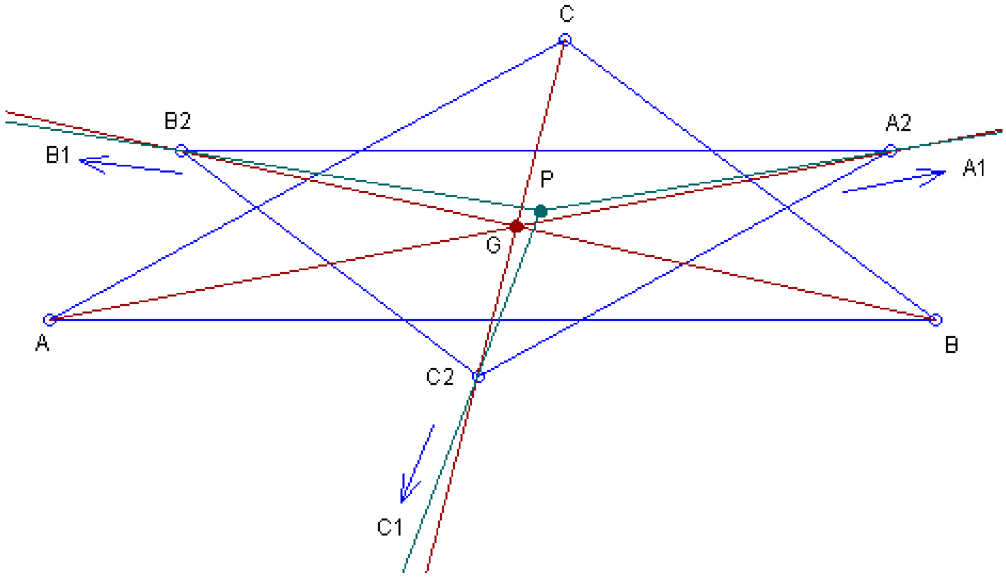
We use the definition and Theorem 5. See the Figure:



G - Centroid;
P - de Longchamps Point;
$A_1B_1C_1$ - Triangle of the Circumcenters of the Triangulation Triangles of the Orthocenter;
A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

Solution 6

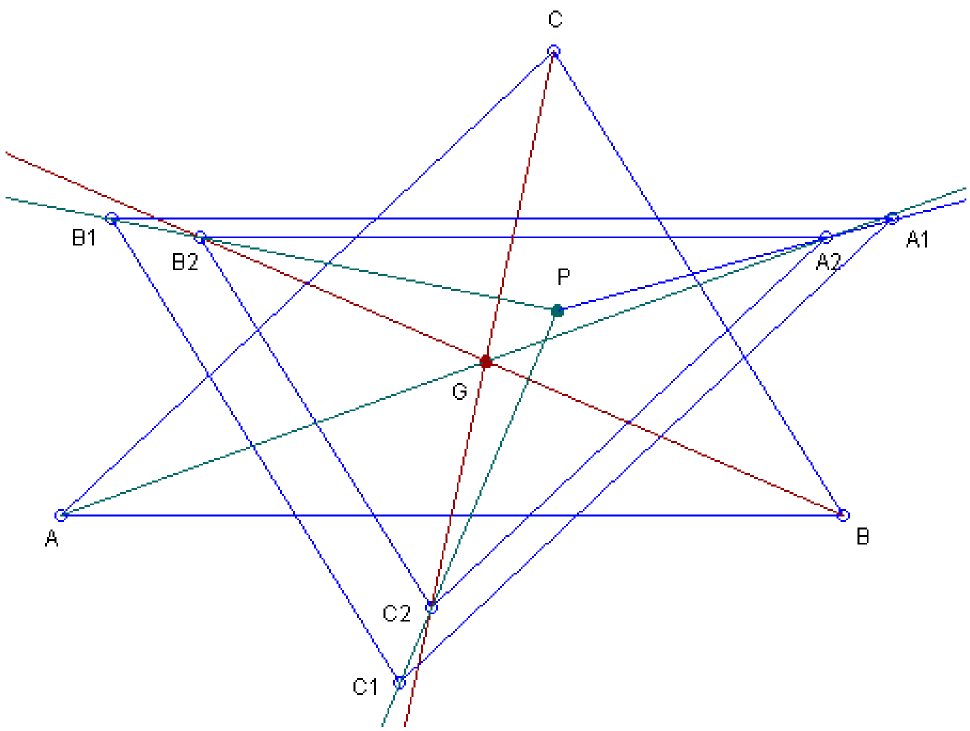
We use the definition and Theorem 6. See the Figure:



G - Centroid;
P - Grinberg Point;
$A_1B_1C_1$ - Triangle of reflections of the Equal Parallelians Point in the vertices of the Anticomplementary Triangle (Outside the picture);
A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

Solution 7

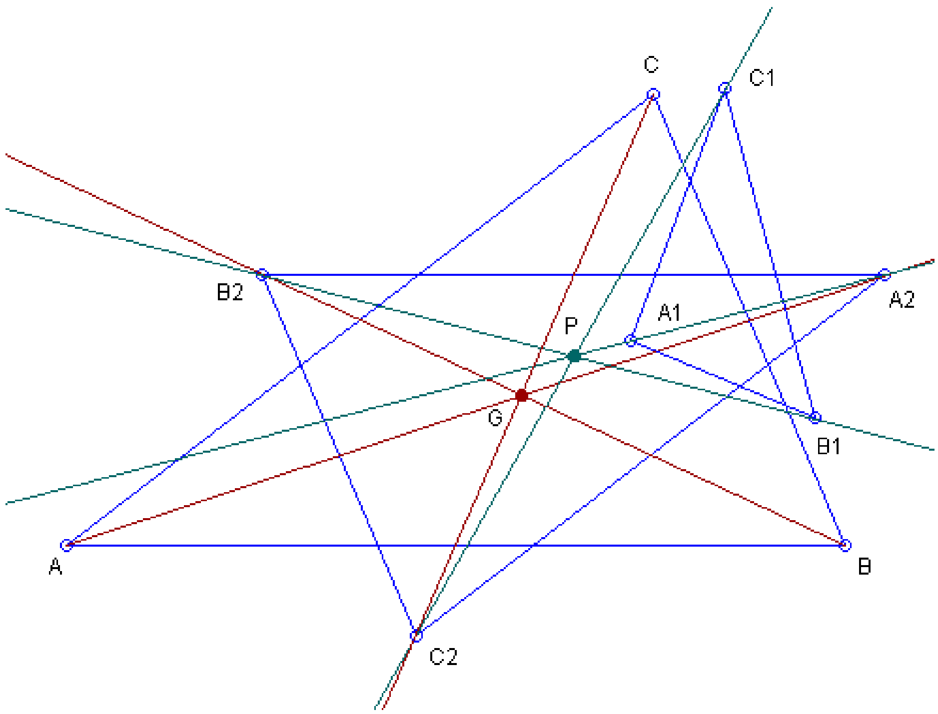
We use the definition and Theorem 7. See the Figure:



G - Centroid;
P - Equal Parallelians Poin;
$A_1B_1C_1$ - Triangle of reflections of the Grinberg Point in the vertices of the Medial Triangle;
A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

Solution 8

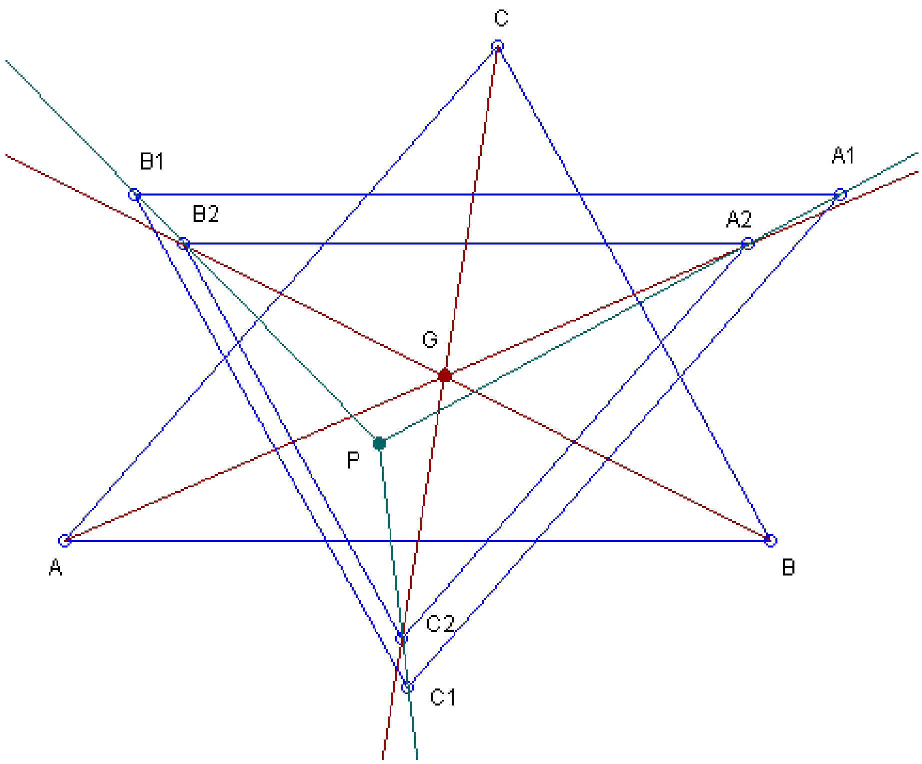
We use the definition and Theorem 8. See the Figure:



G - Centroid;
P - Skordev Point;
$A_1B_1C_1$ - Triangle of reflections of the Skordev Point in the sides of the Orthic Triangle;
A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

Solution 9

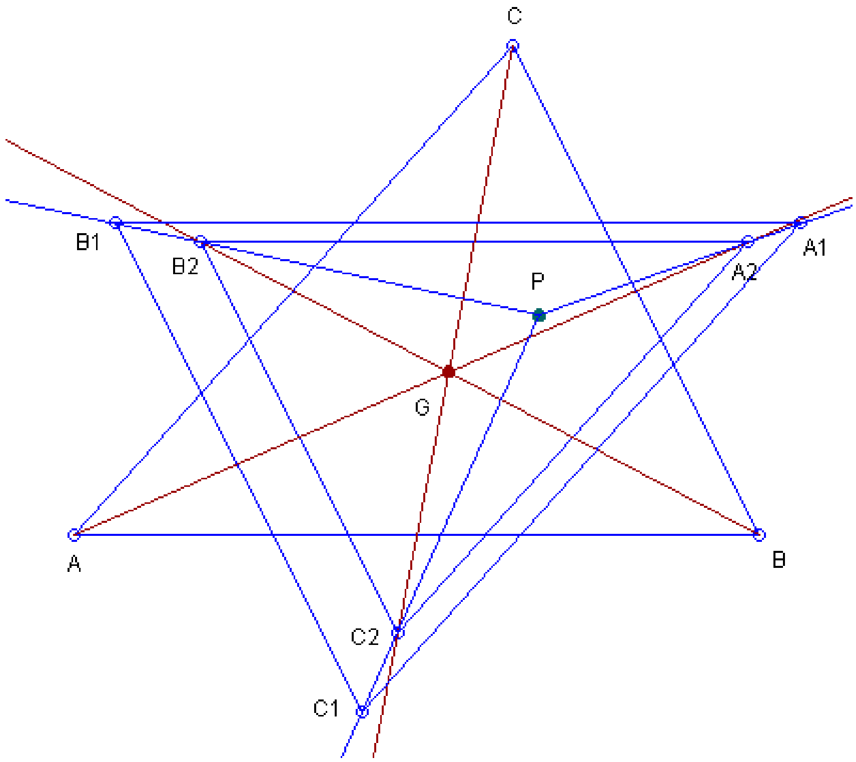
We use the definition and Theorem 9. See the Figure:



G - Centroid;
P - Gergonne Point of the Anticomplementary Triangle;
$A_1B_1C_1$ - Triangle of reflections of the Mittenpunkt in the vertices of the Medial Triangle;
A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

Solution 10

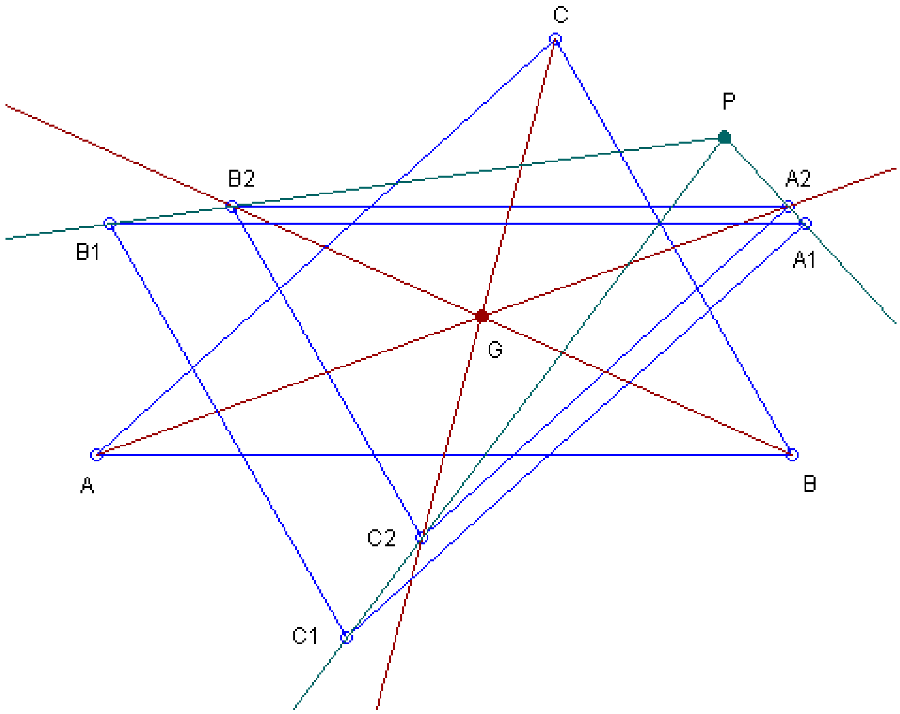
We use the definition and Theorem 10. See the Figure:



G - Centroid;
P - Nagel Point of the Anticomplementary Triangle;
$A_1B_1C_1$ - Triangle of reflections of the Incenter in the vertices of the Medial Triangle;
A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

Solution 11

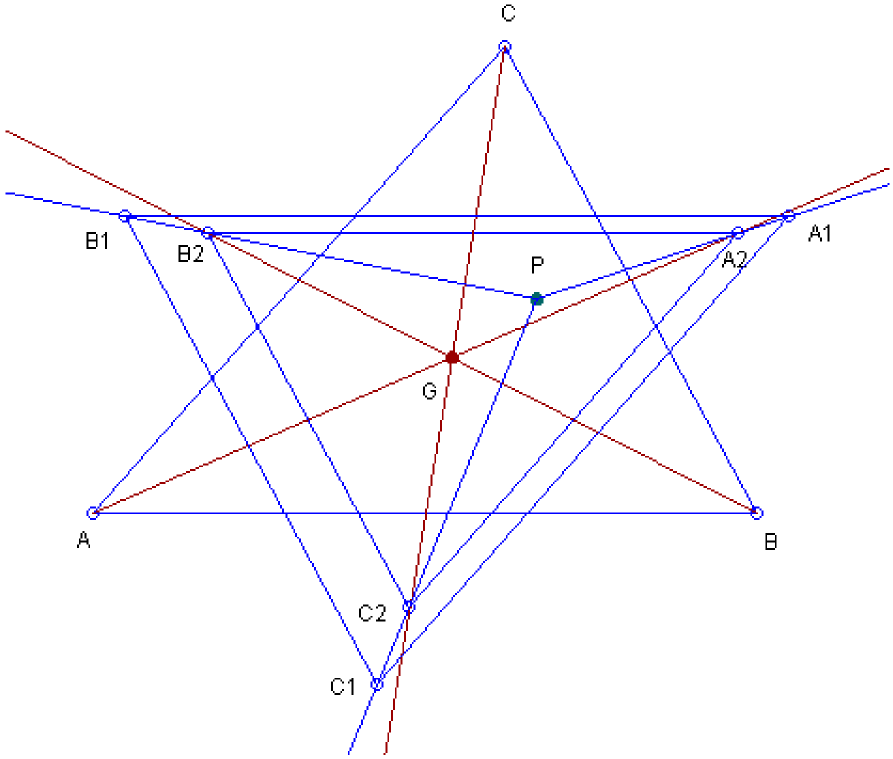
We use the definition and Theorem 11. See the Figure:



G - Centroid;
P - Perspector of the Orthic Triangle and the Anticomplementary Triangle;
$A_1B_1C_1$ - Triangle of reflections of the Symmedian Point in the vertices of the Medial Triangle;
A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

Solution 12

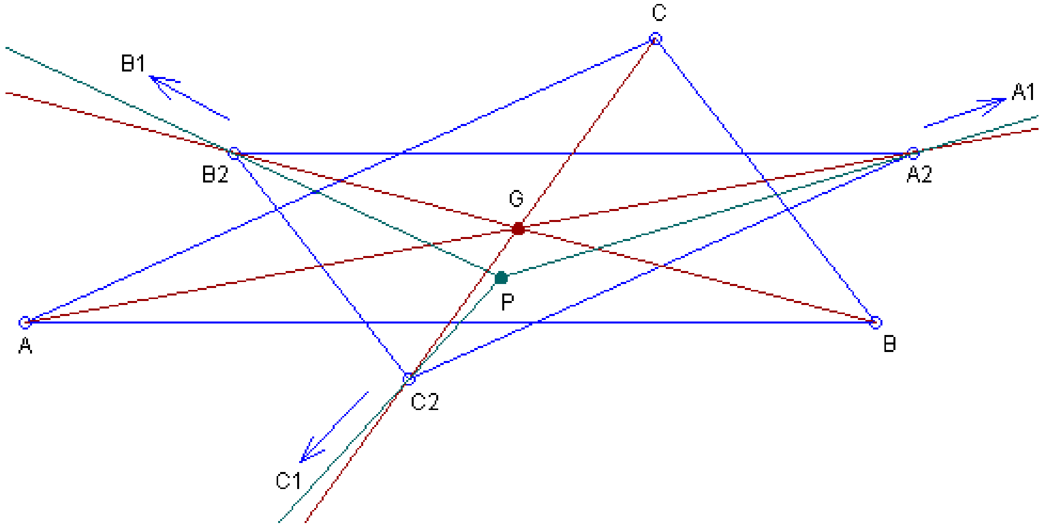
We use the definition and Theorem 12. See the Figure:



G - Centroid;
P - Perspector of the Symmedial Triangle and the Anticomplementary Triangle;
$A_1B_1C_1$ - Triangle of reflections of the Brocard Midpoint in the vertices of the Medial Triangle;
A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

Solution 13

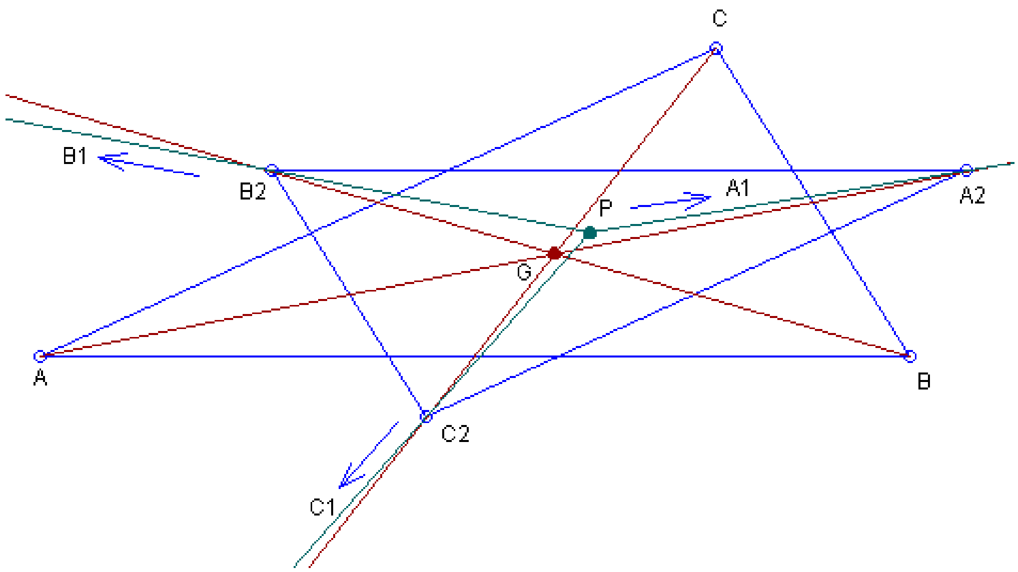
We use the definition and Theorem 13. See the Figure:



G - Centroid;
P - Complement of the Nine-Point Center;
$A_1B_1C_1$ - Triangle of reflections of the Circumcenter in the vertices of the Anticomplementary Triangle (Outside the picture);
A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

Solution 14

We use the definition and Theorem 14. See the Figure:

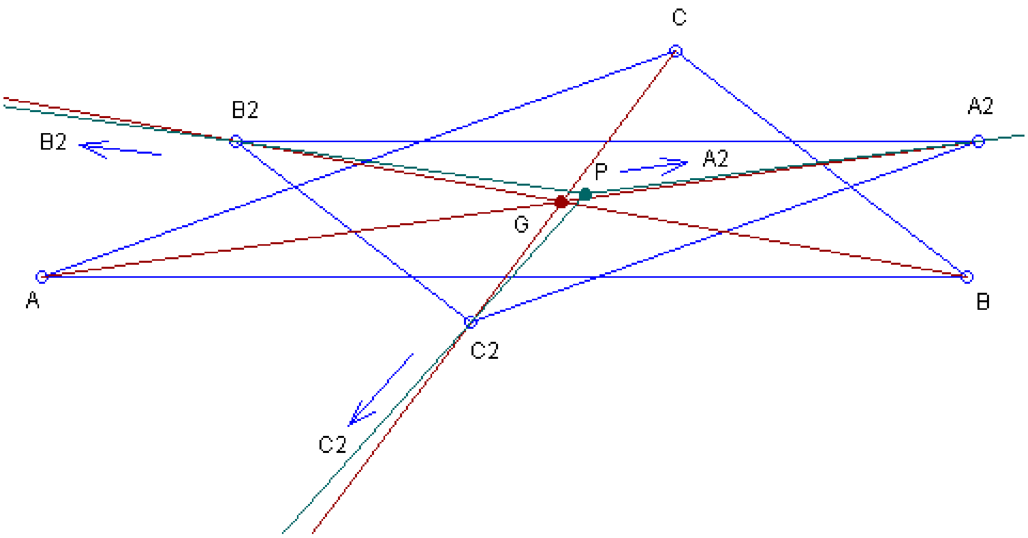


G - Centroid;
P - Complement of the Mittenpunkt;
$A_1B_1C_1$ - Triangle of reflections of the Gergonne Point in the vertices of the Anticomplementary Triangle (Outside the picture);

A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

Solution 15

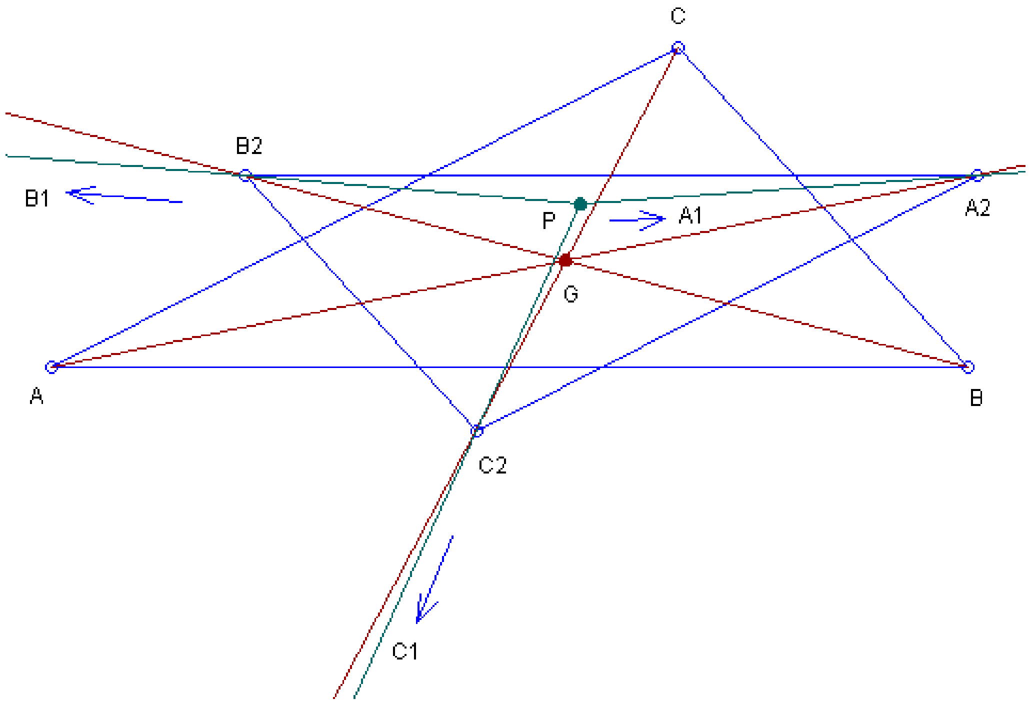
We use the definition and Theorem 15. See the Figure:



G - Centroid;
P - Complement of the Spieker Center;
$A_1B_1C_1$ - Triangle of reflections of the Incenter in the vertices of the Anticomplementary Triangle (Outside the picture);
A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

Solution 16

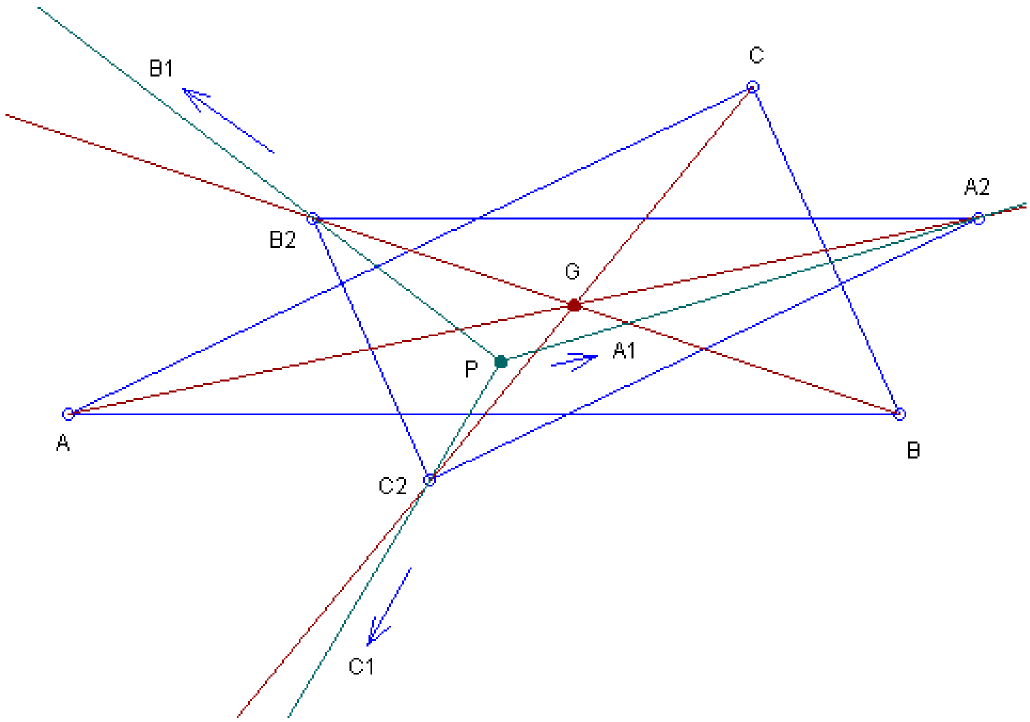
We use the definition and Theorem 16. See the Figure:



G - Centroid;
P - Midpoint of the Centroid and the Nine-Point Center;
$A_1B_1C_1$ - Triangle of reflections of the Center of the Orthocentroidal Circle in the vertices of the Anticomplementary Triangle (Outside the picture);
A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

Solution 17

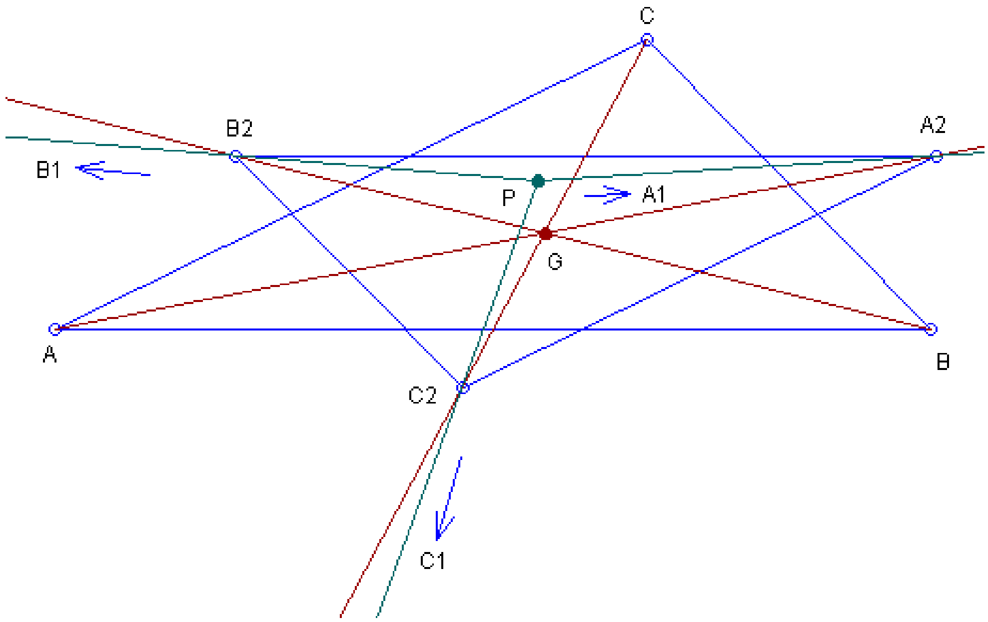
We use the definition and Theorem 17. See the Figure:



G - Centroid;
P - Midpoint of the Circumcenter and the Spieker Center;
$A_1B_1C_1$ - Triangle of reflections of the Bevan Point in the vertices of the Anticomplementary Triangle (Outside the picture);
A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

Solution 18

We use the definition and Theorem 18. See the Figure:



G - Centroid;
P - Midpoint of the Nine-Point Center and the Spieker Center;
$A_1B_1C_1$ - Triangle of reflections of the Center of the Fuhrmann Circle in the vertices of the Anticomplementary Triangle (Outside the picture);
A_2 - intersection point of lines GA and PA_1;
B_2 - intersection point of lines GB and PB_1;
C_2 - intersection point of lines BC and PC_1;
$A_2B_2C_2$ - Stanilov Triangle.

We leave the other solutions to the reader. To obtain the other solutions, we have to use Theorems 1 and 2, Theorems 1 and 3, etc. It is clear that if we have n perspectives, we obtain $n(n-1)/2$ different ways how to construct the desired triangle. In this case $n = 19$ (the definition and theorems 1 to 18), hence we have 171 solutions.

Thanks

The figures in this note are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download in the Web: Rene Grothmann's C.a.R.. It is free and open source. The reader may verify easily the statements of this paper by using C.a.R. Many thanks to Rene Grothmann for his wonderful program.

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