Journal of Computer-Generated Euclidean Geometry

# Towards the first computer-generated encyclopedia

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Abstract. We are going into a new era in which the computers will replace the people as researchers. Here we demonstrate the beginning of the new era. The first achievement of the coming new era is the production of the first computer-generated encyclopedia.

Keywords: computer-generated mathematics, encyclopedia, Euclidean Geometry

"Within ten years a digital computer will discover and prove an important mathematical theorem." (Simon and Newell, 1958).

This is the famous prediction by Simon and Newell [1]. Now is 2008, 50 years later. The first computer program able easily to discover new deep mathematical theorems - The *Machine for Questions and Answers* (The *Machine*) [2,3] has been created in 2006, that is, 48 years after the prediction. The Machine is the first computer program able to create an encyclopedia. The new theorems discovered by now by the Machine (approximately 100 to 500 new mathematical theorems) are the between the first mathematical theorems discovered by a computer. The Machine could be easily improved so that it will be able to rediscover the current Euclidean Geometry and then to extend it at least three times. Further extensions of the Machine in many directions are possible. Hence, the prediction by Simon and Newell of 1958 now goes into the realty.

Computer algebra systems like Maple, Mathematica, GAP, etc. do not have the ability to discover new mathematical theorems. Mathematical provers like Isabelle (Prof. Lawrence Paulson, University of Cambridge, and Prof. Tobias Nipkow, Technical University of Munich) do not discover new theorems. The programs created by Douglas Lenat (Austin, USA) have not discovered by now a new mathematical theorem. Today the main research centers in Europe, like Research Institute for Symbolic Computation, RISC, Austria, Cambridge and Oxford, UK; Aachen and Munich, Germany, even do not conduct any research in the area of automated discovery in mathematics. The same centers have not produced by now even one new mathematical theorem discovered by a computer. My search at the Web could not find any person in Germany, working in the area of automated discovery in mathematics.

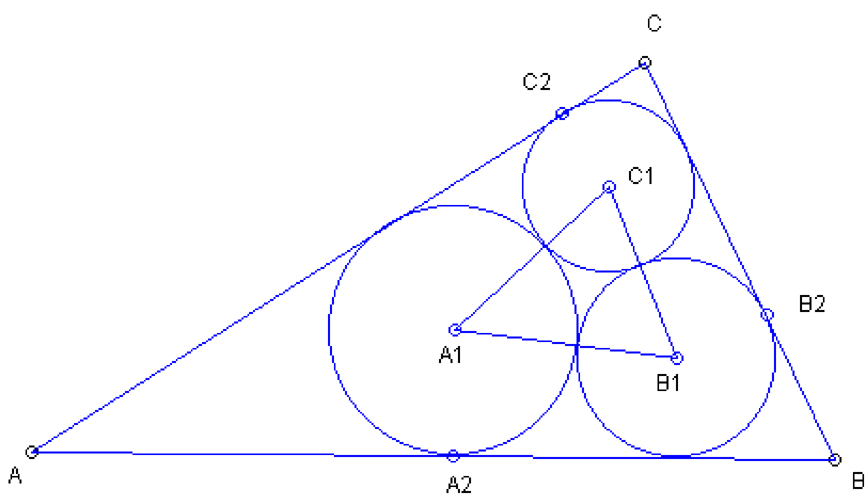
In 2006 the Machine produced the first computer-generated encyclopedia [2]. In order to publish the results produced by the Machine, In 2006 I founded the Journal of Computer-Generated Euclidean Geometry - the first journal devoted to mathematics created by computers. By the moment in the journal are published more than 100 new mathematical theorems, discovered by a computer.

### An example

In [2] we find the following three computer-generated theorems:

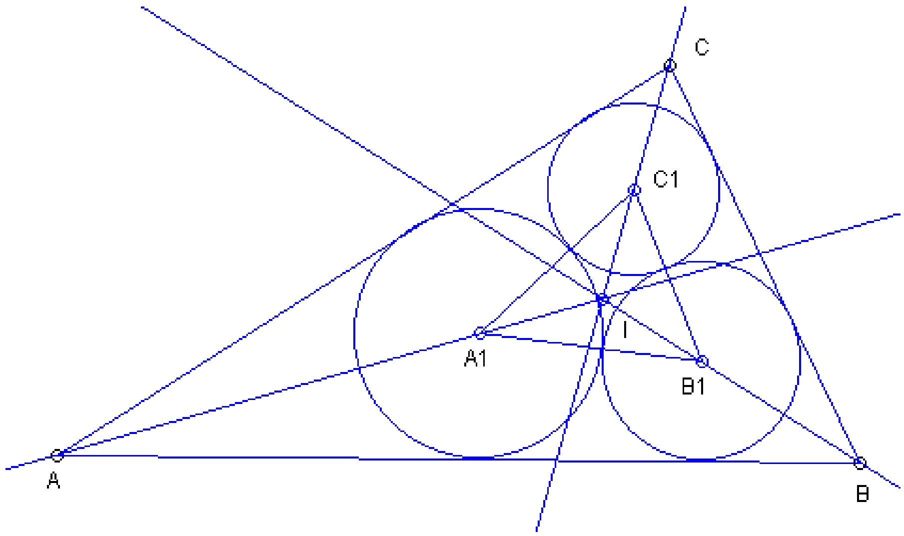
1. The Incenter is the Perspector of Triangle ABC and the Malfatti Triangle. ([2], Theorems, Points, Incenter).
2. The Center of the Radical Circle of the Malfatti Circles is the Perspector of the BCI Triangle and the Malfatti Triangle. ([2], Theorems, Points, Center of the Radical Circle of the Malfatti Circles).
3. The Center of the Radical Circle of the Malfatti Circles is the Outer Eppstein Point of the Tangential Triangle of the BCI Triangle. ([2], Theorems, Points, Center of the Radical Circle of the Malfatti Circles).

We invite the reader to prove the above theorems. The reader may find all definitions in [2]. Recall that ([2], Definitions, Circles, Malfatti Circles) the *Malfatti circles* are the three circles packed inside a triangle such that each is tangent to the other two and to two sides of the triangle. The centers of the Malfatti circles form a triangle known as the Malfatti Triangle ([2], Definitions, Triangles, Malfatti triangle; Malfatti Central Triangle in [3]). We could construct easily the Malfatti circles by using straightedge and compass (in accordance with the ancient Greek traditions), if we construct the Malfatti Triangle. Indeed, suppose we have constructed the Malfatti triangle. Then construct projections of the vertices of the triangle to the sides of the given triangle ABC. The Malfatti circles are the circles centered at the vertices of the Malfatti triangle and passing through the corresponding projections. See the Figure:




$A_1B_1C_1$ - the Malfatti triangle;
$A_2$ - projection of $A_1$ on side AB;
$B_2$ - projection of $A_1$ on side BC;
$C_2$ - projection of $A_1$ on side CA;
The Malfatti circles are the circles centered at the vertices of the Malfatti triangle and passing through the corresponding projections.

Here we will illustrate the first of the above quoted computer-generated theorems: "The Incenter is the Perspector of Triangle ABC and the Malfatti Triangle." We leave illustrations of the second and the third theorems to the reader. The first theorem states that if we draw lines through the corresponding vertices of Triangle ABC and the Malfatti Triangle, then the lines concur at a point, and the point of concurrence is the Incenter of triangle ABC. See the Figure:

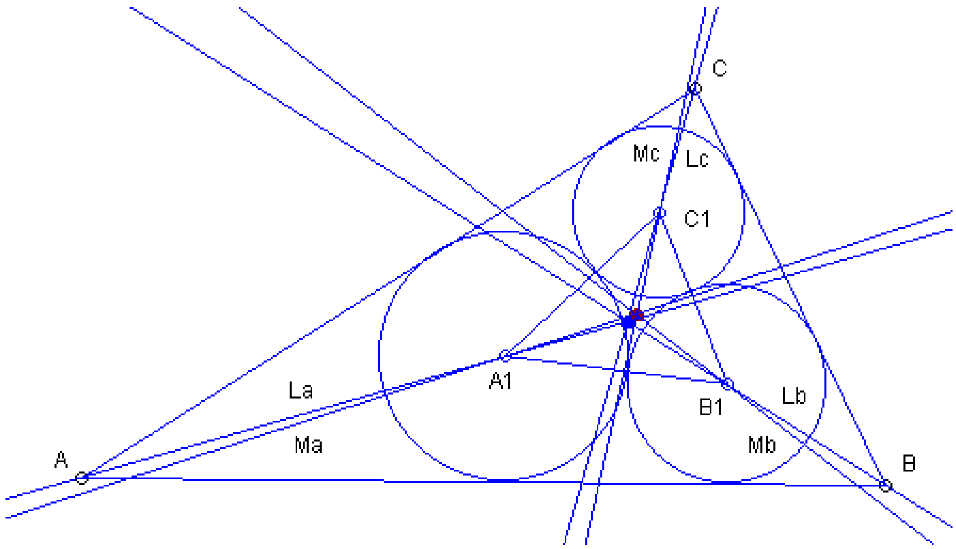



$A_1B_1C_1$ - the Malfatti triangle;
Lines $AA_1$, $BB_1$, and $CC_1$ concur at a point, and the point of concurrence is the Incenter I of triangle ABC.

We can construct the Malfatti circles by using the above three computer-generated theorems as follows.

1. Construct the Incenter and the lines connecting the Incenters and the vertices of the given triangle ABC. Denote by La (resp.Lb, Lc) the line passing through the Incenter and vertex A (resp. B, C).
2. Construct the Center of the Radical Circle of the Malfatti Circles by using the third theorem, that is, construct the Center of the Radical Circle of the Malfatti Circles as the Outer Eppstein Point of the Tangential Triangle of the BCI Triangle.
3. Cosntruct the BCI triangle and the lines connecting the Center of the Radical Circle of the Malfatti Circles with the vertices of the BCI triangle DEF. Denote by Ma (resp.Mb, Mc) the line passing through the Center of the Radical Circle of the Malfatti Circles and vertex D (resp. E, F).
4. Construct the Malfatti triangle $A_1B_1C_1$ as follows: $A_1$ is the point of intersection of lines La and Ma. Similarly, $B_1$ is the point of intersection of lines Lb and Mb. $C_1$ is the point of intersection of lines Lc and Mc.
5. By using the Malfatti triangle, construct the Malfatti circles (see above).

See the Figure:




$A_1$ = intersection point of lines La and Ma;
$B_1$ = intersection point of lines Lb and Mb;
$C_1$ = intersection point of lines Lc and Mc;
Then $A_1B_1C_1$ is the Malfatti triangle.

**The royal road to geometry**

Euclid is said to have said to the first Ptolemy who inquired if there was a shorter way to learn geometry than the Elements: ...there is no royal road to geometry.

Now we have a royal road to geometry. It is not necessary we to be inventive. The Machine will tell us what is necessary. All which we have to do is to write our problem and to go to drink coffee. We will drink coffee and the Machine will work for us. It is easy.

I do not know whether the above construction of the Malfatti circles is a new construction - Euclidean Geometry has a long history and many sources are not available. Possibly, the third of the above theorem is a new theorem, so that, possibly the above construction of the Malfatti circles is a new construction. But, no doubt that the Machine for Questions and Answers has produces at least 100 new theorems, even in its today's fairly primitive version - see [2,3]. The importance of the Machine is in its potential - the Machine could be easily extended and improved, so that it could easily rediscover all known theorems in Euclidean Geometry, and then it could extend the set of these theorems a few times - possibly 3 times, possibly 10 times, possibly 100 times. It depends how much resources will be included in the project.

The title of this article is "Towards the first computer-generated encyclopedia", although the first computer-generated encyclopedia is already available [2]. The title means that the encyclopedia [2] is at a very early stage. There would be very important we to produce a valuable computer-generated encyclopedia. A valuable computer-generated encyclopedia of Euclidean Geometry has to contain at least 20,000 new valuable theorems and systematic description of all important compass-and-straightedge constructions, in accordance with the ancient Greek traditions. Since Euclidean Geometry is a part of school education around the world, the encyclopedia has to be produces in a form suitable for use by school students.

Hereby I invite everybody interested to contribute, to join.

Competition in the game Chess are well known. It is time we to begin the first competitions between people and computers in the area of science. As always, the most suitable participants are school students. The Machine gives us the possibility we to begin these competitions. Today's students will produce the next computer-generated encyclopedias. Hereby I invite everybody interested to contribute, to join.

The Machine for Questions and Answers could produce thousands valuable theorems, but today it is at a very early stage, and it is not in a form suitable for use by the large public. It is time the researchers, as well as the large public, to have the opportunity to use a computer program, able to make scientific discoveries, that is, an improved version of the Machine. Such a program would demonstrate to the large public that the automated reasoning is a really useful area of science. Hereby I invite everybody interested to contribute, to join.

## Thanks

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