

A new simple root-finding method

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Abstract. In this paper we apply the method, first described in [1], to the problem of finding the roots of an equation. The method is designed for use in high schools and colleges.

Keywords: root-finding method; high school and college mathematics

In this paper we offer a new simple root-finding method. In fact, the method is an application to the problem of root-finding of the method, published in [1]. The method is as follows. Suppose that $F(x)$ is a continuous function. Define $f(x) = |F(x)|$. Suppose that a root of the function $f(x)$ is within the segment $[a,b]$. We divide the segment $[a,b]$ by N equal parts by using the points $x_0 = a, x_1, x_2, \dots, x_N = b$. Then we evaluate $f(x_0), f(x_1), f(x_2), \dots, f(x_N)$, and select the minimal of these values. We use the minimal value as the midpoint of a new segment, whose length is 10 times smaller than the length of the previous segment. The process is repeated until the root is found. The method works well, if $N \geq 10$.

The described method is fast, because it needs small numbers of iterations. Since each iteration adds one true digit to the answer, we need only 100 iterations to receive an answer with 100 true digits. The method uses only basic notions. The Newton's method is faster than the described method, but it uses derivatives. Hence, the described method is suitable for high schools as a root-finding method.

The method is simple, so that it allows a simple implementation. I have created a simple computer program by using PHP. Below we give an example.

Example. Solve the equation $x^5 - 3x + 1 = 0$.

Solution. The polynomial $x^5 - 3x + 1$ has three roots, which are within the segments $[-1.5, -1]$ (Root 1), $[0, 0.5]$ (Root 2) and $[1, 1.5]$ (Root 3). By using the computer program, we find the roots. If we require 5 digits after the decimal point, the answer is as follows: $x_1 = -1.38879$, $x_2 = 0.33473$ and $x_3 = 1.21465$. We receive these answers immediately, for less than one second, by using a desktop personal computer.

We could record the calculations, made by the computer. The following files, containing records of calculations made by the computer program, are available for download as supplementary materials:

JGCM201201_Root1_5_digits.pdf (Root 1, 5 digits, 8 KB, 4 pages);
JGCM201201_Root2_5_digits.pdf (Root 2, 5 digits, 7 KB, 3 pages);
JGCM201201_Root3_5_digits.pdf (Root 3, 5 digits, 6 KB, 3 pages);
JGCM201201_Root1_1000_digits.pdf (Root 1, 1000 digits, 3.12 MB, 1260 pages).

References

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