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A new approach for finding the extrema of a function without use of derivatives

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Abstract. . In this paper we offer a new simple numerical method for finding the extrema of a function without use of derivatives. The method is an application of the method, first described in [1], to the problem of finding the extrema. The method is designed for use in high schools and colleges. The method gives the possibility the extremal problems to be included in the high school mathematics education at an early stage.

Keywords: numerical method, extrema, education

The method for finding the extrema of a function by using derivatives has an alternative. In this paper we offer a new simple numerical method able to solve the problems for finding extrema from textbooks for high school and university students. The method is an application of the method, first described in [1], to the problem of finding the extrema

The method for finding the extrema of a function by using the derivatives has three weaknesses. First, it requires large preliminary studying of derivatives. Secondly, in many problems the answer must be in integers, but in these cases the method is not able to find the solution. The authors of textbooks are not able to avoid the first problem. They avoid the second problem as follows. The authors of textbooks create problems whose answers are integers. But such an approach is unnatural, and in fact it deprives the students of a method for solving the problems. And thirdly, in many cases the symbolic manipulations are not able to solve the problems. In such cases we need numerical method. The authors of textbooks avoid the third problem as follows. They include in the textbooks only problems whose solutions could be found by using symbolic manipulations.

The approach of this paper solves the above three problems. First, the method of this paper uses only the definition of a function and the comparison of two numbers. The approach of this paper could be used by university professors and students to solve the extremal problems from textbooks without studying of derivatives, and in fact without any studying. Secondly, the method of this paper solves the integer extremal problems.

Thirdly, since the method is numerical, it is universal. But if we want to receive the answer immediately, we have to use a computer program. Hence, the approach of this paper could be used as a supplementary simple numerical method in the universities and colleges.

But the main advantages of the proposed method are in the high school education. If we look at the extremal problems, we see that about 90% of them could be included in high school education at an early stage, provided we have a suitable method to solve them. The personal opinion of the author of this paper is as follows. The high school teachers could include extremal problems in almost any topic of high school mathematics. They could use the proposed method to solve the problems. Such an approach would make the mathematics high school education more comprehensive and useful during the grades from 5 to 12.

The method is as follows. Suppose that f(x) is a continuous function and suppose that we have to find the minimum of f(x). First, we localize the minimum of f(x). Suppose that the minimum of the function f(x) is within the segment [a,b] We divide the segment [a,b] by N equal parts by using the points x0 = a, x1, x2,...,xN = b. Then we evaluate f(x0), f(x1), f(x2),...,f(xN), and select the minimal of these values. We use the minimal value as the midpoint of a new segment, whose length is 10 times smaller than the previous segment. The process is repeated until the minimum is found. Similarly, we find a maximum of the function. The method works well, if $N \geq 10$. Note that the extension of the method to the case in which the function has many variables is straightforward.

I have created a simple computer program by using PHP. The program is used in the examples given below. Note that the program easily solves all high school and college problems for finding the extrema of functions of one variable. It takes less than 1 second we to receive the answers.

Example 1. A bus company will charter a bus that holds 40 people to groups of 26 or more. If a group contains exactly 26 people, each person pays $52. In larger groups, everybody’s fare is reduced by $1 for each person in excess of 26. Determine the size of the group for which the bus company’s revenue will be greatest.

Solution. Let x denote the total number of people in excess of 26. Denote by n the number of people in the group and by p the fare per person. We have to find the maximum of function f(x) = n.p, where n = 26 + x, p = 52 – x and x is an integer, $0 \leq x \leq 14$. We seek the solution within the closed interval [0,14]. Set N = 14. At the first step the computer program calculates f(x) for x = 0, 1, 2, …,14, and then it select the maximal of the calculated values. The function has maximum if x = 13 and then f(13) = 1521.

Example 2. Find two nonnegative numbers whose sum is 11 and so that the product of one number and the square of the other number is a maximum.

Solution. Let variables x and y represent two nonnegative numbers. The sum of the two numbers is given to be 11 = x + y, so that y = 11 - x. We have to maximize the function $f(x) = x.(11 - x)^2$ where $0 \leq x \leq 11$. We use the computer program. We seek the solution within the closed interval [0,11]. Set N = 11. Then the answer is as follows: x ≈

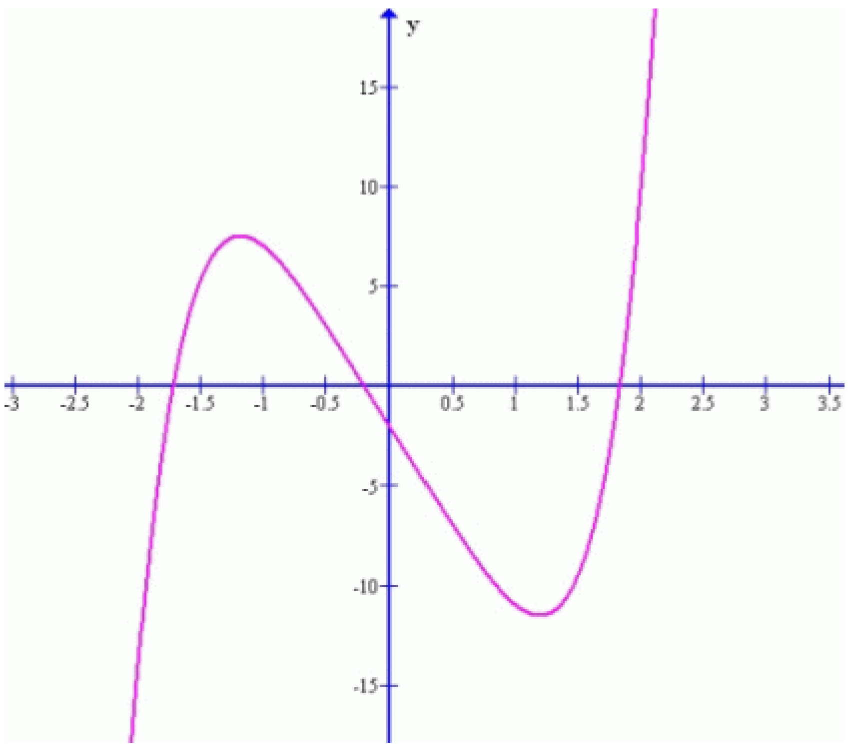
3.667 and f ≈ 197.185. The same answer we receive if we use the approach based on derivatives.

Suppose we have to solve the problem provided we require x and y to be integers. If we use approach based on derivatives, we receive the answer x ≈ 3.667 and f ≈ 197.185, so that an additional analysis is necessary. If we use the computer program, we could see the output of the first step. The output of the first step is the answer to the problem: x = 4 and f = 196.

If we seek the solution within the closed segment [0,10], and we set N = 10, we receive the same answers as above.

Example 3. Find the extrema of function $f(x) = x^5 - 10x - 2$.

Solution. We use the Ivan Johansen's computer program Graph to draw the graph of the function:



From the graph of the function we see that it has a maximum within the segment [-2,0] and a minimum within the segment [0,2]. Set N = 10. If we need in the answer numbers having 5 true digits after the decimal point, we obtain the following answer: x_max ≈ -1.18921, f_max ≈ 7.51366, x_min ≈ 1.18921, f_min ≈ -11.51366.

If we need in the answer for the values of f_max and f_min numbers having 100 true digits after the decimal point, we easily obtain the answer. The answer for the maximum is as follows:
x_max ≈ -1.189207115002721066717499970560475915292972092463817416,
f_max≈7.513656920021768533739999764483807322343776739710539304152017797755733345815337278966250756305101389 7.

It takes less than 1 second we to obtain these answers. Note that we could find without problems also the answer in numbers having 1000 true digits after the decimal point.

We could record the calculations, made by the computer. For the above examples, the files containing records of calculations are available for download as supplementary materials.

References

1. Deko Dekov, A numerical method for solving the horizontal resection problem in Surveying, Journal of Geodetic Science (to appear).

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