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The use of the brute-force method for finding the extrema of a function

Deko Dekov

Abstract. . In this paper we propose the brute-force method as a suitable method for solving extremal problems. We show that the brute-force method easily solves the extremal problems from textbooks. The brute-force method could be a supplementary method for finding the extrema in the universities, and it gives the possibility the extremal problems to be included in high school mathematics education during the grades from 7 to 12.

Keywords: numerical method, extrema, education

The method for finding the extrema of a function by using derivatives has an alternative. In this paper we show that the ordinary brute-force method effectively solves the extremal problems from textbooks for high school and university students.

The method for finding the extrema of a function by using the derivatives has three weaknesses. First, it requires large preliminary studying of derivatives. Secondly, in many problems the answer must be in integers, but in these cases the method is not able to find the solution. And thirdly, in many cases the symbolic manipulations are not able to solve the problems. The authors of textbooks are not able to avoid the first problem. They avoid the second problem as follows. The authors of textbooks create problems whose answers are integers. But such an approach is unnatural, and in fact it deprives the students of a method for solving the problems The authors of textbooks avoid the third problem as follows. They include in the textbooks only problems whose solutions could be found by using symbolic manipulations.

The brute-force method solves the above three problems. First, the brute-force method uses only the definition of a function and the comparison of two numbers. But if we want to receive the answer immediately, we have to use a computer program. Secondly, the brute-force method solves the integer extremal problems. Thirdly, since the brute-force method is a numerical method, it is universal.

The experts in the area of optimization do not like and do not recommend the brute-force method. But for the case of the university textbook problems the method is suitable. It takes less than one second we to receive the answer, if we use a desktop personal computer.

But the main advantages of the proposed method are in the high school education. If we look at the extremal problems, we see that about 90% of them could be included in high school education at an early stage, provided we have a suitable method to solve them. The personal opinion of the author of this paper is as follows. The high school teachers could include extremal problems in almost any topic of high school mathematics. They could use the proposed method to solve the problems. Such an approach would make the mathematics high school education more comprehensive and useful during the grades from 5 to 12.

The method is as follows. Suppose that we have to find the minimum of a function f(x). First, we localize the minimum of f(x). Suppose that the minimum of the function f(x) is within the segment [a,b] We divide the segment [a,b] by N equal parts by using the points x0 = a, x1, x2,...,xN = b. Then we evaluate f(x0), f(x1), f(x2),…,f(xN), and select the minimal of these values. The minimal of the calculated values is the answer. Similarly, we find a maximum of the function.

I have created a simple computer program by using PHP. The program is used in the examples given below. Note that the program easily solves all high school and college problems for finding the extrema of functions of one variable. It takes less than 1 second we to receive the answer.

Example 1. A bus company will charter a bus that holds 40 people to groups of 26 or more. If a group contains exactly 26 people, each person pays $52. In larger groups, everybody’s fare is reduced by $1 for each person in excess of 26. Determine the size of the group for which the bus company’s revenue will be greatest.

Solution. Let x denote the total number of people in excess of 26. Denote by n the number of people in the group and by p the fare per person. We have to find the maximum of function f(x) = n.p, where n = 26 + x, p = 52 – x and x is an integer, $0 \leq x \leq 14$. We use the computer program. The computer program calculates f(0), f(1), f(2),…, f(14) and then it selects the maximal of these values. The function has maximum if x = 13 and the maximal value is f(13) = 1521.

Example 2. Find two nonnegative integers whose sum is 11 and so that the product of one integer and the square of the other integer is a maximum.

Solution. Let variables x and y represent two nonnegative integers. The sum of the two numbers is given to be 11 = x + y, so that y = 11 - x. We have to maximize the function $f(x) = x.(11-x)^2$ where x is an integer, $0 \leq x \leq 11$. We use the computer program. The computer program calculates f(0), f(1), f(2),…, f(14) and then it selects the maximal of these values. The function has maximum if x = 4 and the maximum is f(4) = 196.

Note that if we use the approach based on derivatives, consider the function f(x) as a continuous function within the segment [0,14]. We receive the answer $x \approx 3.667$ and $f \approx 197.185$, so that an additional analysis is necessary.

Example 3. Find the extrema of function $f(x) = x^5 - 10x - 2$.

Solution. From the graph of the function we see that it has a maximum within the segment [-2,-1]. We divide this segments by 1000. Then we calculate the values of f(x) at the initial point of each subsegment and select the maximal of these values. We obtain the following answer: $x_{max} \approx -1.189$ and $f_{max} \approx 7.51366$. Similarly, we obtain the minimum $x_{min} \approx 1.189$ and $f_{min} \approx -11.51366$. It takes less than 1 second we to obtain these answers. Note that we could find without problems also the answer in numbers having up to 5 true digits after the decimal point.

We could record the calculations, made by the computer. For the above examples, the file containing records of calculations is available for download as supplementary material.

Dr.Deko Dekov
Zahari Knjazeski 81
6000 Stara Zagora
Bulgaria
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