Journal of Computer-Generated Mathematics

A new simple numerical method for solving nonlinear systems

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Abstract. In this paper we offer a new simple numerical method for solving nonlinear systems. The method is described for first time in the paper [1] as a numerical method for solving a problem from geodesy. Here we note that the method could be applied to any system of two equations with two unknowns. The method is designed for use in high schools and colleges.

Keywords: Numerical method, nonlinear system, education

In this paper we note that the method, first described in [1], is applicable to any system of two equations with two unknowns.

The method is as follows. Suppose that we have to solve the following system:

$$\begin{cases} f_1(x,y) = 0 \\ f_2(x,y) = 0 \end{cases}$$

where $f_1(x,y)$ and $f_2(x,y)$ are continuous functions. We define $f(x,y) = |f_1(x,y)| + |f_2(x,y)|$. Suppose that a root of the function f(x,y) is within the rectangle [a,b] x [c,d]. We divide the segments [a, b] and [c, d] by N equal parts by using the points x0 = a, x1, x2 , ... , xN = b, and y0 = c, y1, y2,... ,yN = d. Then we evaluate f(x,y) for each x in {x0, x1, …, xN}and y in {y0, y1, …,yN}, and we select the minimal of these values. We use the minimal value as the center of a new rectangle, whose sides have 10 times smaller length than the previous rectangle. The process is repeated until the root is found. The method works well, if $N \geq 100$, but in many cases it is enough we to set smaller N.

The described numerical method has the following advantages. The method is fast, provided we solve a system of two equations with two unknowns.. We receive the answer for less than 1 second, if we use a desktop personal computer. The method is universal: We can solve also systems with transcendental functions, which in many cases cannot be solved by using algebraic methods. The described method is simple, so that the school students could understand and use it. Hence, the method is suitable for high schools and colleges as a numerical method for solving systems of two equations with two unknowns.

The described method is fast, because it needs small numbers of iterations. Since each iteration adds one true digit to the answer, we need only 100 iterations to receive an answer with 100 true digits.

The method is simple, so that it allows a simple implementation. I have created a simple computer program by using PHP.

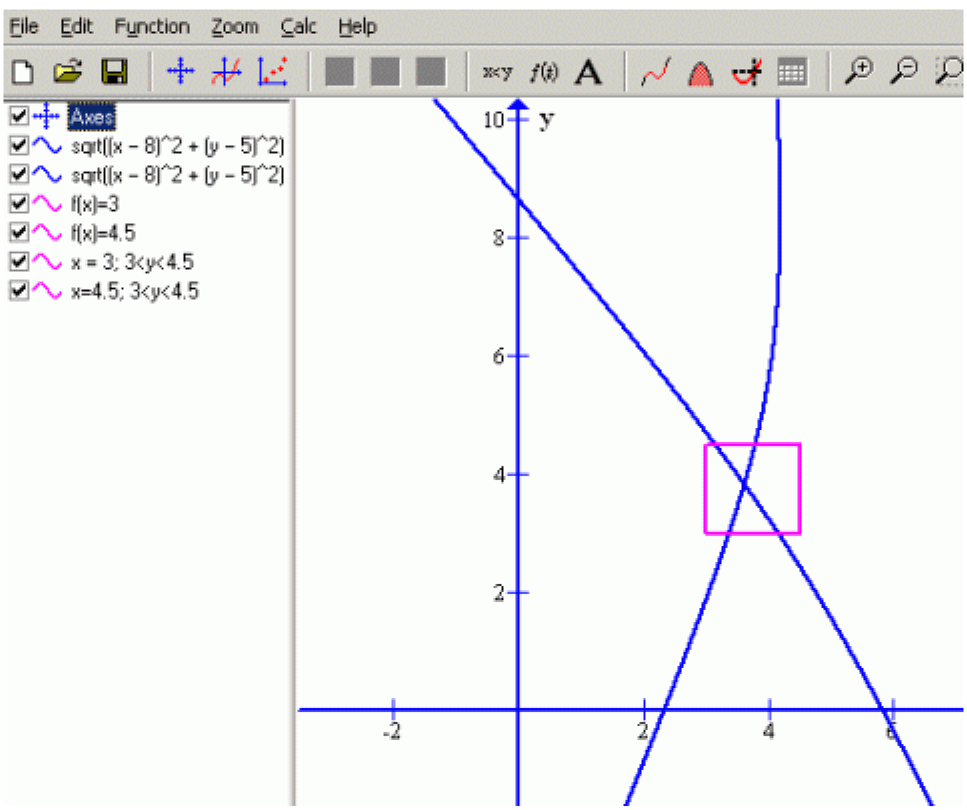
Below we give an example.

Example. Given three circles: circle A with center (x1,y1) and radius r1, circle B with center (x2,y2) and radius r2, and circle C with center (x3,y3) and radius r3. Construct circle that is externally tangent to the given three circles. Solve the problem for x1 = 8, y1 = 5, r1 =2.5; x2 = 1.5, y2 = 1, r2 = 1.5; x3 = 1.5, y3 = 6, r3 = 1.

Solution. Denote by k the desired circle, and by (x,y) and r its center and radius. Denote by d1, d2 and d3 the distances from the center of circle k to the center of circles A, B and C, respectively. Since r = d1 - r1 = d2 - r2 = d3 - r3, we have d1 - d2 - r1 + r2 = 0 and d1 - d3 - r1 + r3 = 0. Hence, we have to solve the following system:

$$\left|\begin{array}{l} \sqrt{(x-x_1)+(y-y_1)} - \sqrt{(x-x_2)+(y-y_2)} - r_1 + r_2 = 0 \\ \sqrt{(x-x_1)+(y-y_1)} - \sqrt{(x-x_3)+(y-y_3)} - r_1 + r_3 = 0 \end{array}\right.$$

We will solve the above system by using the computer program. Each of the equations of the system defines y as an implicit function of x. Graph these functions, e.g. by using the Ivan Johansen's computer program Graph. Any intersection point of the graphs is a root of the system:



From the graphs we see that the system has one root, which is within the square [3, 4.5] x [3, 4.5] (the red square at the above picture). We use the computer program to find the root of the function f(x,y) = |f1| + |f2|, where f1 and f2 are the left sides of the equations of the system. Set N = 15. We require 10 true digits after the decimal point. The answer is as follows: x = 3.6195014038, y = 3.8284457768, so that we obtain r = 2.0344577679.

We could record the calculations, made by the computer. The file containing record of calculation in the above Example is available for download as supplementary material.

References

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Submitted on 1 October 2011
Publication date: 1 February 2012