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The use of the brute-force method for solving nonlinear systems

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Abstract. . In this paper we propose the brute-force method as a suitable method for solving nonlinear systems. We show that the brute-force method easily solves the nonlinear systems from textbooks. The brute-force method could be used as a numerical method for solving of nonlinear systems in universities and in high schools.

Keywords: numerical method, nonlinear systems, education

The methods for solving nonlinear systems studied in the high schools have two weaknesses. First, in many cases the solution needs complicated symbolic manipulations. Secondly, in many cases the symbolic manipulations are not able to solve the problems.

The brute-force method solves the above problems. First, the brute-force method is simple. It uses only the definition of a function and the comparison of two numbers. But if we want to receive the answer immediately, we have to use a computer program. Secondly, since the brute-force method is a numerical method, it is universal. Hence, the professors and students could use the brute-force method as a supplementary method for solving nonlinear systems. Also, the method could be used for fast and easy check of the answers. The experts in the area of optimization do not like and do not recommend the brute-force method. But for the case of the high school and university education the method is suitable. It takes less than one second we to receive the answer, if we use a desktop personal computer.

The brute-force method is as follows. Suppose that we have to solve the following system:

$$\left| \begin{array}{l} f_1(x,y) = 0 \\ f_2(x,y) = 0 \end{array} \right.$$

where $f_1(x,y)$ and $f_2(x,y)$ are continuous functions. We define $f(x,y) = |f_1(x,y)| + |f_2(x,y)|$. Suppose that a root of the function f(x,y) is within the rectangle [a,b] x [c,d]. We divide the segments [a, b] and [c, d] by N equal parts by using the points x0 = a, x1, x2 , ... , xN = b, and y0 = c, y1, y2,... ,yN = d. Then we evaluate f(x,y) for each x in {x0, x1, …, xN}and y in {y0, y1, …,yN}, and we select the minimal of these values. The minimal of the calculated values is the answer.

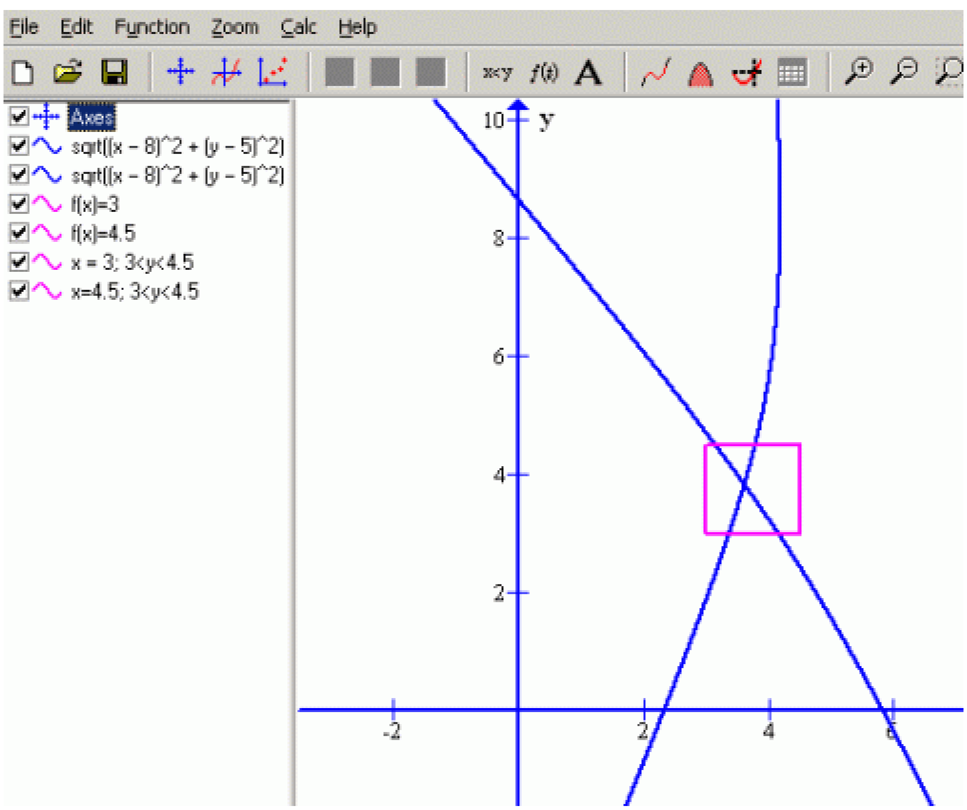
I have created a simple computer program by using PHP. The program is used in the example given below. It takes less than 1 second we to receive the answer.

Example. Given three circles: circle A with center (x1,y1) and radius r1, circle B with center (x2,y2) and radius r2, and circle C with center (x3,y3) and radius r3. Construct circle that is externally tangent to the given three circles. Solve the problem for x1 = 8, y1 = 5, r1 =2.5; x2 = 1.5, y2 = 1, r2 = 1.5; x3 = 1.5, y3 = 6, r3 = 1.

Solution. Denote by k the desired circle, and by (x,y) and r its center and radius. Denote by d1, d2 and d3 the distances from the center of circle k to the center of circles A, B and C, respectively. Since r = d1 - r1 = d2 - r2 = d3 - r3, we have d1 - d2 - r1 + r2 = 0 and d1 - d3 - r1 + r3 = 0. Hence, we have to solve the following system:

$$\begin{cases} \sqrt{(x-x_1)+(y-y_1)} - \sqrt{(x-x_2)+(y-y_2)} - r_1 + r_2 = 0 \\ \sqrt{(x-x_1)+(y-y_1)} - \sqrt{(x-x_3)+(y-y_3)} - r_1 + r_3 = 0 \end{cases}$$

Each of the equations of the system defines y as an implicit function of x. Graph these functions, e.g. by using the Ivan Johansen’s computer program Graph. Any intersection point of the graphs is a root of the system:



From the graphs we see that the system has one root, which is within the square [3, 4.5] x [3, 4.5] (drawn in the figure).We use the computer program to find the root of the function f(x,y) = |f1| + |f2|, where f1 and f2 are the left sides of the equations of the system. Set N = 150. The answer is as follows: $x \approx 3.62$, $y \approx 3.83$, so that we obtain $r \approx 2.03$. Note that the minimal of the calculated values of function f(x,y) is about 0.00000589, that is, the answer is correct, provided we want two true digits after the decimal point.

We could record the calculations, made by the computer. The file containing record of calculation in the above example is available for download as supplementary material.

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