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A new approach to the approximation of a data set by a continuous function

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Abstract. In this paper we offer a new simple numerical method for finding the least squares approximation of a data set by a continuous function without the Gauss' formulae. The method is designed for use in high schools and colleges. The method gives the possibility the approximation problems to be included in the high school mathematics education at an early stage.

Keywords: numerical method, least squares, approximation

The Gauss' method for finding the least squares approximation of a data set has an alternative. In this paper we offer a new simple numerical method able to solve the approximation problems from textbooks. The method first is described in [1]. Here we show that the method is applicable to the problem for finding the least squares approximation of a data set.

The Gauss' method for a least squares approximation of a data set has two weaknesses. First, it requires large preliminary studying of derivatives. Secondly, in many cases the symbolic manipulations are not able to solve the problems. In such cases we need a numerical method.

The approach of this paper solves the above two problems. First, the method of this paper uses only the definition of a function and the comparison of two numbers. The approach of this paper could be used by university professors and students to solve the approximation problems from textbooks without studying of derivatives, and in fact without any studying. Secondly, since the method is numerical, it is universal. But if we want to receive the answer immediately, we have to use a computer program. Hence, the approach of this paper could be used as a supplementary simple numerical method in the universities and colleges. If a professor insists the students to be familiar with the Gauss' method, the approach of this paper could be used at least the professor and the students to be able easily to check the answers.

But the main advantages of the proposed method are in the mathematics high school education. The personal opinion of the author of this paper is as follows. The high school teachers could include approximation problems in high school mathematics. They could use the proposed method to solve the problems. Such an approach would make the mathematics high school education more comprehensive and useful during the grades from 9 to 12.

The method is as follows. Suppose data consisting of n points (x1,y1), (x2,y2),…,(xn,yn) are known and the goal is to find a function y = F(x) that fits the data reasonably well. We will use the least squares criterion. We suppose that the reader is familiar with the least squares criterion. Suppose that f(x) is the objective function, so that we have to find the minimum of f(x). We use the data set, in order to localize the minimum of f(x). Suppose that the minimum of the function f(x) is within the segment [x_a,x_b]. We divide the segment [x_a,x_b] by N equal parts by using the points x0 = x_a, x1, x2, ... , xN = x_b. Then we evaluate f(x0), f(x1), f(x2), ... ,f(xN), and select the minimal of these values. We use the minimal value as the midpoint of a new segment, whose length is 10 times smaller than the previous segment. The process is repeated until the minimum is found. The method works well if N ≥ 100, but in many cases it is enough we to set smaller N.

I have created a simple computer program by using PHP. The program is used in the examples given below. Note that the program easily solves all high school and college problems for finding the least squares approximation of a data set by a continuous function. It takes less than 1 second we to receive the answers.

Example 1. Use the least-squares criterion to find a and b such that the line y = ax + b that is closest to the points (1, 1.4), (2, 2.3), (3, 3) and (4, 3.6).

Solution. We have to minimize the function

$$f(a,b) = (a + b - 1.4)^2 + (2a + b - 2.3)^2 + (3a + b - 3)^2 + (4a + b - 3.6)^2$$

We use the computer program. Set N = 20. We take the segment [0,2] as initial segment for a and b. We receive the following answer: a = 0.73 and b = 0.75. Hence, the least squares line has equation y = 0.73 x + 0.75.

We could use the Ivan Johansen’s computer program Graph in order to draw the graphs of the data and the line. See Fig.1.

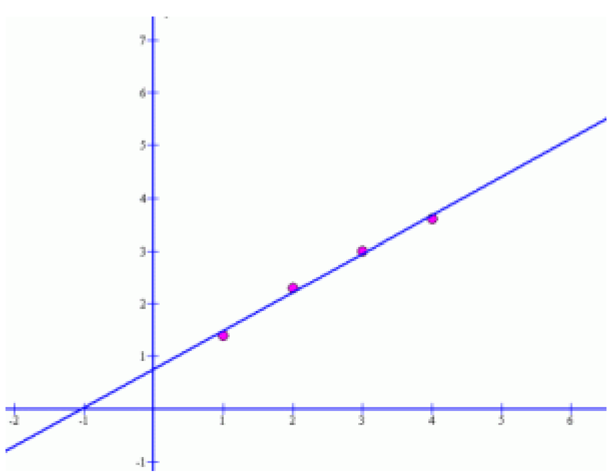

Fig.1

Example 2. Use the least-squares criterion to find a and b such that the curve $y = ax^b$ is closest to the points given in Example1.

Solution. We have to minimize the function

$$f(a,b) = (a - 1.4)^2 + (a2^b - 2.3)^2 + (a3^b - 3)^2 + (a4^b - 3.6)^2$$

We use the computer program. Set N = 100. We take the segment [0,2] as initial segment for a and b. If we require 10 true digits after the decimal point, we receive the following answer: a = 1.4281616576 and b = 0.6706946001. Hence, the least squares curve has equation $y = (1.4281616576)x^{0.6706946001}$.

The graphs of the data and the curve are given at Fig.2.

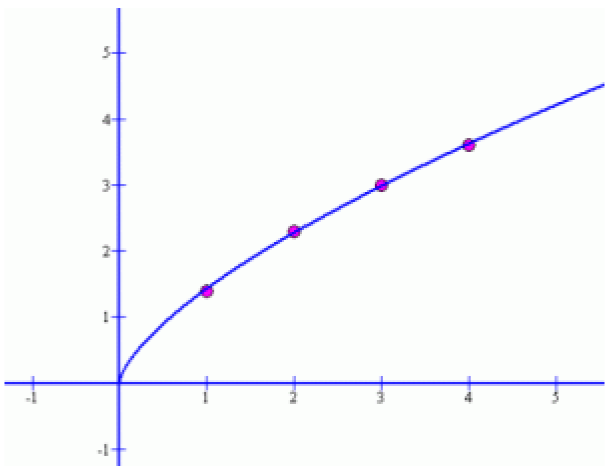

Fig.2

Note that the sum of the least squares in Example 1 is equal to 0.023, and in Example 2 the sum of the least squares is approximately equal to 0.0021. We see that the curve in Example 2 fits the data approximately 11 times better than the line in Example 1, if we use the sum of the least squares as criterion.

Example 3. Use the least-squares criterion to find a and b such that the curve $y = ae^{bx}$ is closest to the points (1, 2.5), (2, 3.5), (3, 4.5) and (4, 6).

Solution. We have to minimize the function

$$f(a,b) = (ae^b - 2.5)^2 + (ae^{2b} - 3.5)^2 + (ae^{3b} - 4.5)^2 + (ae^{4b} - 6)^2$$

We use the computer program. Set N = 100. We take the segment [0,5] as initial segment for a and b. If we require 10 true digits after the decimal point, we receive the following answer: a = 1.9340254881 and b = 0.2832622978. Hence, the least squares curve has equation: $y = (1.9340254881)e^{(0.2832622978)x}$.

The graphs of the data and the curve are given at Fig.3.

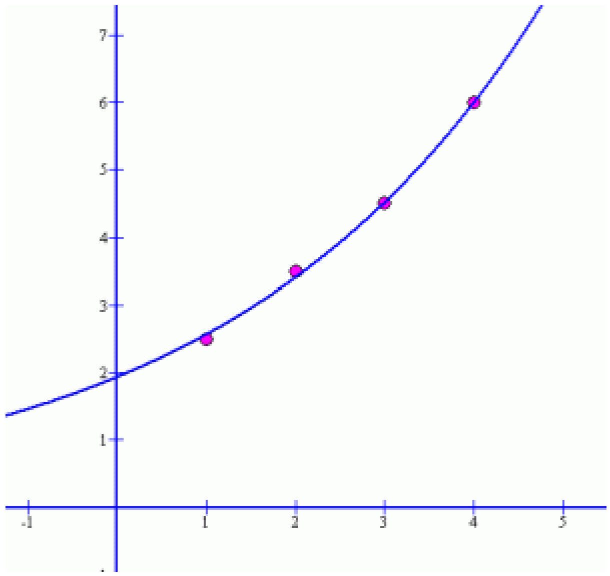

Fig.3

Example 4. Use the least-squares criterion to find a and b such that the curve $y = a + b\ln x$ is closest to the points (0.1, -0.4). (0.3, -0.15), (0.5, -0.03), (0.7, 0.03).

Solution. We have to minimize the function

$$f(a,b) = (a + b\ln 0.1 + 0.4)^2 + (a + b\ln 0.3 + 0.15)^2 + (a + b\ln 0.5 + 0.03)^2 + (a + b\ln 0.7 - 0.03)^2$$

We use the computer program. Set N = 100. We take the segment [0,2] as initial segment for a and b. If we require 10 true digits after the decimal point, we receive the following answer: a = 0.1173279063 and b = 0.2237108451. Hence, the least squares curve has equation: $y = 0.1173279063 + (0.2237108451)\ln x$.

The graphs of the data and the curve are given at Fig.4.

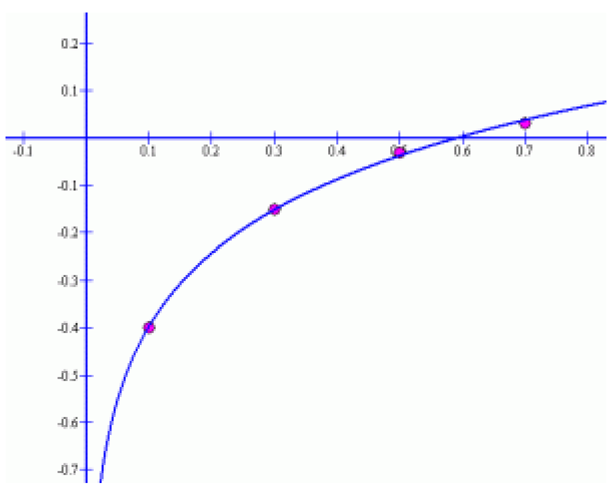

Fig.4

Example 5. Use the least-squares criterion to find a, b and c such that the curve $y = a + b\sin(cx)$ is closest to the points (1, 2.8). (2, 3.2), (3, 3) (4, 2.4).

Solution. We have to minimize the function

$$f(a,b,c) = (a + b\sin(c) - 2.8)^2 + (a + b\sin(2c) - 3.2)^2 + (a + b\sin(3c) - 3)^2 + (a + b\sin(4c) - 2.4)^2$$

We use the computer program. Set N = 50. We take the segment [0,5] as initial segment for a and b. If we require 3 true digits after the decimal point, we receive the following answer: a = 2.072, b = 1.123 and c = 0.713. Hence, the least squares curve has equation: $y = 2.072 + (1.123)\sin(0.713x)$.

The graphs of the data and the curve are given at Fig.5.

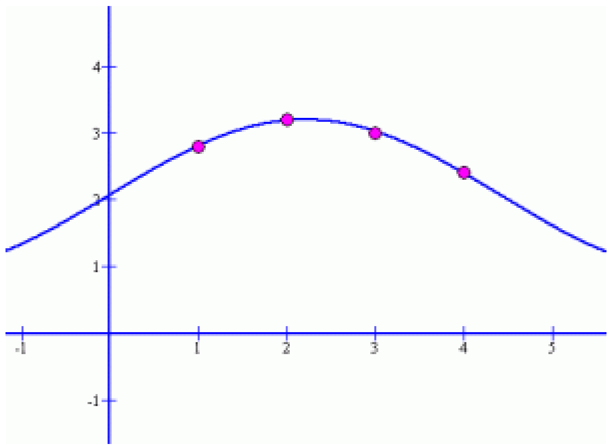

Fig.5

Files containing records of calculation of the above examples are available for download as supplementary materials.

References

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