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The use of the brute-force method for finding the least squares approximation of a data set by a continuous function

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Abstract. In this paper we propose the brute-force method for finding the least squares approximation of a data set by a continuous function. The brute-force method gives the possibility the approximation problems to be included in the high school mathematics education at an early stage.

Keywords: brute-force method, least squares, approximation

The Gauss' method for finding the least squares approximation of a data set has an alternative. In this paper we show that the brute-force method is able to solve the approximation problems from textbooks.

The Gauss' method for a least squares approximation of a data set has two weaknesses. First, it requires large preliminary studying of derivatives. Secondly, in many cases the symbolic manipulations are not able to solve the problems. In such cases we need a numerical method.

The brute-force method solves the above two problems. First, the brute-force method uses only the definition of a function and the comparison of two numbers. The brute-force method could be used by university professors and students to solve the approximation problems from textbooks without studying of derivatives, and in fact without any studying. Secondly, since the method is numerical, it is universal. But if we want to receive the answer immediately, we have to use a computer program. Hence, the brute-force method could be used as a supplementary simple numerical method in the universities and colleges. If a professor insists the students to be familiar with the Gauss' method, the brute-force method could be used at least the professor and the students to be able easily to check the answers.

But the main advantages of the brute-force method are in the mathematics high school education. The personal opinion of the author of this paper is as follows. The high school teachers could include approximation problems in high school mathematics. They could use the brute-force method to solve the problems. Such an approach would make the mathematics high school education more comprehensive and useful during the grades from 7 to 12.

The brute-force method is as follows. Suppose that we have to find the minimum of a function f(x). First, we localize the minimum of f(x). Suppose that the minimum of the function f(x) is within the segment [a,b] We divide the segment [a,b] by N equal parts by using the points x0 = a, x1, x2,...,xN = b. Then we evaluate f(x0), f(x1), f(x2),…,f(xN), and select the minimal of these values. The minimal of the calculated values is the answer.

I have created a simple computer program by using PHP. The program is used in the examples given below. Note that the program easily solves all high school and college problems for finding the least squares approximation of a data set by a continuous function. It takes less than 1 second we to receive the answer.

Example 1. Use the least squares criterion to find a and b such that the line y = ax + b is closest to the points (1, 1.4), (2, 2.3), (3, 3) and (4, 3.6).

Solution. We have to minimize the function

$$f(a,b) = (a + b - 1.4)^2 + (2a + b - 2.3)^2 + (3a + b - 3)^2 + (4a + b - 3.6)^2$$

We use the computer program. We take the segment [0,2] as initial segment for a and b. We divide this segment by 200 equal parts. Then we calculate the values of f(x) at the initial point of each subsegment and select the minimal of these values. We receive the following answer: a ≈ 0.73 and b 0.75. Hence, the least squares line has equation y = 0.73 x + 0.75. Also, we obtain that the sum of the least squares is about 0.023. It takes less than 1 second we to obtain the answer.

We could use the Ivan Johansen's computer program Graph in order to draw the graphs of the data and the line. See Fig.1.

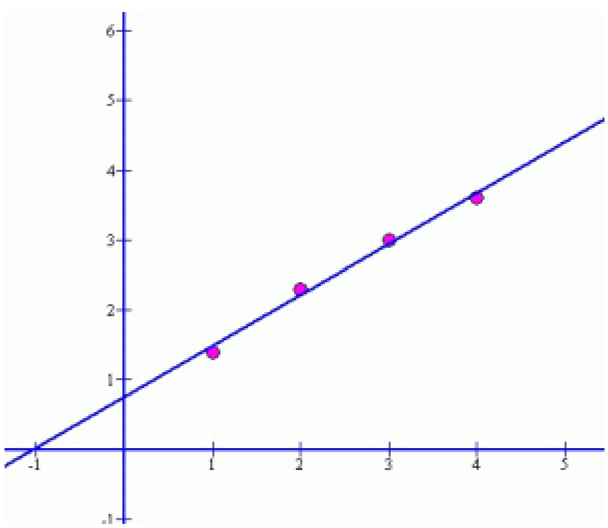

Fig.1

Example 2. Use the least squares criterion to find a and b such that the curve $y = ax^b$ is closest to the points given in Example1.

Solution. We have to minimize the function

$$f(a,b) = (a - 1.4)^2 + (a2^b - 2.3)^2 + (a3^b - 3)^2 + (a4^b - 3.6)^2$$

We use the computer program. We take the segment [0,2] as initial segment for a and b. We divide this segment by 200 equal parts. We receive the following answer: a ≈ 1.43 and b ≈ 0.67. Hence, the least squares curve has equation $y = (1.43)x^{0.67}$. Also, we obtain that the sum of the least squares is about 0.0021.

The graphs of the data and the curve are given at Fig.2.

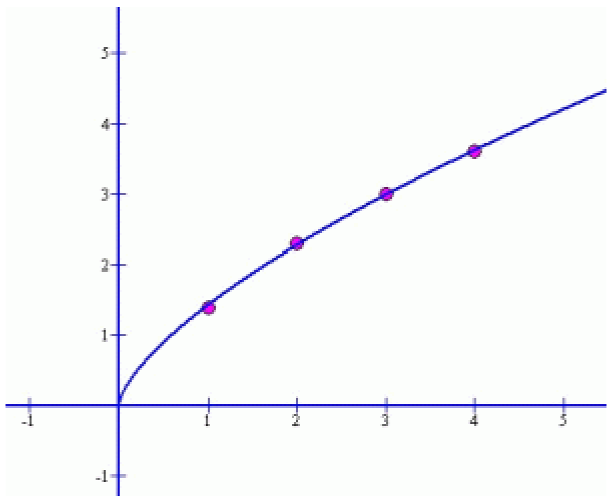

Fig.2

Example 3. Use the least squares criterion to find a and b such that the curve $y = ae^{bx}$ is closest to the points (1, 2.5), (2, 3.5), (3, 4.5) and (4, 6).

Solution. We have to minimize the function

$$f(a,b) = (ae^b - 2.5)^2 + (ae^{2b} - 3.5)^2 + (ae^{3b} - 4.5)^2 + (ae^{4b} - 6)^2$$

We use the computer program. We take the segment [0,5] as initial segment for a and b. We divide this segment by 250 equal parts. We receive the following answer: a ≈ 1.96 and b ≈ 0.28. Hence, the least squares curve has equation $y = (1.96)e^{0.28}$ Also, we obtain that the sum of the least squares is about 0.0151.

The graphs of the data and the curve are given at Fig.3.

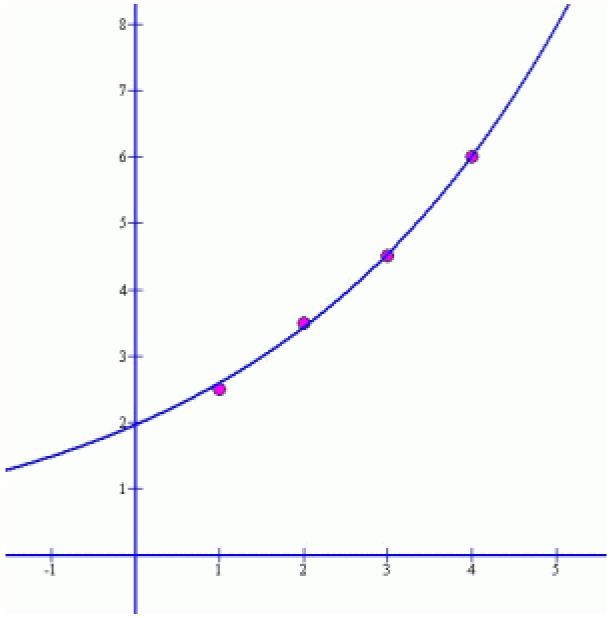

Fig.3

Example 4. Use the least squares criterion to find a and b such that the curve $y = a + b\ln x$ is closest to the points (0.1, -0.4). (0.3, -0.15), (0.5, -0.03), (0.7, 0.03).

Solution. We have to minimize the function

$$f(a,b) = (a + b\ln 0.1 + 0.4)^2 + (a + b\ln 0.3 + 0.15)^2 + (a + b\ln 0.5 + 0.03)^2 + (a + b\ln 0.7 - 0.03)^2$$

We use the computer program. We take the segment [0,1] as initial segment for a and b. We divide this segment by 100 equal parts. We receive the following answer: a ≈ 0.11 and b ≈ 0.22. Hence, the least squares curve has equation $y = 0.11 + (0.22)\ln x$. Also, we obtain that the sum of the least squares is about 0.00019.

The graphs of the data and the curve are given at Fig.4.

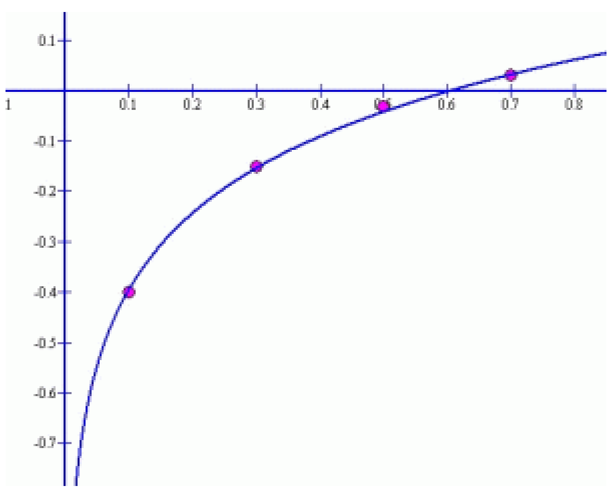

Fig.4

Example 5. Use the least squares criterion to find a, b and c such that the curve $y = a + b\sin(cx)$ is closest to the points (1, 2.8). (2, 3.2), (3, 3) (4, 2.4).

Solution. We have to minimize the function

$$f(a,b,c) = (a + b\sin(c) - 2.8)^2 + (a + b\sin(2c) - 3.2)^2 + (a + b\sin(3c) - 3)^2 + (a + b\sin(4c) - 2.4)^2$$

We use the computer program. We take the segment [0,5] as initial segment for a, b and c. We divide this segment by 50 equal parts. We receive the following answer: a ≈ 2.0, b ≈ 1.2 and c ≈ 0.7. Hence, the least squares curve has equation $y = 2 + (1.2)\sin(0.7x)$. Also, we obtain that the sum of the least squares is about 0.0023.

The graphs of the data and the curve are given at Fig.5.

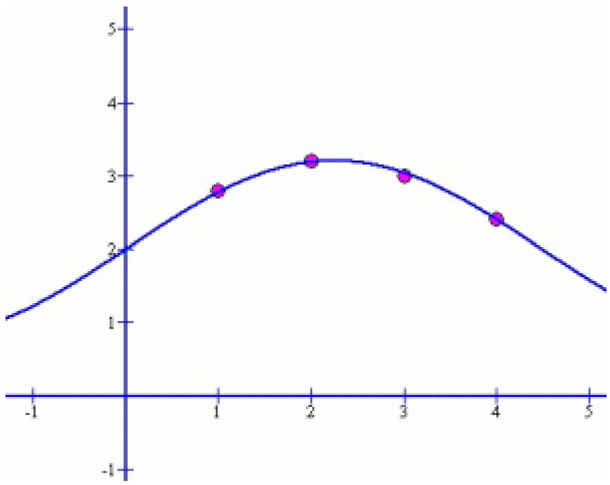

Fig.5

We could record the calculations, made by the computer. For the above examples, the file containing records of calculations is available for download as supplementary material.

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