

Least-Squares Line, Example 1.

Objective function:

$$f(a,b) = (a + b - 1.4)^2 + (2a + b - 2.3)^2 + (3a + b - 3)^2 + (4a + b - 3.6)^2$$

Record of calculations:

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Interval = [0, 2].

$$f(0, 0) = 29.21$$

$$f(0, 0.01) = 29.0044$$

$$f(0, 0.02) = 28.7996$$

$$f(0, 0.03) = 28.5956$$

$$f(0, 0.04) = 28.3924$$

$$f(0, 0.05) = 28.19$$

$$f(0, 0.06) = 27.9884$$

$$f(0, 0.07) = 27.7876$$

$$f(0, 0.08) = 27.5876$$

$$f(0, 0.09) = 27.3884$$

$$f(0, 0.1) = 27.19$$

$$f(0, 0.11) = 26.9924$$

$$f(0, 0.12) = 26.7956$$

$$f(0, 0.13) = 26.5996$$

$$f(0, 0.14) = 26.4044$$

$$f(0, 0.15) = 26.21$$

etc.

Answer:

$$f_{\min} = 0.023$$

$$a = 0.73$$

$$b = 0.75$$

Least-Squares Curve  $y = ax^b$ . Example 2.

Objective function:

$$f(a,b) = (a - 1.4)^2 + (a2^b - 2.3)^2 + (a3^b - 3)^2 + (a4^b - 3.6)^2$$

Record of calculations:

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Interval = [0, 2].

$$f(0, 0) = 29.21$$

$$f(0, 0.01) = 29.21$$

$$f(0, 0.02) = 29.21$$

$$f(0, 0.03) = 29.21$$

$$f(0, 0.04) = 29.21$$

$$f(0, 0.05) = 29.21$$

$$\begin{aligned}
f(0, 0.06) &= 29.21 \\
f(0, 0.07) &= 29.21 \\
f(0, 0.08) &= 29.21 \\
f(0, 0.09) &= 29.21 \\
f(0, 0.1) &= 29.21 \\
f(0, 0.11) &= 29.21 \\
f(0, 0.12) &= 29.21 \\
f(0, 0.13) &= 29.21 \\
f(0, 0.14) &= 29.21 \\
f(0, 0.15) &= 29.21
\end{aligned}$$

etc.

Answer:

$$f_{\min} = 0.0021281038572061$$

$$a = 1.43$$

$$b = 0.67$$

Least-Squares Curve  $y = ae^{bx}$  Example.

Objective function:

$$f(a, b) = (ae^b - 2.5)^2 + (ae^{2b} - 3.5)^2 + (ae^{3b} - 4.5)^2 + (ae^{4b} - 6)^2$$

Record of calculations:

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Interval = [0, 5].

$$\begin{aligned}
f(0, 0) &= 74.75 \\
f(0, 0.02) &= 74.75 \\
f(0, 0.04) &= 74.75 \\
f(0, 0.06) &= 74.75 \\
f(0, 0.08) &= 74.75 \\
f(0, 0.1) &= 74.75 \\
f(0, 0.12) &= 74.75 \\
f(0, 0.14) &= 74.75 \\
f(0, 0.16) &= 74.75 \\
f(0, 0.18) &= 74.75 \\
f(0, 0.2) &= 74.75 \\
f(0, 0.22) &= 74.75 \\
f(0, 0.24) &= 74.75 \\
f(0, 0.26) &= 74.75 \\
f(0, 0.28) &= 74.75 \\
f(0, 0.3) &= 74.75
\end{aligned}$$

etc.

Answer:

$$f_{\min} = 0.015085480171014$$

$$a = 1.96$$

$$b = 0.28$$

Least-Squares Curve  $y = a + b \ln x$ . Example 4.

Objective function:

$$f(a, b) = (a + b \ln 0.1 + 0.4)^2 + (a + b \ln 0.3 + 0.15)^2 + (a + b \ln 0.5 + 0.03)^2 + (a + b \ln 0.7 - 0.03)^2$$

Record of calculations:

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Interval = [0, 1].

$$f(0, 0) = 0.1843$$

$$f(0, 0.01) = 0.16280142936646$$

$$f(0, 0.02) = 0.14277468246363$$

$$f(0, 0.03) = 0.12421975929152$$

$$f(0, 0.04) = 0.10713665985013$$

$$f(0, 0.05) = 0.091525384139452$$

$$f(0, 0.06) = 0.077385932159495$$

$$f(0, 0.07) = 0.064718303910255$$

$$f(0, 0.08) = 0.053522499391732$$

$$f(0, 0.09) = 0.043798518603927$$

$$f(0, 0.1) = 0.035546361546839$$

$$f(0, 0.11) = 0.028766028220468$$

$$f(0, 0.12) = 0.023457518624814$$

$$f(0, 0.13) = 0.019620832759878$$

$$f(0, 0.14) = 0.01725597062566$$

$$f(0, 0.15) = 0.016362932222159$$

etc.

Answer:

$$f_{\min} = 0.00019393480171204$$

$$a = 0.11$$

$$b = 0.22$$

Least-Squares Curve  $y = a + b \sin(cx)$ . Example 5.

Objective function:

$$f(a, b, c) = (a + b \sin(c) - 2.8)^2 + (a + b \sin(2c) - 3.2)^2 + (a + b \sin(3c) - 3)^2 + (a + b \sin(4c) - 2.4)^2$$

Record of calculations:

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Interval = [0, 5].

$$f(0, 0, 0) = 32.84$$

$$f(0, 0, 0.1) = 32.84$$

$$f(0, 0, 0.2) = 32.84$$

$$f(0, 0, 0.3) = 32.84$$

$$f(0, 0, 0.4) = 32.84$$

$$f(0, 0, 0.5) = 32.84$$

$$f(0, 0, 0.6) = 32.84$$

$$f(0, 0, 0.7) = 32.84$$

$$f(0, 0, 0.8) = 32.84$$

$$f(0, 0, 0.9) = 32.84$$

$$f(0, 0, 1) = 32.84$$

$$f(0, 0, 1.1) = 32.84$$

$$f(0, 0, 1.2) = 32.84$$

$$f(0, 0, 1.3) = 32.84$$

$$f(0, 0, 1.4) = 32.84$$

$$f(0, 0, 1.5) = 32.84$$

etc.

Answer:

$$f_{\min} = 0.0023198152610863$$

$$a = 2.0$$

$$b = 1.2$$

$$c = 0.7$$