

Least-Squares Normal Distribution, Example.

Objective function:

$$f(\mu, \sigma) = \left( \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(155-\mu)^2}{2\sigma^2}} - 0.0125 \right)^2 + \left( \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(165-\mu)^2}{2\sigma^2}} - 0.0375 \right)^2 \\ + \left( \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(175-\mu)^2}{2\sigma^2}} - 0.04 \right)^2 + \left( \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(185-\mu)^2}{2\sigma^2}} - 0.01 \right)^2$$

Record of calculations:

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dig = 10 // The number of the digits after the decimal point in the answer.

N = 10

Step 1:

mu = 170;

sigma = 8;

Initial segment for mu: [165, 175].

Initial segment for sigma: [3, 13].

The length of the segments = 10.

The length of the subsegments = 1.

Output:

Minimal value of the objective function: f = 7.4887942262736E-6

mu = 170

sigma = 9

Step 2:

Segment for mu: [169.5, 170.5].

Segment for sigma: [8.5, 9.5].

The length of the segments = 1.

The length of the subsegments = 0.1.

Output:

Minimal value of the objective function: f = 7.0393863076995E-6

mu = 170

sigma = 8.9

Step 3:

Segment for mu: [169.95, 170.05].

Segment for sigma: [8.85, 8.95].

The length of the segments = 0.1.

The length of the subsegments = 0.01.

Output:

Minimal value of the objective function: f = 6.9653871853635E-6

mu = 170.05

sigma = 8.87

Step 4:

Segment for mu: [170.045, 170.055].

Segment for sigma: [8.865, 8.875].

The length of the segments = 0.01.

The length of the subsegments = 0.001.

Output:

Minimal value of the objective function:  $f = 6.9646319222571E-6$

mu = 170.048

sigma = 8.865

Step 5:

Segment for mu: [170.0475, 170.0485].

Segment for sigma: [8.8645, 8.8655].

The length of the segments = 0.001.

The length of the subsegments = 0.0001.

Output:

Minimal value of the objective function:  $f = 6.9646300546795E-6$

mu = 170.0479

sigma = 8.8652

Step 6:

Segment for mu: [170.04785, 170.04795].

Segment for sigma: [8.86515, 8.86525].

The length of the segments = 0.0001.

The length of the subsegments =  $1.0E-5$ .

Output:

Minimal value of the objective function:  $f = 6.964629976249E-6$

mu = 170.04786

sigma = 8.86524

Step 7:

Segment for mu: [170.047855, 170.047865].

Segment for sigma: [8.865235, 8.865245].

The length of the segments =  $1.0E-5$ .

The length of the subsegments =  $1.0E-6$ .

Output:

Minimal value of the objective function:  $f = 6.9646299759093E-6$

mu = 170.047861

sigma = 8.865243

Step 8:

Segment for mu: [170.0478605, 170.0478615].

Segment for sigma: [8.8652425, 8.8652435].

The length of the segments =  $1.0E-6$ .

The length of the subsegments =  $1.0E-7$ .

Output:

Minimal value of the objective function:  $f = 6.9646299759034E-6$   
 $\mu = 170.0478607$   
 $\sigma = 8.8652434$

Step 9:

Segment for  $\mu$ : [170.04786065, 170.04786075].

Segment for  $\sigma$ : [8.86524335, 8.86524345].

The length of the segments =  $1.0E-7$ .

The length of the subsegments =  $1.0E-8$ .

Output:

Minimal value of the objective function:  $f = 6.9646299759034E-6$

$\mu = 170.04786074$

$\sigma = 8.86524342$

Step 10:

Segment for  $\mu$ : [170.047860735, 170.047860745].

Segment for  $\sigma$ : [8.865243415, 8.865243425].

The length of the segments =  $1.0E-8$ .

The length of the subsegments =  $1.0E-9$ .

Output:

Minimal value of the objective function:  $f = 6.9646299759033E-6$

$\mu = 170.047860742$

$\sigma = 8.865243415$

Step 11:

Segment for  $\mu$ : [170.0478607415, 170.0478607425].

Segment for  $\sigma$ : [8.8652434145, 8.8652434155].

The length of the segments =  $1.0E-9$ .

The length of the subsegments =  $1.0E-10$ .

Output:

Minimal value of the objective function:  $f = 6.9646299759033E-6$

$\mu = 170.047860742$

$\sigma = 8.865243415$

Step 12:

Segment for  $\mu$ : [170.04786074195, 170.04786074205].

Segment for  $\sigma$ : [8.86524341495, 8.86524341505].

The length of the segments =  $1.0E-10$ .

The length of the subsegments =  $1.0E-11$ .

Output:

Minimal value of the objective function:  $f = 6.9646299759033E-6$

$\mu = 170.047860742$

$\sigma = 8.86524341504$

Answer:

$$\mu = 170.047860742$$

$$\sigma = 8.865243415$$