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A new approach for solving 2 and 3-dimensional linear optimization problems

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Abstract. In this note we offer a new simple numerical method for solving 2 and 3-dimensional linear optimization problems. The method is designed for use in high schools and colleges.

Keywords: numerical method, linear optimization

The simplex method for solving 2 and 3-dimensional linear optimization problems has an alternative. In this paper we offer a new simple numerical method able to solve the problems from textbooks for university students in the area of 2 and 3-dimensional linear optimization. The method first is described in [1]. In this paper we show that the method could be applied to solve linear optimization problems.

The simplex method has two weaknesses. First, it is too complicated. Secondly, in many problems the answers must be in integers, but in these cases the simplex method is not able to find the solutions. The authors of textbooks are not able to avoid the first problem. They avoid the second problem as follows. The authors of textbooks create problems whose answers are integers. But such an approach is unnatural, and in fact it deprives the students of a method for solving the problems.

The approach of this paper solves the above two problems. First, the method of this paper uses only the definition of a function and the comparison of two numbers. The approach of this paper could be used by university professors and students to solve the 2 and 3-dimensional linear optimization problems from textbooks without studying the simplex method and in fact without any studying. If a professor insists the students to be familiar with the simplex method, the approach of this paper could be used at least the professor and to students to be able easily to check the answers. Since the method of this paper is simple, the approach of this paper gives the possibility the 2 and 3-dimensional linear optimization problems to be included also in the high-school education. The high-school students could understand and use the method without any difficulties. But if we want to receive the answer immediately, we have to use a computer program.

Secondly, the method of this paper solves the integer 2 and 3-dimensional linear optimization problems. In this case the solution is even simpler.

The personal opinion of the author of this paper is as follows. The professors could replace the simplex method by the described in this paper method, accompanied by a computer program. Such an approach would save lots of time. Also, the students will be able to solve the integer optimization problems. But the professors have to explain clearly to the students that for the real world problems which contain many variables, possibly hundreds variables, the computer programs use improved and sophisticated methods.

The method is as follows. Suppose we have to solve a linear optimization problem. Suppose that f(x,y) is the objective function, and suppose that we have to find the minimum of f(x,y). First, we localize the minimum of f(x,y). Suppose that we seek the minimum of the function f(x,y) within the square [a,b] x [a,b]. We divide the segment [a,b] by N equal parts by using the points x0 = a, x1, x2, ... ,xN = b and then by points y0 = a, y1, y2, ... ,yN = b. Then we calculate f(x,y) for each x in {f(x0), f(x1), f(x2), ... ,f(xN)} and each y in {f(y0), f(y1), f(y2), ... ,f(yN)}, and select the minimal of these values. We use the minimal value as the center of a new square, whose length is 10 times smaller than the length of the previous square. The process is repeated until the minimum is found. Similarly, we can find the maximum of the objective functions. In both cases, we could add arbitrary constraints. The method works well if $N \geq 100$, but in many cases it is enough we to set smaller N. The extension of the method to the case in which the objective function has three variables is straightforward.

I have created a simple computer program by using PHP. The program is used in the examples given below.

Example 1. A manufacturer makes two grades of concrete. Each bag of the high-grade concrete contains 11 kilograms of gravel and 6 kilograms of cement, while each bag of low-grade concrete contains 13 kilograms of gravel and 4 kilograms of cement. There are 1,850 kilograms of gravel and 600 kilograms of cement currently available. The manufacturer can make a profit of $2.50 on each bag of the high-grade and $1.70 on each bag of the low-grade concrete. How many bags of each grade should be made up from available supplies to generate the largest possible profit?

Solution. For the convenience of the reader, we rewrite the input data of the problem in the following table.

	bag of the high-grade	bag of the low-grade	Available supplies
Gravel	11	13	1,850
Cement	6	4	600
Price	2.50	1.70	

We have to maximize f(x,y) = 2.5x + 1.7y subject to

$$\begin{cases} 11x + 13y \leq 1850 \\ 6x + 4y \leq 600 \\ x, y \geq 0 \end{cases}$$

We have to find integers as solution. We use the computer program. Set N = 100. From the system of inequalities it follows that it is enough we to consider x and y such that $0 \leq x,y \leq 200$. If we want to find a solution with numbers having 3 digits after the decimal point, we receive the following answer: x = 11.768, y = 132.348 and f = 254.412. If we use the simplex method, we will obtain the same answer. We could round x and y to integers, so that we could accept x = 12 and y = 132 as an answer. In this case this is the correct answer, but such a situation is an exception.

We could use the computer program to obtain the integer solution. At the first step of calculations we obtain x = 12, y = 132, f = 254.4. This is the integer solution.

Example 2. A toy manufacturer makes three different kinds of model cars, the Porsche, the Ferrari, and the Jaguar, which are made of aluminum and steel. Each Porsche requires 5 units of steel and 3 units of aluminum, while the Ferrari and the Jaguar require 3 units and 7 units of steel, and 2 units and 6 units of aluminum, respectively. The company has available 78 units of steel and 80 units of aluminum. If $12 profit is made on each Porsche, $10 on each Ferrari and $8 on each Jaguar, what is the maximum possible profit?

Solution. For the convenience of the reader, we rewrite the input data of the problem in the following table.

	Porsche	Ferrari	Jaguar	Units available
Steel	5	3	2	78
Aluminum	3	7	6	80
Profit	12	10	8	

We have to maximize f(x,y,z) = 12x + 10y + 8z subject to

$$\left| \begin{array}{l} 5x + 3y + 2z \leq 78 \\ 3x + 7y + 6z \leq 80 \\ x, y \geq 0 \end{array} \right.$$

We have to find integers as solution. From the system of inequalities it follows that it is enough we to consider x, y and z such that $0 \leq x,y,z \leq 50$. If we want a solution with numbers having 3 digits after the decimal point, we receive the following answer: x = 12.823, y = 0.059, z = 6.853, and f = 209.290. If we use the simplex method, we will obtain the same answer. We could round x, y and z to integers, that is, x = 13, y = 0 and z = 7, but these rounded values are not the answer. We could easily see that these integers do not satisfy the system of inequalities.

We could use the computer program to obtain the integer solution. At the first step of calculations we obtain x = 13, y = 1, z = 5, and f = 206. This is the integer solution. Note that by using the simplex method we cannot obtain the integer solution.

Example 3. A farmer mixes feed for her dairy herd using three different grains, G1, G2 and G3. The feed mix must meet certain minimum nutritional standards. In particular, it must contain at least 35 units of nutrient N1, at least 40 units of nutrient N2, and at least 50 units of nutrient N3. The nutritional content (in units) and the cost of each kilogram of each grain is given in the following table:

	G1	G2	G3	ration
N1	5	2	3	20
N2	4	5	4	30
N3	2	3	5	25
cost	35	40	50	

How can the mix be made in order to satisfy the nutritional requirements at minimal cost?

Solution. We have to minimize f(x,y,z) = 35x + 40y + 50z subject to

$$\begin{cases} 5x + 2y + 3z \geq 20 \\ 4x + 5y + 4z \geq 30 \\ 2x + 3y + 5z \geq 25 \\ x, y, z \geq 0 \end{cases}$$

We have to find real numbers as solution. We use the computer program. Set N = 50. From the system of inequalities it follows that it is enough we to consider x, y and z such that $0 \leq x,y,z \leq 50$. If we want to find a solution with numbers having 3 digits after the decimal point, we receive the following answer: x = 1.171, y = 2.766, z = 2.872 and f = 295.225. If we use the simplex method, we will obtain the same answer. We could round x, y and z to integers, but x = 1, y = 3 and z = 3 is not a solution. We could easily see that these integers do not satisfy the system of inequalities.

If we want to find a rounded answer, we have to see the outputs of the steps in the computer program. After the first step we obtain x = 2, y = 2, z = 3, and f = 300. This is the answer, if we want to use integers in the answer. We cannot obtain this answers, if we use the simplex method. After the second step we obtain x = 1.3, y = 2.8, z = 2.8, and f = 297.5. This is the answer, if we want to use in the answer with real numbers having one digit after the decimal point. We cannot obtain this answer, if we use the simplex method.

We could record the calculations, made by the computer. For the above examples, the files containing records of calculations are available for download as supplementary materials.

References

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