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The use of the brute-force method for solving 2 and 3-dimensional linear optimization problems

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Abstract. In this paper we propose the brute-force method as a suitable method for solving 2 and 3-dimensional linear optimization problems. We show that the brute-force method essentially simplifies and improves the solutions of 2 and 3-dimensional linear optimization problems.

Keywords: numerical method, linear optimization

The simplex method for solving 2 and 3-dimensional linear optimization problems has an alternative. In this paper we show that the ordinary brute-force method effectively solves the problems from textbooks for university students in the area of 2 and 3-dimensional linear optimization.

The simplex method has two weaknesses. First, it is too complicated. Secondly, in many problems the answer must be in integers, but in these cases the simplex method is not able to find the solutions. The authors of textbooks are not able to avoid the first problem. They avoid the second problem as follows. The authors of textbooks create problems whose answers are integers. But such an approach is unnatural, and in fact it deprives the students of a method for solving the problems.

The brute-force method solves the above two problems. First, the brute-force uses only the definition of a function and the comparison of two numbers. The brute-force method could be used by university professors and students to solve the 2 and 3-dimensional linear optimization problems from textbooks without studying the simplex method and in fact without any studying. If a professor insists the students to be familiar with the simplex method, the brute-force method could be used at least the professor and the students to be able easily to check the answers. Since the brute-force method is simple, it gives the possibility the 2 and 3-dimensional linear optimization problems to be included also in the high-school education. The high-school students could understand and use the brute-force method without any difficulties. But if we want to receive the answer immediately, we have to use a computer program.

Secondly, the brute-force method solves the integer 2 and 3-dimensional linear optimization problems.

The experts in the area of discrete optimization do not like and do not recommend the brute-force method. But for the case of the university textbook problems the method is suitable. It takes less than one second we to receive the answer, if we use a desktop personal computer.

The personal opinion of the author of this paper is as follows. The professors could replace the simplex method by the brute-force method, accompanied by a computer program. Such an approach would save lots of time and efforts. Also, the students will be able to solve the integer optimization problems. But the professors have to explain clearly to the students that for the real world problems which contain many variables, possibly hundreds variables, the computer programs use improved and sophisticated methods.

The brute-force method is as follows. Suppose we have to solve an integer linear optimization problem. Suppose that f(x,y) is the objective function, and suppose that we have to find the minimum of f(x,y). First, we localize the minimum of f(x,y). Suppose that we seek the minimum of the function f(x,y) within the square [a,b] x [a,b], where a and b are integers. Let N = b – a. We divide the segment [a,b] by N equal parts by using the points x0 = a, x1, x2, ... ,xN = b and then by points y0 = a, y1, y2, ... ,yN = b. Then we calculate f(x,y) for each x in {f(x0), f(x1), f(x2), ... ,f(xN)} and each y in {f(y0), f(y1), f(y2), ... ,f(yN)}, and select the minimal of these values. The minimal value is the answer to the problem. Similarly, we can find the maximum of the objective functions. In both cases, we could add arbitrary constraints. The extension of the method to the case in which the objective function has three variables is straightforward. If we have to solve an optimization problem whose answer requires real numbers, we could modify the problem to an integer optimization problem.

I have created a simple computer program by using PHP. The program is used in the examples given below.

Example 1. A manufacturer makes two grades of concrete. Each bag of the high-grade concrete contains 11 kilograms of gravel and 6 kilograms of cement, while each bag of low-grade concrete contains 13 kilograms of gravel and 4 kilograms of cement. There are 1,850 kilograms of gravel and 600 kilograms of cement currently available. The manufacturer can make a profit of $2.50 on each bag of the high-grade and $1.70 on each bag of the low-grade concrete. How many bags of each grade should be made up from available supplies to generate the largest possible profit?

Solution. For the convenience of the reader, we rewrite the input data of the problem in the following table.

	bag of the high-grade	bag of the low-grade	Available supplies
Gravel	11	13	1,850
Cement	6	4	600
Price	2.50	1.70	

We have to maximize f(x,y) = 2.5x + 1.7y subject to

$$\begin{cases} 11x + 13y \le 1850 \\ 6x + 4y \le 600 \\ x, y \ge 0 \end{cases}$$

We have to find integers as solution. We use the computer program. From the system of inequalities it follows that it is enough we to consider integers x and y such that $0 \le x,y \le 200$. We obtain x = 12, y = 132, f = 254.4. The computer finds the solution for less than 1 second.

Example 2. A toy manufacturer makes three different kinds of model cars, the Porsche, the Ferrari, and the Jaguar, which are made of aluminum and steel. Each Porsche requires 5 units of steel and 3 units of aluminum, while the Ferrari and the Jaguar require 3 units and 7 units of steel, and 2 units and 6 units of aluminum, respectively. The company has available 78 units of steel and 80 units of aluminum. If $12 profit is made on each Porsche, $10 on each Ferrari and $8 on each Jaguar, what is the maximum possible profit?

Solution. For the convenience of the reader, we rewrite the input data of the problem in the following table.

	Porsche	Ferrari	Jaguar	Units available
Steel	5	3	2	78
Aluminum	3	7	6	80
Profit	12	10	8	

We have to maximize f(x,y,z) = 12x + 10y + 8z subject to

$$\begin{cases} 5x + 3y + 2z \le 78 \\ 3x + 7y + 6z \le 80 \\ x, y \ge 0 \end{cases}$$

We have to find integers as solution. We use the computer program. From the system of inequalities it follows that it is enough we to consider integers x, y and z such that $0 \le x,y,z \le 30$. We obtain x = 13, y = 1, z = 5, and f = 206. The computer finds the solution for less than 1 second. Note that by using the simplex method we cannot obtain the integer solution.

Example 3. A farmer mixes feed for her dairy herd using three different grains, G1, G2 and G3. The feed mix must meet certain minimum nutritional standards. In particular, it must contain at least 35 units of nutrient N1, at least 40 units of nutrient N2, and at least 50 units of nutrient N3. The nutritional content (in units) and the cost of each kilogram of each grain is given in the following table:

	G1	G2	G3	ration
N1	5	2	3	20

N2	4	5	4	30
N3	2	3	5	25
cost	35	40	50	

How can the mix be made in order to satisfy the nutritional requirements at minimal cost?

Solution. We have to minimize f(x,y,z) = 35x + 40y + 50z subject to

$$\begin{cases} 5x + 2y + 3z \geq 20 \\ 4x + 5y + 4z \geq 30 \\ 2x + 3y + 5z \geq 25 \\ x, y, z \geq 0 \end{cases}$$

We use the computer program. From the system of inequalities it follows that it is enough we to consider integers x, y and z such that 0 ≤ x,y,z ≤ 10. If we want to have an integer solution, we obtain x = 2, y = 2, z = 3, and f = 300. The computer finds the solution for less than 1 second. Note that by using the simplex method we cannot obtain this integer solution.

If we want to use in the answer real numbers with one digit after the decimal point, first we have to modify the problem to an integer problem. To modify the problem, we use the following modified table:

	G1	G2	G3	ration
N1	0.5	0.2	0.3	20
N2	0.4	0.5	0.4	30
N3	0.2	0.3	0.5	25
cost	3.5	4.0	5.0	

Now we have to minimize f(x,y,z) = 3.5x + 4.0y + 5.0z subject to

$$\begin{cases} 0.5x + 0.2y + 0.3z \geq 20 \\ 0.4x + 0.5y + 0.4z \geq 30 \\ 0.2x + 0.3y + 0.5z \geq 25 \\ x, y, z \geq 0 \end{cases}$$

We use the computer program. From the system of inequalities it follows that it is enough we to consider integers x and y such that 0 ≤ x,y,z ≤ 50. We obtain x = 13, y = 28, z = 28, and f = 297.5, that is, the answer is as follows: x = 1.3 kilogram, y = 2.8 kilogram, z = 2.8 kilogram and f = 297.5. The computer finds the solution for less than 1 second. Note that by using the simplex method we cannot obtain this solution.

We could record the calculations, made by the computer. For the above examples, the files containing records of calculations are available for download as supplementary materials.

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