Journal of Computer-Generated Mathematics

# The least squares normal approximation to the Poisson distribution

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**Abstract.** By using the method, first described in [1] and then used in [2], we find the least squares normal approximation to the Poisson distribution. The method is designed for use in high schools and colleges.

**Keywords:** normal approximation; least squares; Poisson distribution

In this paper, by using the method, first described in [1] and then used in [2], we find the least squares normal approximation to the Poisson distribution.

Recall that the Poisson distribution is given by

$$P(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

where the variable k is an integer (k ≥ 0) and the parameter λ is a real positive quantity. The expected value of the Poisson distribution is μP = λ, and the standard deviation is $\sigma P = \sqrt{\lambda}$. The normal distribution N(μP,σP), defined by μP and σP, is an approximation to the Poisson distribution, which we will call the standard normal approximation to the Poisson distribution.

We use a computer program in order to obtain the expected value μL and the standard deviation σL of the least squares normal approximation N(μL,σL) to the Poisson distribution.. The computer program finds μL and σL as the solution to the following problem. Find μ and σ, which minimize the function

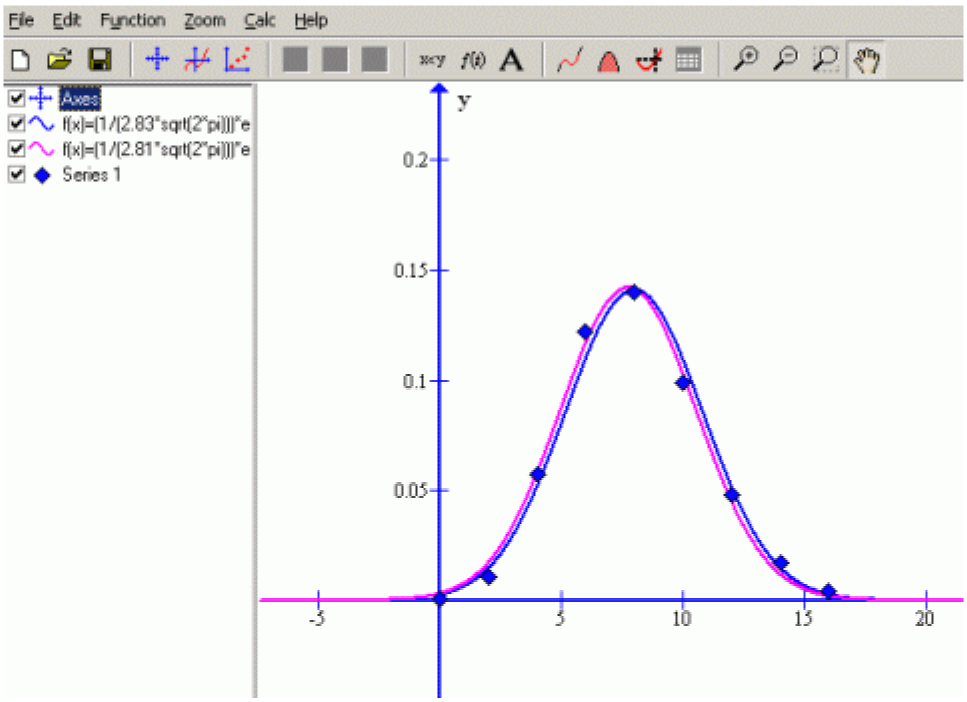
$$f(\mu,\sigma) = \sum_{k=0}^{n} \left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(k-\mu)^2}{2\sigma^2}} - \frac{\lambda^k e^{-\lambda}}{k!}\right)^2$$

**Example 1.** Find the least squares normal approximation to the Poisson distribution P(λ), if λ = 8 and 0 ≤ k ≤ 16.

Set N = 10. As initial intervals for finding μL and σL we take the intervals [μP-0.5, μP+0.5] and [σP-0.5, σP+0.5], where μP = 8 and σP = 2.8 are the rounded values of the expected value and standard deviation of the Poisson distribution, provided λ = 8 and 0 ≤ k ≤ 16. If we require 3 digits after the decimal point, we receive the following answer: μL = 7.747 and σL = 2.809. Hence, the probability density function of the least squares normal distribution is as follows:

$$N(x) = \frac{1}{2.809\sqrt{2\pi}} e^{-\frac{(x-7.747)^2}{2(2.809)^2}}$$

In the graph below of the standard normal curve we take μP = 8 and σP = 2.83. The red curve is the least squares normal curve:

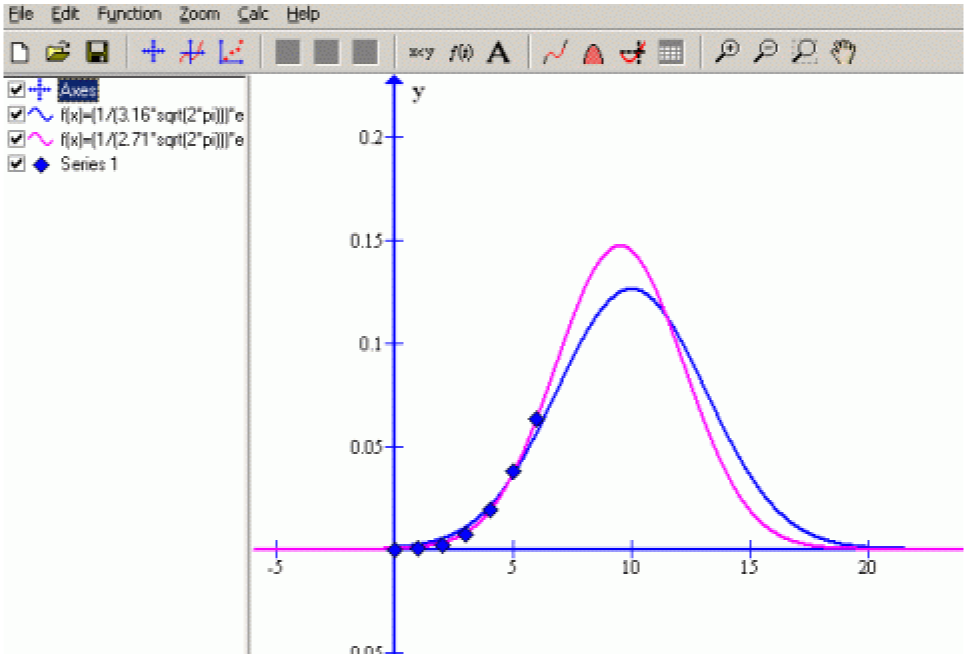



**Example 2.** Find the least squares normal approximation to the Poisson distribution P(λ), if λ = 10 and 0 ≤ k ≤ 6.

Set N = 10. As initial intervals for finding μL and σL we take the intervals [μP-0.5, μP+0.5] and [σP-0.5, σP+0.5], where μP = 10 and σP = 3.2 are the rounded values of the expected value and standard deviation of the Poisson distribution, provided λ = 10 and 0 ≤ k ≤ 6. If we require 3 digits after the decimal point, we receive the following answer: μL = 9.514 and σL = 2.715. Hence, the probability density function of the least squares normal distribution is as follows:

$$N(x) = \frac{1}{2.715\sqrt{2\pi}} e^{-\frac{(x-9.514)^2}{2(2.715)^2}}$$

In the graph below of the standard normal curve we take μP = 10 and σP = 3.16. The red curve is the least squares normal curve:




In this example, the least squares normal distribution approximately 23 times better fits the Poisson distribution than the standard normal distribution. Let P and L are respectively the sum of squares in the standard and in the least squares normal distributions. For the convenience of the reader we give below a table which shows the values of the quotient P / L for some λ and n, $0 \le k \le n$.

| | n = 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|
| λ = 5 | 3.17 | 3.27 | 3.96 | 3.57 | 2.93 | 2.63 | 2.54 | 2.51 |
| 6 | 5.78 | 3.17 | 3.07 | 3.75 | 3.68 | 3.08 | 2.68 | 2.52 |
| 7 | 11.64 | 5.68 | 3.22 | 2.99 | 3.62 | 3.78 | 3.27 | 2.80 |
| 8 | 18.53 | 11.08 | 5.66 | 3.30 | 2.96 | 3.51 | 3.84 | 3.46 |
| 9 | 24.48 | 17.66 | 10.69 | 5.65 | 3.37 | 2.96 | 3.43 | 3.86 |
| 10 | 41.19 | 23.19 | 16.98 | 10.39 | 5.66 | 3.44 | 2.96 | 3.36 |
| 11 | 128.90 | 33.43 | 22.34 | 16.41 | 10.14 | 5.67 | 3.51 | 2.98 |
| 12 | 476.50 | 84.01 | 29.46 | 21.69 | 15.91 | 9.92 | 5.67 | 3.57 |

We could record the calculations, made by the computer. For the above examples, the files containing records of calculations are available for download as supplementary materials.

## References

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2. Deko Dekov, The least squares normal approximation to the binomial distribution, Journal of Computer-Generated Mathematics, vol. 7 (2012), no 3.

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Submitted on 1 October 2011
Publication date: 1 February 2012
Revised publication: 10 April 2012