

## Least-Squares Pareto Approximation

Objective function:

$$f(\alpha) = \left(\frac{\alpha 2^\alpha}{2^{\alpha+1}} - 1.25\right)^2 + \left(\frac{\alpha 2^\alpha}{3^{\alpha+1}} - 0.3\right)^2 + \left(\frac{\alpha 2^\alpha}{4^{\alpha+1}} - 0.1\right)^2 + \left(\frac{\alpha 2^\alpha}{5^{\alpha+1}} - 0.05\right)^2$$

Record of calculations:

dig = 5 // The number of the digits after the decimal point in the answer.

N = 10

Step 1:

alpha = 2.5;

Initial segment for alpha: [2, 3].

The length of the segments = 1.

The length of the subsegments = 0.1.

f(2.1) = 0.040634099282236

f(2.2) = 0.022962570229

f(2.3) = 0.010318818999464

f(2.4) = 0.002702959280226

f(2.5) = 0.00011608959656875

f(2.6) = 0.0025599542544206

f(2.7) = 0.010036683627336

f(2.8) = 0.02254859671627

f(2.9) = 0.040098052557335

f(3) = 0.062687339921125

Output:

Minimal value of the objective function: f = 0.00011608959656875

alpha = 2.5

Step 2:

Segment: [2.45, 2.55].

The length of the segment = 0.1.

The length of the subsegment = 0.01.

f(2.45) = 0.00078080341542181

f(2.46) = 0.00054725814590802

f(2.47) = 0.00036401225958599

f(2.48) = 0.00023106760408469

f(2.49) = 0.00014842607161364

f(2.5) = 0.00011608959656875

f(2.51) = 0.00013406015320243

f(2.52) = 0.00020233975335676

f(2.53) = 0.00032093044425793

f(2.54) = 0.0004898343063706

f(2.55) = 0.00070905345131078

Output:

Minimal value of the objective function:  $f = 0.00011608959656875$   
 $\alpha = 2.5$

Step 3:

Segment: [2.495, 2.505].

The length of the segment = 0.01.

The length of the subsegment = 0.001.

$f(2.495) = 0.00012596957942942$

$f(2.496) = 0.0001229874542095$

$f(2.497) = 0.00012050839133126$

$f(2.498) = 0.00011853239277499$

$f(2.499) = 0.00011705946052484$

$f(2.5) = 0.00011608959656875$

$f(2.501) = 0.00011562280289844$

$f(2.502) = 0.00011565908150939$

$f(2.503) = 0.00011619843440081$

$f(2.504) = 0.00011724086357564$

$f(2.505) = 0.0001187863710405$

Output:

Minimal value of the objective function:  $f = 0.00011562280289844$

$\alpha = 2.501$

Step 4:

Segment: [2.5005, 2.5015].

The length of the segment = 0.001.

The length of the subsegment = 0.0001.

$f(2.5005) = 0.00011579331582324$

$f(2.5006) = 0.00011574915180464$

$f(2.5007) = 0.00011571001850086$

$f(2.5008) = 0.00011567591591391$

$f(2.5009) = 0.00011564684404577$

$f(2.501) = 0.00011562280289844$

$f(2.5011) = 0.00011560379247392$

$f(2.5012) = 0.0001155898127742$

$f(2.5013) = 0.0001155808638013$

$f(2.5014) = 0.00011557694555719$

$f(2.5015) = 0.00011557805804389$

Output:

Minimal value of the objective function:  $f = 0.00011557694555719$

$\alpha = 2.5014$

Step 5:

Segment: [2.50135, 2.50145].

The length of the segment = 0.0001.

The length of the subsegment =  $1.0E-5$ .

$f(2.50135) = 0.00011557827583802$

$$f(2.50136) = 0.00011557790916725$$

$$f(2.50137) = 0.00011557759280378$$

$$f(2.50138) = 0.00011557732674761$$

$$f(2.50139) = 0.00011557711099875$$

$$f(2.5014) = 0.00011557694555719$$

$$f(2.50141) = 0.00011557683042294$$

$$f(2.50142) = 0.000115576765596$$

$$f(2.50143) = 0.00011557675107638$$

$$f(2.50144) = 0.00011557678686406$$

$$f(2.50145) = 0.00011557687295907$$

Output:

Minimal value of the objective function:  $f = 0.00011557675107638$

$\alpha = 2.50143$

meanP = 3.33206

medianP = 2.6386

varianceP = 8.85172

standard\_deviationP = 2.97518

Answer:

$\alpha = 2.50143$