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Computer-Generated Mathematics: Stevanovic Products

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Abstract. The authors define the Stevanovic products, and by using the computer program "Discoverer", give examples of Stevanovic products.

Keywords: triangle geometry, remarkable points, computer-generated mathematics, Euclidean geometry, Discoverer.

In 2003, Milorad Stevanovic (Stevanovic, 2003) gave a nice way how to construct the Yff Center of Congruence. Let I be the Incenter of triangle ABC. Let A_1 = I-of-BCP, B_1 = I-of-CAP, C_1 = I-of-ABP. Denote by A_2 the point of intersection of lines IA_1 and BC. Cyclically define B_2 and C_2. Then triangles ABC and $A_2B_2C_2$ are perspective and the perspector is the Yff Center of Congruence.

The Stevanovic construction generalizes. Suppose that P and Q are remarkable points of triangle ABC. Let A_1 = Q-of-BCP, B_1 = Q-of-CAP, C_1 = Q-of-ABP. Denote by A_2 the point of intersection of lines PA_1 and BC. Cyclically define B_2 and C_2. We call triangle T(P,Q) = $A_2B_2C_2$ the *Stevanovic Triangle of P and Q*, and, if triangles ABC and $A_2B_2C_2$ are perspective, the perspector is called the *Stevanovic product of P and Q*, denoted by S(P,Q). We could generalize this definition, taking any triangle T instead of triangle ABC.

In this paper, by using the computer program "Discoverer", we find examples of Stevanovic products. For the computer program "Discoverer" the reader may see (Grozdev & Dekov, 2013a-h). In this paper we use the standard procedure of the computer program "Discoverer", named *the Partial Identification of Points.* (See (Grozdev & Dekov, 2013h)). Given a set of points, the procedure produces the following files:

1_List_1.php.htm - A list of points to be identified.
2_List_1K.php.htm - Points of List 1, available at the ETC.
3_List_1D.php.htm - Points of List 1, not available at the ETC.
4_List_P-X.php.htm - List of theorems.
5_Table_P-X.php.htm - The previous list as a table.

6_Table_X-P.php.htm - The previous table re-ordered. (This table is available only upon request).

In order to produce examples of Stevanovic Triangles, we use the list of points, given in the file "6_Table_X-P.php.htm" of folder 1. (See the enclosed files). Note that any other set of sample points could be used.

For any P and Q from the above list, the computer program "Discoverer" constructs the Stevanovic triangle T(P,Q) . (We omit the case Q = "Centroid", since it is triavial). Then the "Discoverer" constructs the perspector (that is, the Stevanovic product) S(P,Q) between triangle ABC and the T(P,Q). Finally, it uses the set of Stevanovic products as starting set of points to be identified by using the "Partial Identification of Points" Procedure. The results are the files given at the enclosed folder 2.

Also, we find the generalized Stevanovic products for T = "Excentral Triangle", T = "Antimedial Triangle", T = "Euler Triangle", and we give the results in the enclosed folders 3, 4 and 5.

Below we illustrate theorem 106 of the file "6_Table_X-P.php.htm" of folder 4.

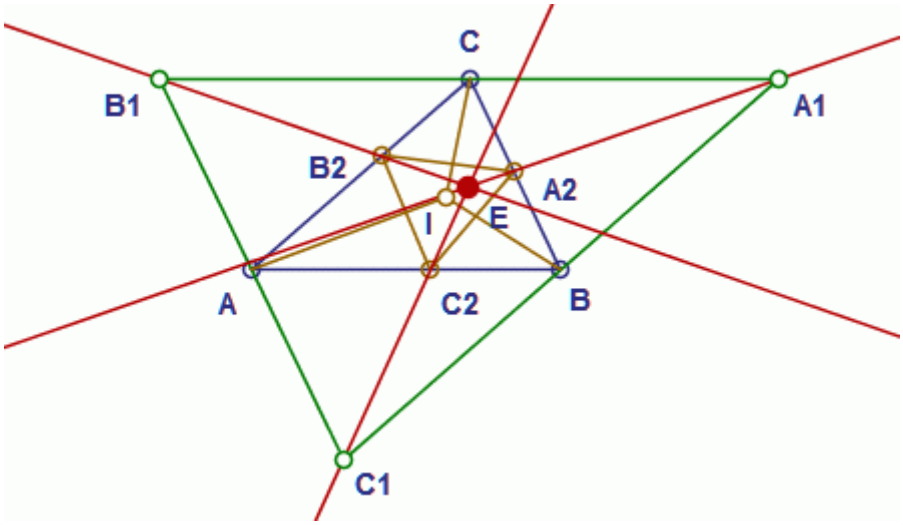

Fig.1. I = Incenter, Triangle $A_1B_1C_1$ = Antimedial Triangle, Triangle $A_2B_2C_2$ = Stevanovic Triangle of the Circumcenters of the Triangulation Triangles of the Incenter, E = Equal Parallelians Point = Perspector of triangles $A_1B_1C_1$ and $A_2B_2C_2$.

Note that many Stevanovic products are not included in the ETC, e.g. the "Prespector of Triangle ABC and the Stevanovic Triangle of the Internal Centers of Similitude of the Incircle and the Circumcircle of the Triangulation Triangles of the Incenter" (See no.6 in the "3_List_1D.php.htm" of folder 2). By using the "Discoverer", we find that this point is also the Square Quotient of the Yff Center of Conguence and the Second Malfatti-Rabinowitz Point. We may use the "Discoverer" in order to find the identification of points (that is, the additional roles of points) which are not in the ETC.

We give one more example. The result no.1 from the file "3_List_1D.php.htm" of folder 5 states that "the Euler Triangle and the Stevanovic Triangle of the de Longchamps Points of the Triangulation Triangles of the Orthocenter" are perspective. We may use the "Discoverer" in order to find a role (roles) of the perspector of these triangles. We obtain that the perspector is the "Midpoint of the Orthocenter and the Isotomic Conjugate of the Circumcenter". Figures 2 and 3 illustrate these results.

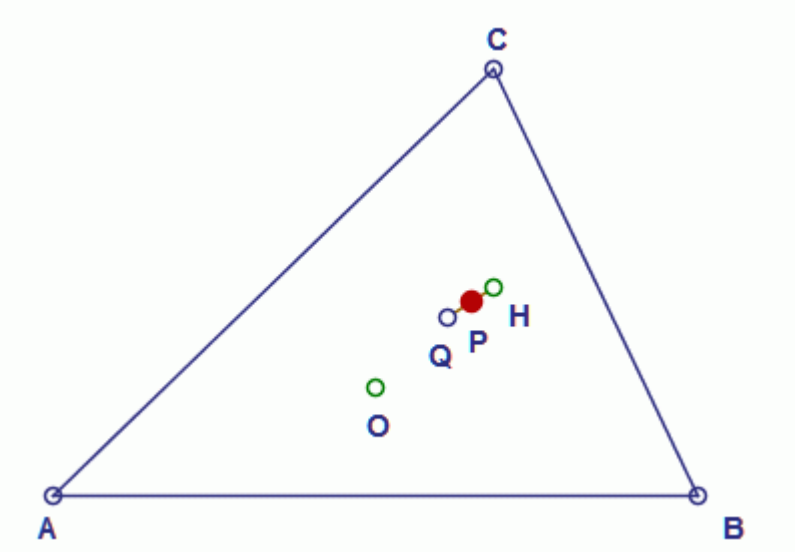

Fig.2. H = Orthocenter, O = Circumcenter, Q = Isotomic Conjugate of the Circumcenter, P = Midpoint of H and Q.

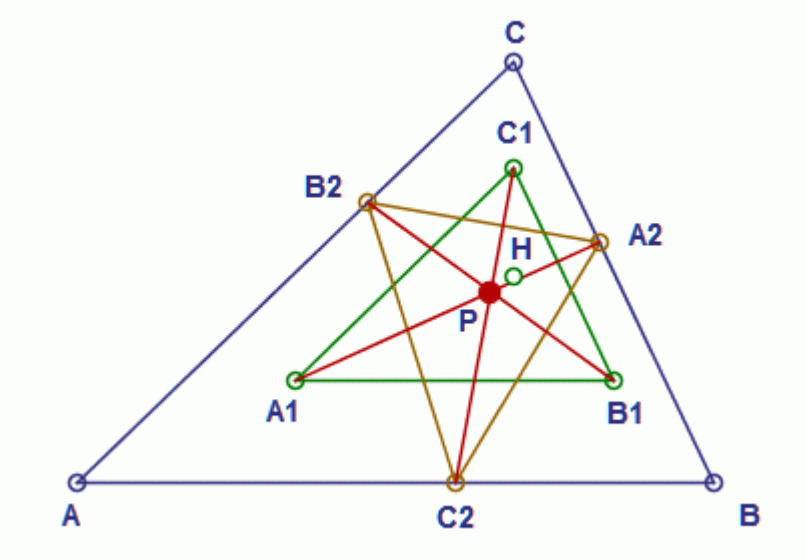

Fig.3. H = Orthocenter, Triangle $A_1B_1C_1$ = Euler Triangle, Triangle $A_2B_2C_2$ = Stevanovic Triangle of the de Longchamps Points of the Triangulation Triangles of the Orthocenter, P = Midpoint of the Orthocenter and the Isotomic Conjugate of the Circumcenter = Perspector of triangles $A_1B_1C_1$ and $A_2B_2C_2$.

The computer program "Discoverer" has discovered the following theorem:

Theorem 1. For any Point P, the Stevanovic Product S(P,Q) exists, provided Q is the Incenter, the Symmedian Point, the Third Brocard Point, the Second Power Point, The Third Power Point or the Fourth Power Point. If P is a point from the List 1 of the file "6_Table_X-P.php.htm" of folder 1, and if it is different form these six points, there exists at least one triangle center Q such that the Stevanovic product S(P,Q) does not exist.

The reader is invited to find other similar results. Proofs of theorems are also welcome.

Thanks

The figures in this note are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download in the Web. It is free and open source. The reader may verify easily the statements of this paper by using C.a.R. Many thanks to Rene Grothmann for his wonderful program.

Enclosed files

The file "2013-1.zip" is enclosed.

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