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Points on the Kiepert Hyperbola

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Abstract. The authors present a list of remarkable points of the triangle, which lie on the Kiepert hyperbola. The list is produced by the computer program "Discoverer", created by the authors.

Keywords: Kiepert hyperbola, triangle geometry, remarkable points, computer-generated mathematics, Euclidean geometry, Discoverer.

In 1869, Ludwig Kiepert introduced a hyperbola, now known as the Kiepert hyperbola. See (Kiepert,1869). For the current state of the art, see (Weisstein, Kiepert Hyperbola), (Yiu, 2001), (Quim, Kiepert Hyperbola).

During the years, a number of remarkable points are discovered to lie on the Kiepert hyperbola. In 1994, Eddy and Fritsch discovered that the Spieker center lies on the Kiepert hyperbola (Eddy and Fritsch, 1994, Theorem 3) and also the Third Brocard point lies on the Kiepert hyperbola (Eddy and Fritsch, 1994, Theorem 4). Eric Weisstein has produced a list of points which lie on the Kiepert hyperbola, and are available at the Kimberling's Encyclopedia of Triangle Centers (Kimberling, ETC). The Weisstein's list contains 44 points.

In this paper, by using the computer program "Discoverer", we find new remarkable points on the Kiepert hyperbola. For the computer program "Discoverer" the reader may see (Grozdev & Dekov, 2013a-h). In this paper we use the standard procedure of the computer program "Discoverer", named *the Partial Identification of Points*. (See (Grozdev & Dekov, 2013h)). Given a set of points, the procedure produces the following files:

1_List_1.php.htm - A list of points to be identified.
2_List_1K.php.htm - Points of List 1, available at the ETC.
3_List_1D.php.htm - Points of List 1, not available at the ETC.
4_List_P-X.php.htm - List of theorems about points, available at the ETC..
5_Table_P-X.php.htm - The previous list as a table.
6_Table_X-P.php.htm - The previous table re-ordered. (This table is available only upon request).

The “Discoverer” selects from its database the points which lie on the Kiepert hyperbola and forms List 1. Then it applies the above described procedure. The result is given in six files in HTML-format, included in the file “2013-2.zip”.

The List 1 contains 2364 remarkable points, which lie on the Kiepert hyperbola. Seventy of these points are available at the current edition of the (Kimberling, ETC). These points are presented in List K (Here “K” is from “Kimberling”) and also in tables P-X and X-P. Note that the current database of “Discoverer” does not contain all points from the (Kimberling, ETC), so that List K does not contain all points, which lie on the Kiepert hyperbola and are included in the (Kimberling, ETC). The rest of 2294 points are not included in the (Kimberling, ETC). These points are presented in the List D (“D” means “difference”, that is, List 1 minus List K).

Below we illustrate one of the theorems from List D. The proofs of the “Discoverer” are non-standard, so that we present another proof.

Theorem 1. (List D, Theorem 168). The Isotomic Conjugate of the Midpoint of the Centroid and the Symmedian Point lies on the Kiepert Hyperbola.

Proof. We use barycentric coordinates. In order to avoid calculations by hand, we use the computer program Maple. We enter the following commands:

```
1 > K:=(b^2-c^2)*y*z+(c^2-a^2)*z*x+(a^2-b^2)*x*y;
2 > u1:=1/3; v1:=1/3; w1:=1/3;
3 > u2:=a^2/(a^2+b^2+c^2);
4 > v2:=b^2/(a^2+b^2+c^2);
5 > w2:=c^2/(a^2+b^2+c^2);
6 > u3:=(u1+u2)/2;u3:=simplify(u3);
7 > v3:=(v1+v2)/2;v3:=simplify(v3);
8 > w3:=(w1+w2)/2;w3:=simplify(w3);
9 > u:=1/u3; v:=1/v3; w:=1/w3;
10 > K:=subs(x=u,y=v,z=w,K);
11 > K:=simplify(K);
```

- Command 1 introduces the left-hand side of the equation “K = 0” of the Kiepert hyperbola. Here x,y,z are the unknowns, a,b,c are the sidelengths of Triangle ABC, a=BC, b=CA and c=AB.
- Command 2 introduces the normalized barycentric coordinates of the Centroid, denoted by u1,v1,w1..
- Commands 3-5 introduce the normalized barycentric coordinates of the Symmedian Point, denoted by u2,v2,w2..
- Commands 6-8 give the coordinates of the Midpoint of Centroid and Symmedian Pointq denoted by u3,v3,w3.
- Command 9 gives the coordinates of Isotomic Conjugate of the Midpoint of Centroid and Symmedian Point, denoted by u,v,w.
- Command 10 substitutes in K the coordinates u,v,w for the unknowns x,y,z, respectively

- Finally, command 11 simplifies the expression for K.

After the execution of the commands, we obtain as output “`K := 0`”, which proves the theorem. The file “2013-2.zip”, enclosed to this paper, contains the file “Theorem1.mws” with the proof.

Fig.1 below illustrates theorem 1.

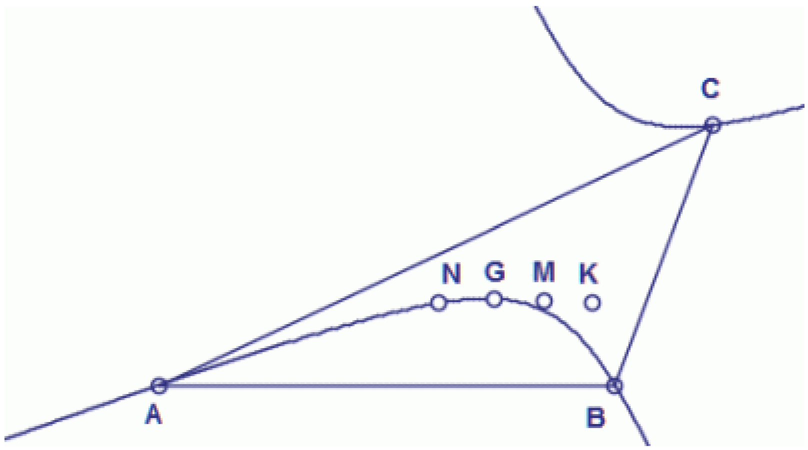

Fig.1. Triangle ABC and the Kiepert hyperbola. Point G = Centroid, Point K = Symmedian Point, Point M = Midpoint of G and K, N = Isotomic Conjugate of M. Points G and N lie on the Kiepert hyperbola.

The reader is invited to find proofs of other theorems from List D, and to submit them to this journal for publication.

Thanks

The figure in this note is produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download at the Web. It is free and open source. The reader may verify easily the statements of this paper by using C.a.R. Many thanks to Rene Grothmann, for his wonderful program.

Enclosed file

The file “2013-2.zip” is enclosed.

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