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Points on the Steiner Circumellipse

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Abstract. The authors present a list of remarkable points of the triangle, which lie on the Steiner circumellipse. The list is produced by the computer program "Discoverer", created by the authors.

Keywords: Steiner circumellipse, triangle geometry, remarkable points, computer-generated mathematics, Euclidean geometry, Discoverer.

The Steiner circumellipse of a triangle ABC is the unique ellipse passing through the vertices of the triangle and having the triangle centroid of the triangle as its center.. It is the circumscribed ellipse of minimum area. For the current state of the art, see (Weisstein, Steiner Circumellipse), (Yiu, 2001), (Quim, Steiner Circumellipse).

During the years, a number of remarkable points are discovered to lie on the Steiner circumellipse. Eric Weisstein has produced a list of points which lie on the Steiner circumellipse, and are available at the Kimberling's Encyclopedia of Triangle Centers (Kimberling, ETC). The Weisstein's list contains 19 points.

In this paper, by using the computer program "Discoverer", we find new remarkable points on the Steiner circumellipse. For the computer program "Discoverer" the reader may see (Grozdev & Dekov, 2013a-h). In this paper we use the standard procedure of the computer program "Discoverer", named *the Partial Identification of Points.* (See (Grozdev & Dekov, 2013h)). Given a set of points, the procedure produces the following files:

1_List_1.php.htm - A list of points to be identified.
2_List_1K.php.htm - Points of List 1, available at the ETC.
3_List_1D.php.htm - Points of List 1, not available at the ETC.
4_List_P-X.php.htm - List of theorems about points, available at the ETC..
5_Table_P-X.php.htm - The previous list as a table.
6_Table_X-P.php.htm - The previous table re-ordered. (This table is available only upon request).

The "Discoverer" selects from its database the points which lie on the Steiner circumellipse and forms List 1. Then it applies the above described procedure. The result is given in six files in HTML-format, included in the file "2013-3.zip".

The List 1 contains 2387 remarkable points, which lie on the Steiner circumellipse. Seventeen of these points are available at the current edition of the (Kimberling, ETC). These points are presented in List K (Here "K" is from "Kimberling") and also in tables P-X and X-P. Note that the current database of "Discoverer" does not contain all points from the (Kimberling, ETC), so that List K does not contain all points, which lie on the Steiner circumellipse and are included in the (Kimberling, ETC). The rest of 2270 points are not included in the (Kimberling, ETC). These points are presented in the List D ("D" means "difference", that is, List 1 minus List K).

Below we illustrate one of the theorems from List D. The proofs of the "Discoverer" are non-standard, so that we present another proof.

Theorem 1. (List D, Theorem 8). The Ceva Product of the Equal Parallelians Point and the Schroder Point lies on the Steiner Circumellipse.

Proof. We use barycentric coordinates. In order to avoid calculations by hand, we use the computer program Maple. We enter the following commands:

```
1 > S:=y*z+z*x+x*y;
2 > u1:=c*a+a*b-b*c;
3 > v1:=a*b+b*c-c*a;
4 > w1:=b*c+c*a-a*b;
5 > u2:=a*((b-c)^2+a*(b+c-2*a));
6 > v2:=b*((c-a)^2+b*(c+a-2*b));
7 > w2:=c*((a-b)^2+c*(a+b-2*c));
8 > u:=(u1*w2+w1*u2)*(u1*v2+v1*u2);
9 > v:=(v1*u2+u1*v2)*(v1*w2+w1*v2);
10 > w:=(w1*v2+v1*w2)*(w1*u2+u1*w2);
11 > S:=subs(x=u,y=v,z=w,S);
12 > S:=simplify(S);
```

- Command 1 introduces the left-hand side of the equation "S = 0" of the Steiner Circumellipse. Here x,y,z are the unknowns, a,b,c are the sidelengths of Triangle ABC, a=BC, b=CA and c=AB.
- Command 2-4 introduce the barycentric coordinates of the Equal Parallelians Point, denoted by u1,v1,w1..
- Commands 5-7 introduce the barycentric coordinates of the Schroder Point, denoted by u2,v2,w2..
- Commands 8-10 give the coordinates of the Ceva Product of the Equal Parallelians Point and the Schroder Point, denoted by u,v,w.
- Command 11 substitutes in S the coordinates u,v,w for the unknowns x,y,z, respectively.

- Finally, command 12 simplifies the expression for S.

After the execution of the commands, we obtain as output "`S := 0`", which proves the theorem. The file "2013-3.zip", enclosed to this paper, contains the file "Theorem1.mws" with the proof.

Note that we may use the output of commands 8-10 as the barycentric coordinates of the Ceva Product of the Equal Parallelians Point and the Schroder Point.

Fig.1 below illustrates theorem 1.

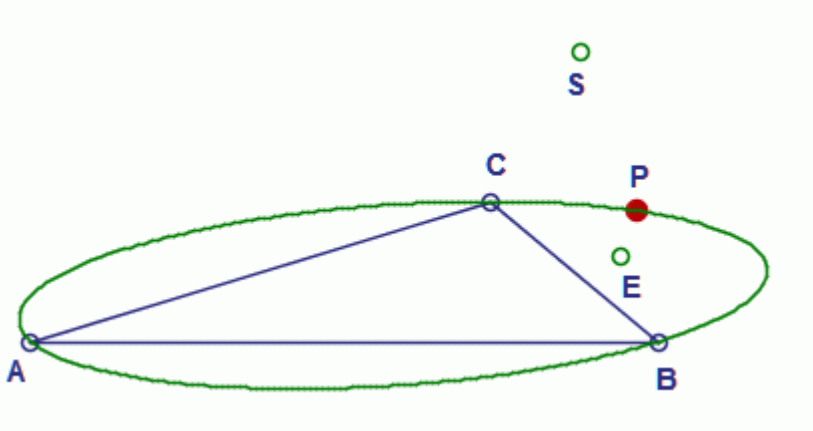

Fig.1. Triangle ABC and the Steiner circumellipse. Point E = Equal Parallelians Point, S = Schroder Point, P = Ceva Product of the Equal Parallelians Point and the Schroder Point. Point P lies on the Steiner circumellipse.

The reader is invited to find proofs of other theorems from List D, and to submit them to this journal for publication.

Thanks

The figure in this note is produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download at the Web. It is free and open source. The reader may verify easily the statements of this paper by using C.a.R. Many thanks to Rene Grothmann, for his wonderful program.

Enclosed file

The file "2013-3.zip" is enclosed.

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