

Problem 4

The Lester Circle is orthogonal to the Orthocentroidal Circle of the Outer Fermat Triangle of the Johnson Triangle

Sava Grozdev and Deko Dekov

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At the present time, there are seven notable circles known to be orthogonal to the Lester circle. See [1]. The below problem introduces a new notable circle which is orthogonal to the Lester circle.

Prove the following problem, produced by the computer program “Discoverer”:

Problem 4. Given $\triangle ABC$. The Lester circle of $\triangle ABC$ is the circle passing through the circumcenter, nine-point center and the outer Fermat point. Let J_a be the center of the circle passing through the orthocenter and vertices B and C . Similarly define points J_b and J_c . Then $\triangle J_a J_b J_c$ is known as the Johnson triangle. Let $\triangle DEF$ is the outer Fermat triangle of the Johnson triangle, G is the centroid of $\triangle DEF$ and H is the orthocenter of $\triangle DEF$. Prove that the circle having as diameter the segment GH is orthogonal to the Lester circle.

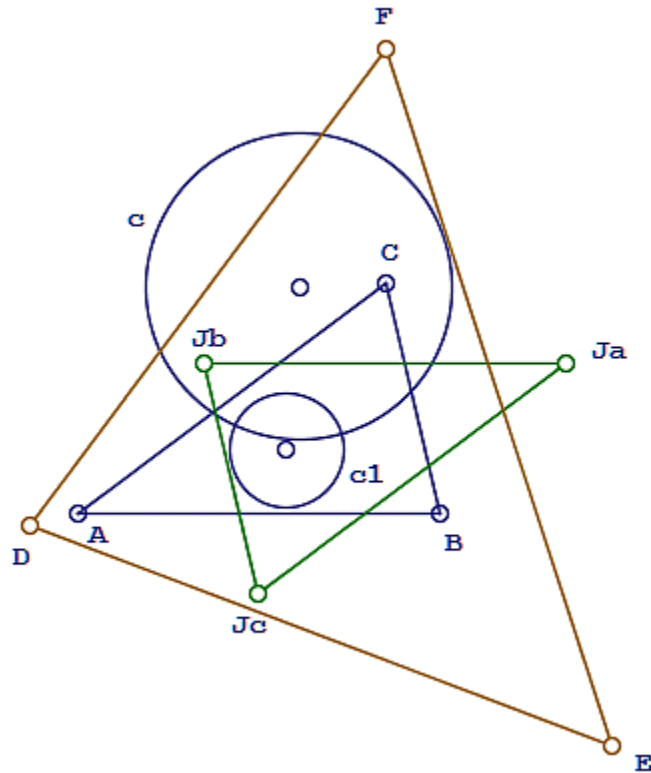
Short form of the problem:

Problem 4. Prove that the Lester circle is orthogonal to the Orthocentroidal Circle of the Outer Fermat Triangle of the Johnson Triangle.

The reader may find the definitions in [2-5].

Please submit the solution of the problem for publication in this journal to the editor of this journal: ddekov@ddekov.eu

See the figure:



In the figure:

c – Lester circle,

Triangle $JaJbJc$ – Johnson Triangle,

Triangle DEF – Outer Fermat Triangle of the Johnson Triangle,

Circle $c1$ – Orthocentroidal Circle of the Outer Fermat Triangle of the Johnson Triangle.

Circle $c1$ is orthogonal to the Lester circle.

References

1. Dekov, D., Computer-Generated Mathematics: Seven Circles orthogonal to the Lester Circle, Didactical Modeling, 2008, http://www.math.bas.bg/omi/DidMod/Articles/D%5B1%5D.Dekov_Lester_Circle.pdf
2. Sava Grozdev and Deko Dekov, Computer-Generated Encyclopedia of Euclidean Geometry, 2014, available at the Web: <http://www.ddekov.eu/e2/>
3. Eric W. Weisstein, MathWorld - A Wolfram Web Resource, <http://mathworld.wolfram.com/>
4. Quim Castellsaguer, The Triangles Web, <http://www.xtec.es/~qcastell/ttw/ttweng/portada.html>
5. P. Yiu, The Circles of Lester, Evans, Parry, and Their Generalizations, Forum Geometricorum, 10 175–209.

Sava Grozdev, Sofia, Bulgaria, sava.grozdev@gmail.com

Deko Dekov, Stara Zagora, Bulgaria, ddekov@ddekov.eu