

## Problem 5

### The Lester Circle is orthogonal to the Orthocentroidal Circle of the Inner Fermat Triangle of the Johnson Triangle

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At the present time, there are seven notable circles known to be orthogonal to the Lester circle. See [1]. The below problem introduces a new notable circle which is orthogonal to the Lester circle.

Prove the following problem, produced by the computer program “Discoverer”:

**Problem 5.** Given  $\triangle ABC$ . The Lester circle of  $\triangle ABC$  is the circle passing through the circumcenter, nine-point center and the outer Fermat point. Let  $J_a$  be the center of the circle passing through the orthocenter and vertices  $B$  and  $C$ . Similarly define points  $J_b$  and  $J_c$ . Then  $\triangle J_a J_b J_c$  is known as the Johnson triangle. Let  $\triangle DEF$  is the inner Fermat triangle of the Johnson triangle,  $G$  is the centroid of  $\triangle DEF$  and  $H$  is the orthocenter of  $\triangle DEF$ . Prove that the circle having as diameter the segment  $GH$  is orthogonal to the Lester circle.

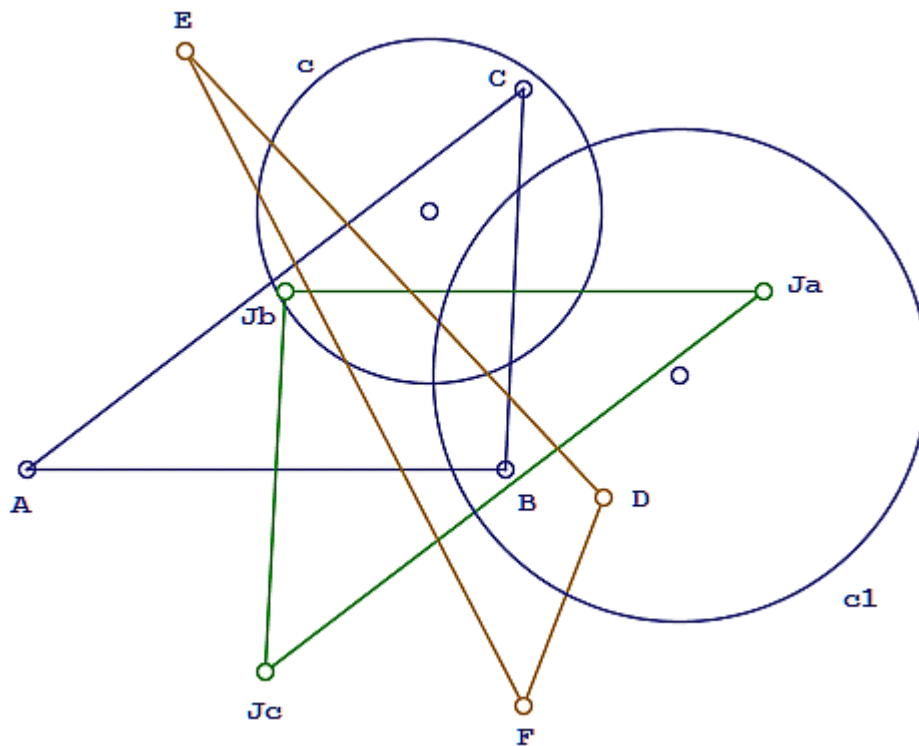
Short form of the problem:

**Problem 5.** Prove that the Lester circle is orthogonal to the Orthocentroidal Circle of the Inner Fermat Triangle of the Johnson Triangle.

The reader may find the definitions in [2-5].

Please submit the solution of the problem for publication in this journal to the editor of this journal: [ddekov@ddekov.eu](mailto:ddekov@ddekov.eu)

See the figure:



In the figure:

$c$  – Lester circle,

Triangle  $JaJbJc$  – Johnson Triangle,

Triangle  $DEF$  – Inner Fermat Triangle of the Johnson Triangle,

Circle  $c1$  – Orthocentroidal Circle of the Inner Fermat Triangle of the Johnson Triangle.

Circle  $c1$  is orthogonal to the Lester circle.

## References

1. Dekov, D., Computer-Generated Mathematics: Seven Circles orthogonal to the Lester Circle, Didactical Modeling, 2008,  
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2. Sava Grozdev and Deko Dekov, Computer-Generated Encyclopedia of Euclidean Geometry, 2014, available at the Web: <http://www.ddekov.eu/e2/>
3. Eric W. Weisstein, MathWorld - A Wolfram Web Resource,  
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4. Quim Castellsaguer, The Triangles Web,  
<http://www.xtec.es/~qcastell/ttw/ttweng/portada.html>
5. P. Yiu, The Circles of Lester, Evans, Parry, and Their Generalizations, Forum Geometricorum, 10 175–209.

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