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The Simon and Newell prediction is realized by the "Discoverer"

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Abstract. In 1958, in a seminal paper predicting future successes of artificial intelligence, Herbert Simon and Alan Newell suggested that: "Within ten years a digital computer will discover an important new mathematical theorem". More that 50 years after the prediction, the computer program "Discoverer", created by the authors, is the first computer program which realizes the prediction.

Keywords: Simon and Newell prediction, artificial intelligence, discovery system, computer-discovered mathematics, Steiner's construction of the Malfatti circles, triangle geometry, Euclidean geometry, Discoverer.

1. Introduction

In 1958, in a seminal paper predicting future successes of artificial intelligence and operational research, Herbert Simon and Alan Newell (Simon & Newell, 1958) suggested that:

"Within ten years a digital computer will discover an important mathematical theorem".

Now is year 2015, and an announcement of a computer-discovered theorems are already available. See e.g. (Grozdev & Dekov, 2013, 2014, 2015). Hence, now we can tick of the prediction by Simon and Newell as realized. Note that the prototype of the "Discoverer" has discovered a number of new theorems beginning with 2006.

We consider a computer-discoverer as effective, if it is an every-day tool for investigation of non-AI experts. The "Discoverer" satisfies this requirement. In this paper we present here theorems, discovered by the "Discoverer".

In 1803 Gian Fancesco Malfatti posed a problem, now known as the Malfatti's construction problem (Malfatti, 1803), (Tabov & Lazarov, 1990). This is one of the famous problems in mathematics. The problem is as follows: By using compass and ruler to construct three circles within a given triangle such that each circle touches externally the other two circles and to two sides of the triangle. In 1826 the famous Swiss geometer Jacob Steiner discovered a simple and elegant synthetic solution to the Malfatti construction problem. The Steiner's solution was studied by many of the greatest mathematicians. Many mathematicians have tried to improve the Steiner's solution, without success. The improvement was discovered by a computer! (Grozdev & Dekov, 2015).

Below we give an improvement of the Steiner's construction, discovered by the "Discoverer", as well as four additional alternatives to the Steiner's construction, discovered by the "Discoverer". Today the "Discoverer" is an infant child. When the "Discoverer" become a teenager, we can expect that he will discover dozens alternatives to the Steiner's construction.

The computer program "Discoverer" is in an early stage of development. We invite interested persons and institutions to join the team and to work together with us on the "Discoverer". Also, we expect that the computers-discoverers working in areas like chemistry and physics will have a great influence to current economics.

2. The Steiner's contruction of the Malfatti circles

Below we give an improvement of the Steiner's construction, discovered by the "Discoverer", as well as four additional alternatives to the Steiner's construction, discovered by the "Discoverer". Today the "Discoverer" is an infant child. When the "Discoverer" become a teenager, we can expect that he will discover dozens alternatives to the Steiner's construction.

The first measure of the complexity of geometric constructions is proposed by Lemoine (Geometrography). In this paper we use the measure of Lazarov and Tabov (Tabov & Lazarov, 1990) which is summarized in Table 1. This measure specifies the Lemoine's measure. The explanation of row 1 in Table 1 is as follows. To place the edge of the ruler in coincidence with a point (Lemoine's operation R_1) – one point. To place the edge of the ruler in coincidence with a second point – one point. To draw a straight line (Lemoine's operation R_2) – one point. Hence, we obtain 3 points for drawing a straight line. The explanation of rows 2 and 3 in Table 1 is similar. Examples are given in (Grozdev and Dekov, 2015).

	Construction	Lazarov-Tabov complexity
1	Construct a line, which passes through two points.	3
2	Construct a circle with a given center and passing through another point.	3
3	Construct a circle with a given center and a radius, given by two points which are different form the center.	4
4	Construct a point, which is the intersection of two lines, circles, or a line and a circle.	1
5	Construct a point, which lies on a geometric figure or outside a geometric figure.	1

Given $\triangle ABC$. The Steiner's construction of the Malfatti circles has the following stages:

Stage 1. Construct the internal angle bisectors and the incenter *I* of $\triangle ABC$.

Stage 2. Construct the de Villiers triangle $\triangle V_aV_bV_c$, that is, the triangle whose vertices are the centers of the de Villiers circles c_1, c_2 and c_3, inscribed in triangles *BCI*, *CAI* and *ABI*, respectively.

Stage 3. Construct the Malfatti-Steiner point *S*, that is the point of intersection of the internal tangent to c_2 and c_3, not passing through point *A*, and the internal tangent to c_1 and c_3, not passing through point *B*.

Stage 4. Construct the Malfatti central triangle $O_aO_bO_c$, that is, the triangle whose vertices are the centers of the Malfatti circles.

Stage 5. Construct the Malfatti circles.

The Steiner's construction of stage 4 is as follows. Construct the point of intersection *X* of line *BC* and the internal tangent to c_2 and c_3 not passing through point *A*. Similarly, construct points *Y* and *Z*. Construct point O_a as the intersection of the angle bisectors of $\angle SYA$ and $\angle SZA$. Similarly construct points O_b and O_c.

The improvement of the stage 4, discovered by the "Discoverer" is as follows (Fig.1). The line through points V_a and *S* intersects the line *AI* in point O_a. Similarly construct points O_b and O_c.

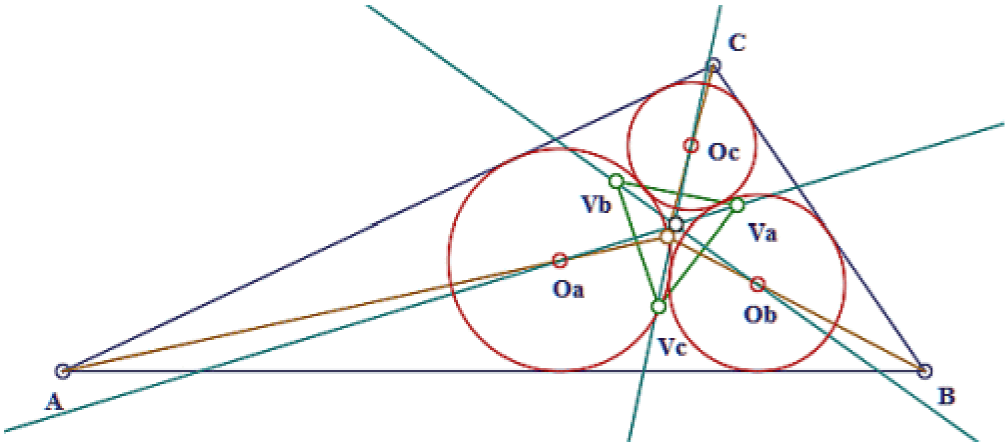

Figure 1.

As it is proved in (Grozdev & Dekov, 2015), the Lazarov-Tabov measure of stage 4 of Steiner's construction is 50, while the improved by "Discoverer" stage 4 has measure 12. The complexity of the improved by "Discoverer" stage 4 is 24% of the complexity of the Steiner's stage 4. Hence, the computer program "Discoverer" has discovered an essential improvement of stage 4 of the Steiner's construction of the Malfatti circles.

3. Alternatives to stage 3 of the Steiner's construction

The third stage of the Steiner's construction is the construction of the Malfatti-Steiner point. An alternative of stage 3 of the Steiner's construction is given by R.K.Guy (Guy 2007), (Malfatti Circles), (Grozdev & Dekov 2015). The "Discoverer" has discovered three alternatives to the stage 3 of the Steiner's construction. For the first two alternatives see (Grozdev & Dekov, 2013).

Alternative 1. The Malfatti-Steiner point is the intersection point of the line which is the reflection about the internal angle bisector of $\angle V_aV_bV_c$, and the line which is the reflection about the internal angle bisector of $\angle V_bV_cV_a$.

Alternative 2. The Malfatti-Steiner point is the external center of similitude of the circumcircle of $\triangle V_aV_bV_c$ and the cosine circle of $\triangle V_aV_bV_c$.

Alternative 3. Construct the centers J_a and J_b of the excircles inscribed in $\angle A$ and $\angle B$, respectively. Let P_a be the point of intersection of lines V_aJ_a and *BC*, and let P_b be the point of intersection of lines V_bJ_b and *CA*. Then the Malfatti-Steiner point is the point of intersection of lines AP_a and BP_b.

4. An alternative to stages 3 and 4 of the Steiner's construction

The "Discoverer" has discovered also an alternative to the stages 3 and 4 of the Steiner's construction (Fig.2). In this alternative we do not construct the Malfatti-Steiner point. The alternative is as follows.

Construct de Villiers circles c_1, c_2 and c_3. Construct the tangency point *X* of circle c_1 and line *BC*. Similarly, construct points *Y* and *Z*. Construct lines *XY*, *YZ* and *ZX*. Construct the intersection point D_a of lines BV_c and CV_b, and the intersection point D_b of lines AV_c and CV_a. Construct the intersection point *P* of lines V_aD_a and V_bD_b. Construct the line L_a through point *P* and perpendicular to line *YZ*. Similarly, construct lines L_b and L_c. Then O_a is the intersection point of lines *AI* and L_a. Similarly, O_b is the intersection point of lines *BI*, and L_b, and O_c is the intersection point of lines *CI* and L_c.

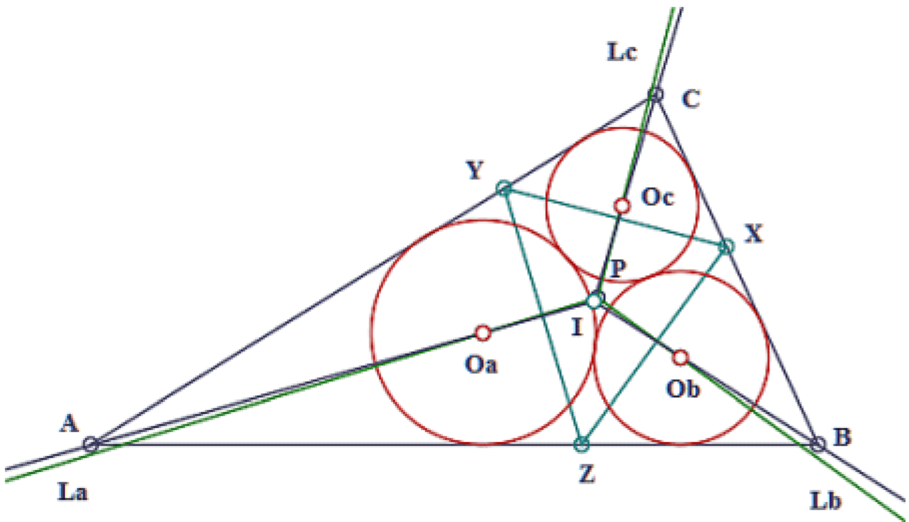

Figure 2

5. The future

Euclid is said to have said to the first Ptolemy who inquired if there was a shorter way to learn geometry than the Elements: ...there is no royal road to geometry.

Now we have a royal road to geometry. It is not necessary we to be inventive. The "Discoverer" will tell us what is necessary. All which we have to do is to write our problem and to go to drink coffee. We will drink coffee and the "Discoverer" will work for us. It is easy.

6. Conclusion

The era of new knowledge discovered only by human being is over. We are going into a new era – the era of computers-discoverers. The computers-discoverers are important. They could extend effectively the current science and technology.

In this paper there are debatable statements. But this is necessary debate. It is time we to open our eyes and to see the advantages of the computers-discoverers. They work fast, they work hard, they work day and night, they do not need salaries, they do not do errors, and most important, they can do what the people cannot do. Today "Discoverer" is the only computer program, able to discover new theorems in mathematics. We hope that tomorrow pleiads of computers-discoverers will help the people to move faster in science. Possibly, the computers-discoverers will discover new cheap fuel which will change the world's economics? But the people must be careful – the computers evolve much faster than the people. Tomorrow the computers could want to take the place of the people.

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