Problem.
Prasolov Circumcevian Products

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Given points U and P in the plane of $\triangle ABC$. Let $U_aU_bU_c$ be the circumcevian triangle of U. Denote by R_a, R_b and R_c the reflections of U_a, U_b and U_c in P. If the lines AR_a, BR_b and CR_c concur in a point, we say that the Prasolov circumcevian product is defined. In this case the intersection point of the lines is the *Prasolov circumcevian product of points U and P*.

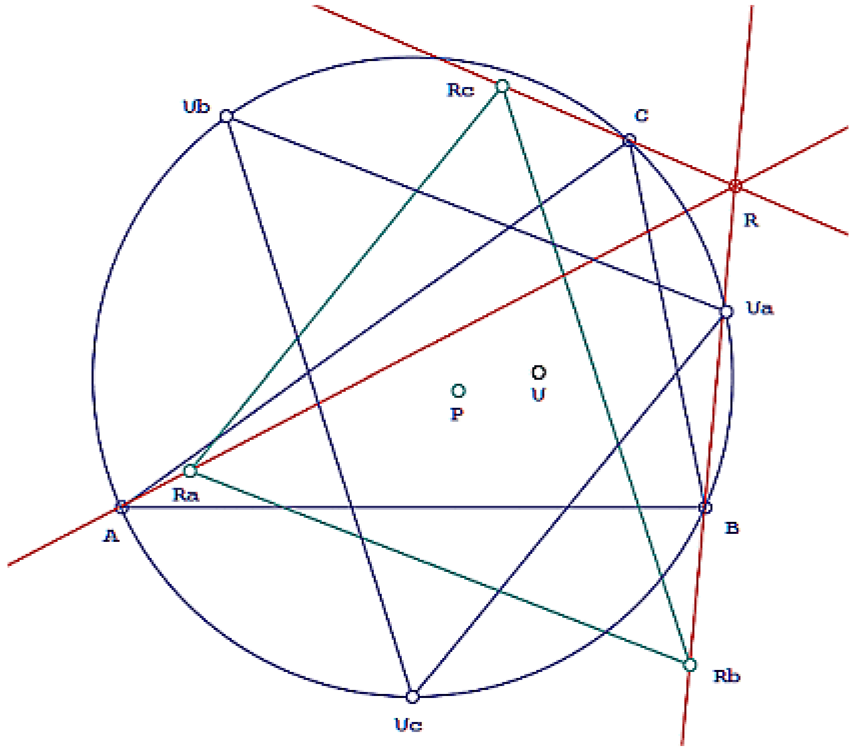
Prove the following problem, produced by the computer program "Discoverer":

Problem. Prove that if U is the Incenter of $\triangle ABC$ and P is the Spieker center of $\triangle ABC$, then the Prasolov circumcevian product is defined.

The reader may find the definitions in [1,3,4].

About the Prasolov cevian products, see [2].The reader is invited to submit a synthetic solution of this problem for publication in this journal.

See the Figure:



In the Figure:
U is the Incenter of $\triangle ABC$,
$\triangle U_aU_bU_c$ is the circumcevian triangle of U,
P is the Spieker Center of $\triangle ABC$,
R_a is the reflection of point U_a in point P,
R_b is the reflection of point U_b in point P,
R_c is the reflection of point U_c in point P.
Then the lines AR_a, BR_b and CR_c concur in a point, the point S in the Figure.

Enclosed file

The enclosed file "p001_Solution.pdf" contains the solution of Problem 1 by using barycentric coordinates, as well as five additional problems about the Prasolov circumcevian products.

References

1. Sava Grozdev and Deko Dekov, Computer-Generated Encyclopedia of Euclidean Geometry, 2014, available at the Web: http://www.ddekov.eu/e2/
2. Sava Grozdev and Deko Dekov, Problem 97, MathProblems, 2014, vol. 4, no.2, p.263-264.
3. Eric W. Weisstein, MathWorld - A Wolfram Web Resource, http://mathworld.wolfram.com/

4. Quim Castellsaguer, The Triangles Web, http://www.xtec.es/~qcastell/ttw/ttweng/portada.html

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