Computer-Generated Mathematics: Seven Circles orthogonal to the Lester Circle

Deko Dekov

Abstract. By using the computer program "Machine for Questions and Answers", we find seven circles orthogonal to the Lester circle. We use this result to give new straightedge-and-compass constructions of the Lester circle.

Keywords: computer-generated mathematics, Eucñidean geometry

Constructions of geometric objects in the plane by using straightedge and compass only, is an essential part of school education in Geometry. In this paper we illustrate the use of the computer program "Machine for Questions and Answers" (The Machine), created by the author of the paper, for discovering a theorem useful for the straightedge-and-compass constructions. We use a theorem created by the Machine to find new straightedge-and-compass constructions of the Lester circle. The reader may find a few thousands computer generated theorems, created by the Machine, as well as a few applications of computer-generated theorems, in [4,5].

The *Lester circle* of a triangle is the circle passing through the Circumcenter, the Nine-Point Center and the Fermat Point of the triangle. The Second Fermat Point also lies on the Lester circle. The Lester circles was discovered by June Lester in 1996. A list of papers about the Lester circle is available at the Web [1].

In 2000, Paul Yiu [2] discovered that the orthocentroidal circle is orthogonal to the Lester circle. Then John H. Conwey [3] has asked: "What other interesting properties does this circle have?" Here we show that besides the orthocentroidal circle, six named circles are orthogonal to the Lester circle.

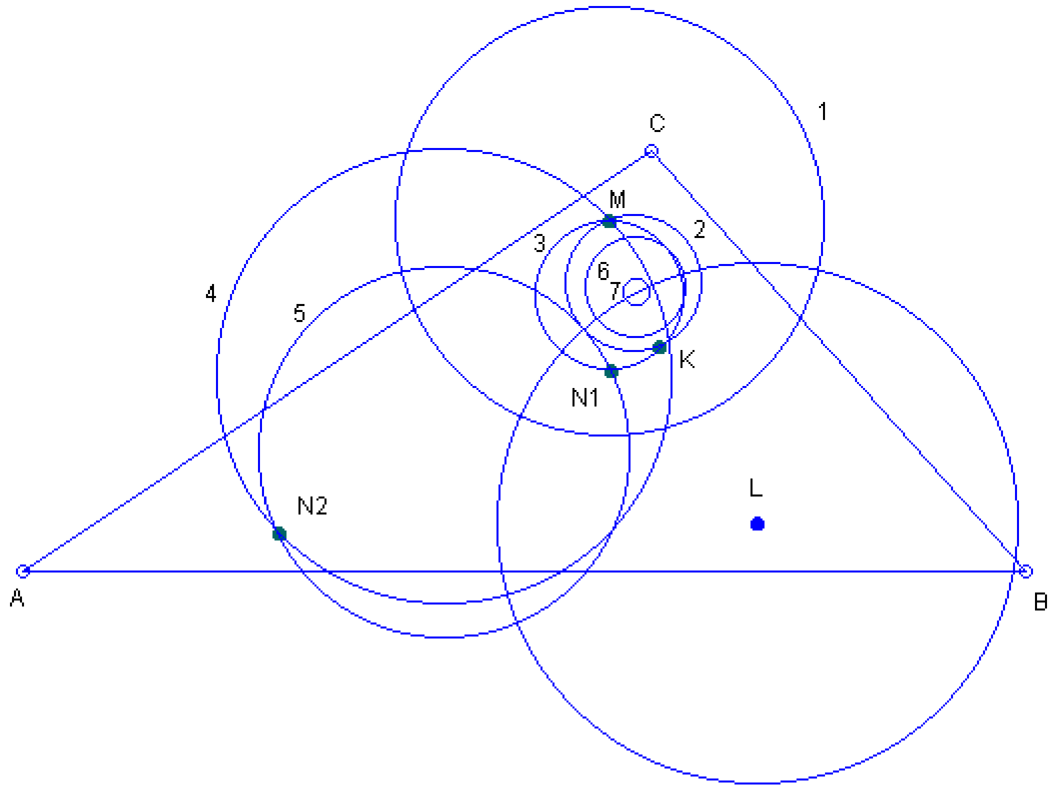
To construct the Lester circle by using orthogonal circles, we need at least thee circles with non-collinear centers, orthogonal to the Lester circle. Given an object (circle, point, triangle, line, etc.) the Machine produces theorems related to the object. We ask the Machine to find examples of circles orthogonal to the Lester circle. We obtain the following theorem:

THEOREM. The Lester Circle is orthogonal to the following circles:

1. Orthocentroidal Circle.
2. Circle having as its diameter the line segment connecting the Center of the Orthocentroidal Circle and the Symmedian Point.
3. Circle having as its diameter the line segment connecting the Center of the Orthocentroidal Circle and the Outer Napoleon Point.
4. Circle having as its diameter the line segment connecting the Center of the Orthocentroidal Circle and the Inner Napoleon Point.
5. Circle having as its diameter the line segment connecting the Inner Napoleon Point

to the Outer Napoleon Point.
6. Inner Lucas Circle of the Fourth Brocard Triangle.
7. Radical Circle of the Lucas Circles of the Fourth Brocard Triangle.

We invite the reader to prove the above theorems. The reader may find the definitions in [4-6]. Recall that the Fourth Brocard circle is also known as the D-triangle. The Outer (resp. Inner) Napoleon point is also known as the First (resp. Second) Napoleon point. Here we will illustrate the circles from the above theorems. See the Figure:

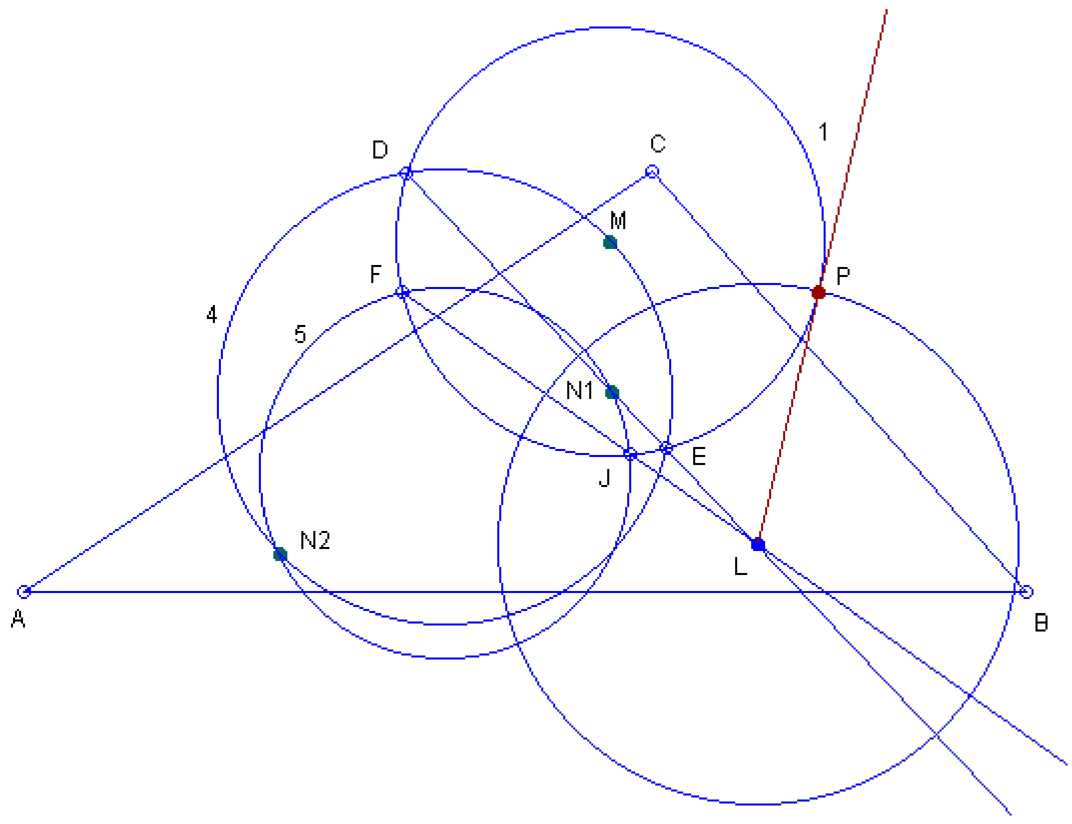


L - Center of the Lester circle;
M - Center of the orthocentroidal circle;
K - Symmedian point;
N1 - Outer Napoleon point;
N2 - Inner Napoleon point;
Circle centered at L - Lester circle;
1 - Orthocentroidal circle;
2 - Circle having as its diameter the line segment connecting the Center of the Orthocentroidal Circle and the Symmedian Point;
3 - Circle having as its diameter the line segment connecting the Center of the Orthocentroidal Circle and the Outer Napoleon Point;
4 - Circle having as its diameter the line segment connecting the Center of the Orthocentroidal Circle and the Inner Napoleon Point;
5 - Circle having as its diameter the line segment connecting the Inner Napoleon Point to the Outer Napoleon Point;

6 - Inner Lucas Circle of the Fourth Brocard Triangle;
7 - Radical Circle of the Lucas Circles of the Fourth Brocard Triangle.

The Lester circle could be constructed by a straightedge and compass as a circle passing through three points. An alternative construction is given below.

We will construct the Lester circle by using circles orthogonal to the Lester circle. We use the well-known method for construction of a circle, given three circles with non-collinear center, orthogonal to the given circle. We select circles 1, 4 and 5 to illustrate the method. Construct the center of the Lester circle L as the radical center of circles 1, 4 and 5 as follows. Construct the intersection points of circles 1 and 4, and label them by D and E. Construct the intersection points of circles 1 and 5, and label them by F and J. Construct L as the intersection point of lines DE and FJ. Construct a tangent line from point L to one of the given circles, 1 say. Label by P the tangency point. Then the circle centered at L and passing through point P is the Lester circle. See the Figure (Points M, N1, N2 are the same as in the previous figure):



The figures in this note are produced by using the program C.a.R. (Compass and Ruler), an amazing program created by Rene Grothmann. The Grothmann's program is available for download at the Web. It is free and open source. Many thanks to Rene Grothmann for his wonderful program.

References

1. Bibliography about the Lester circle, available at the Web: http://thejuniverse.org/PUBLIC/LesterCircle/LCbiblio.html

2. Paul Yiu, The Lester circle is orthogonal to the orthocentroidal circle, Hyacinthos message 1258, 21 August 2000, available at the Web: http://tech.groups.yahoo.com/group/Hyacinthos/messages/1258
3. John H. Conway, Hyacinthos message 1284, 24 August 2000, available at the Web: http://tech.groups.yahoo.com/group/Hyacinthos/messages/1284
4. D. Dekov, Computer-Generated Encyclopedia of Euclidean Geometry, First Edition, 2006, available at the Web: http://www.dekovsoft.com/e1/.
5. D. Dekov, papers in the Journal of Computer-Generated Euclidean Geometry, 2006, 2007, 2008, available at the Web: http://www.dekovsoft.com/j/.
6. Eric W. Weisstein, MathWorld - A Wolfram Web Resource, available at the Web: http://mathworld.wolfram.com/.

Deko Dekov
Zahari Knjazheski 81
6000 Stara Zagora
Bulgaria
ddekov@dekovsoft.com.