Triangles Homothetic with the Intouch Triangle

by

Sava Grozdev, Hiroshi Okumura, and Deko Dekov

Abstract. We present problems for students and teachers about triangles homothetic with the Intouch triangle. The problems were discovered by the computer program "Discoverer" created by the authors.

The Intouch triangle of a triangle ABC, also called the Contact triangle, is the triangle formed by the points of tangency of the incircle of triangle ABC with triangle ABC. The Intouch triangle is also the Cevian triangle of triangle ABC with respect to the Gergonne point. See also Contact triangle in [7].

We present problems for students and teachers about triangles homothetic with the Intouch triangle. The problems are discovered by the computer program "Discoverer" [4], [5], created by the authors.

We denote the side lengths of triangle ABC by $a = BC, b = CA$ and $c = AB$. We denote by $PaPbPc$ the Intouch triangle, by $QaQbQc$ a triangle homothetic with the Intouch triangle, by X the center of the homothety, and by $k = \dfrac{XQa}{XPa}$ the ratio (coefficient) of the homothety.

References for Problem 1: Excentral triangle, Mittenpunkt and Isogonal conjugate in [7], point X(57) Isogonal Conjugate of the Mittenpunkt in [6].

Problem 1. *The Intouch triangle PaPbPc is homothetic with the Excentral triangle QaQbQc. The center of the homothety X is the Isogonal Conjugate of the Mittenpunkt. The ratio of the homothety k is*

$$k = \frac{4abc}{(a+b-c)(b+c-a)(c+a-b)} > 0.$$

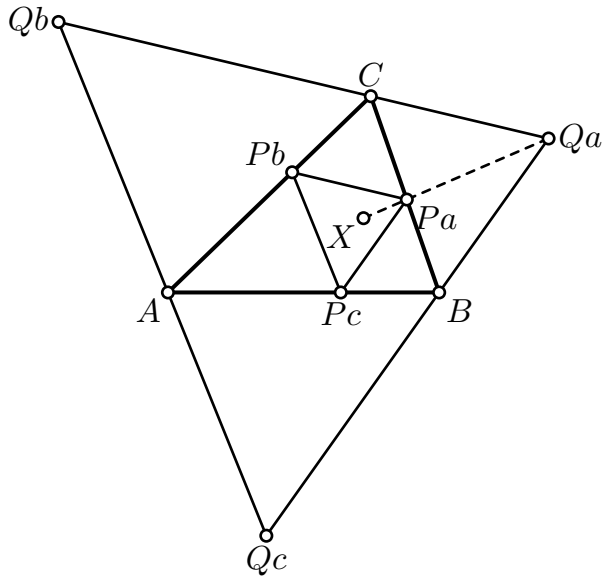

Figure 1.

Figure 1 illustrates Problem 1.

Reference for Problem 2: Hexyl triangle in [7].

Problem 2. *The Intouch triangle PaPbPc is homothetic with the Hexyl triangle QaQbQc. The center of the homothety X is the Incenter. The ratio of the homothety k is*

$$k = -\frac{4abc}{(a+b-c)(b+c-a)(c+a-b)} < 0.$$

Figure 2 illustrates Problem 2.

References for Problem 3: The Circum-Incentral triangle is the Circum-cevian triangle of the Incenter. See Circumcevian Triangle in [7], and in [1], Chap.10.

Problem 3. *The Intouch triangle PaPbOc is homothetic with the Circum-Incentral triangle QaQbQc. The center of the homothety X is the External Center of Similitude of Circumcircle and Incircle. The ratio of the homothety k is*

$$k = \frac{2abc}{(a+b-c)(b+c-a)(c+a-b)} > 0.$$

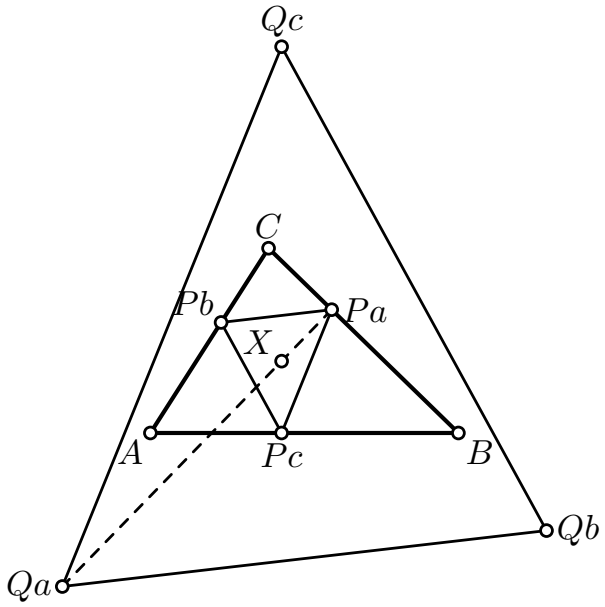

FIGURE 2.

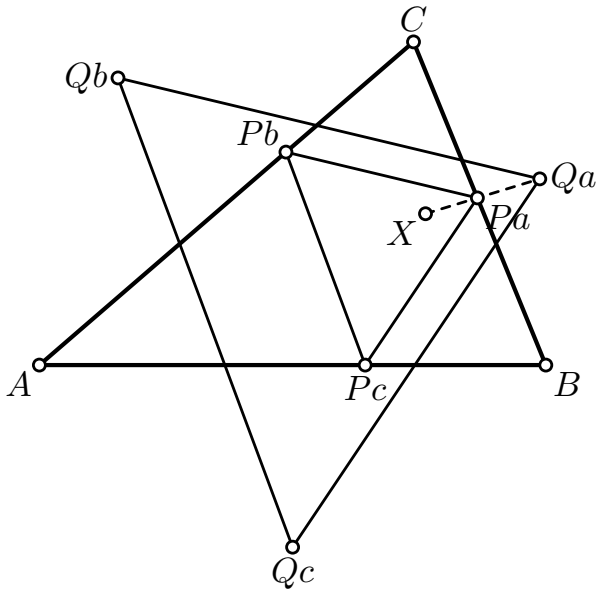

FIGURE 3.

Figure 3 illustrates Problem 3.

References for Problem 4: The Circum-Excentral triangle is the Circum-anticevian triangle of the Incenter. See Circum-anticevian triangle in [1], Chap.10.

Problem 4. *The Intouch triangle PaPbPc is homothetic with the Circum-Excentral triangle QaQbQc. The center of the homothety X is the External Center of Similitude of Circumcircle and Incircle. The ratio of the homothety k is*

$$k = -\frac{2abc}{(a+b-c)(b+c-a)(c+a-b)} < 0.$$

Figure 4 illustrates Problem 4.

References for Problem 5: Half-cevian triangles in [2], Nagel Point in [7].

Problem 5. *The Intouch triangle PaPbPc is homothetic with the Half-cevian triangle of the Nagel point QaQbQc. The center of the homothety X is the Centroid. The ratio of the homothety k is*

$$k = -\frac{1}{2} < 0.$$

Figure 5 illustrates Problem 5. In 5 *Na* is the Nagel point.

References for Problem 6: Nine-Point Center in [7], point X(942) Nine-Point Center of the Intouch Triangle in [6].

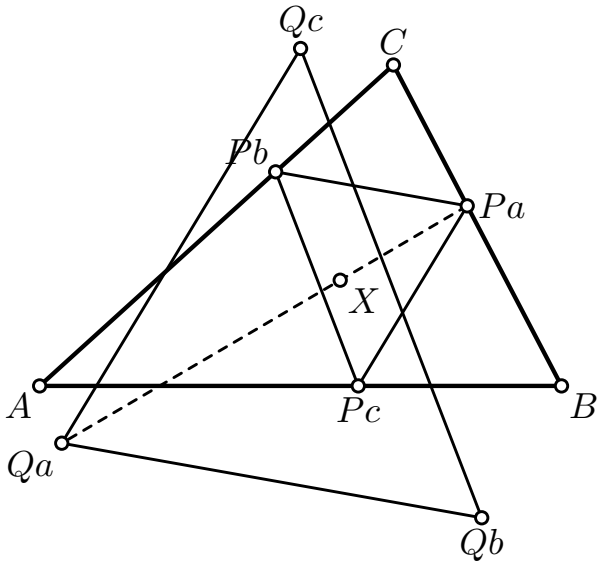

FIGURE 4.

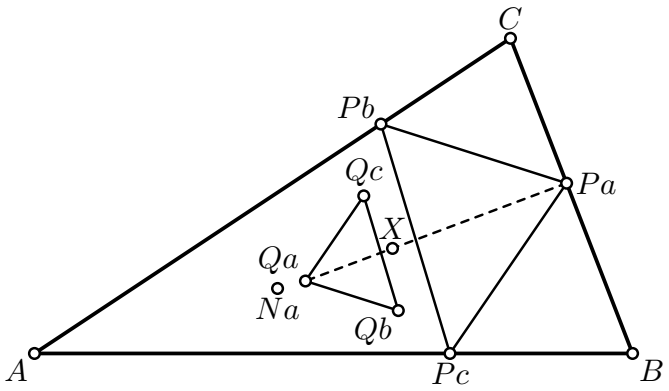

FIGURE 5.

Problem 6. *The Intouch triangle PaPbPc is homothetic with the Triangle of Reflections of the Incenter in the Sidelines of the Intouch Triangle QaQbQc. The center of the homothety X is the Nine-Point Center of the Intouch Triangle. The ratio of the homothety k is*

$$k = -1 < 0.$$

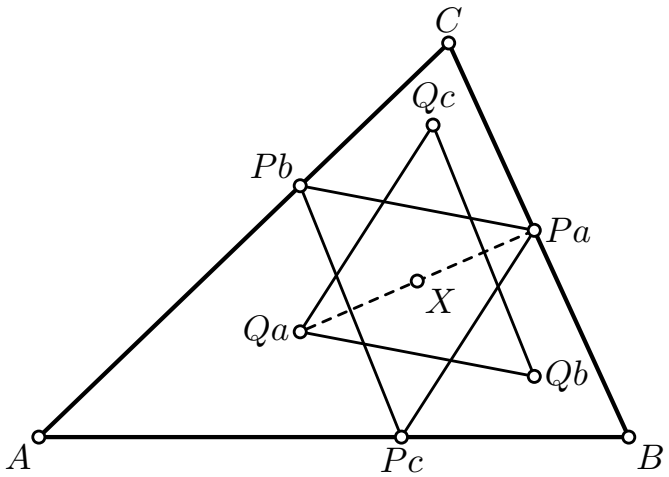

FIGURE 6.

Figure 6 illustrates Problem 6.

References for Problem 7: Spieker Center in [7], Complement in [7], point X(1125) Complement of the Spieker Center in [6].

Problem 7. *The Intouch triangle PaPbOc is homothetic with the Triangle of Reflections of the Spieker center in the Sidelines of the Medial Triangle QaQbQc. The center of the homothety X is the Complement of the Spieker*

Center. The ratio of the homothety k is

$$k = -1 < 0.$$

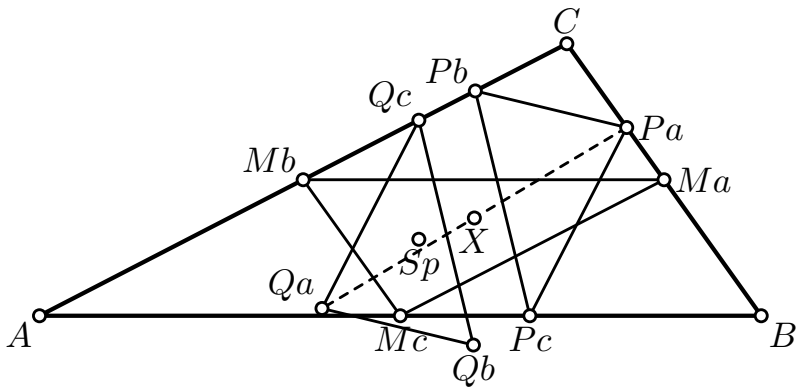

FIGURE 7.

Figure 7 illustrates Problem 7. In figure 7, *MaMbMc* is the Medial triangle, and *S p* is the Spieker center.

References for Problem 8: Nagel point in [7], Gergonne point in [7], Schiffler point in [7], point X(3616) Intersection of the line through the Centroid and the Incenter and the line through the Gergonne point and the Schiffler point in [6].

Problem 8. *The Intouch triangle PaPbPc is homothetic with the Triangle of Reflections of the Nagel Point in the Sidelines of the Antimedial Triangle QaQbQc. The center of the homothety X is the Intersection of the line through the Centroid and the Incenter and the line through the Gergonne point and the Schiffler point. The ratio of the homothety k is*

$$k = -4 < 0.$$

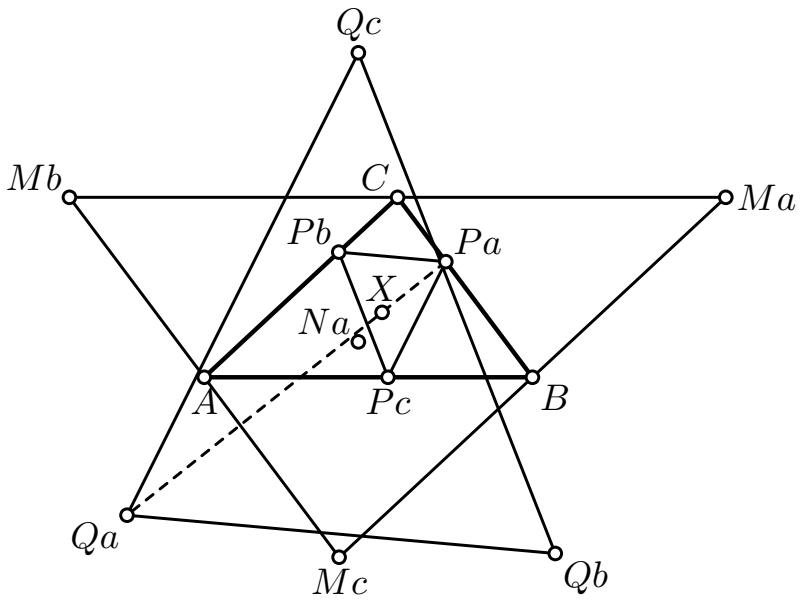

FIGURE 8.

Figure 8 illustrates Problem 8. In figure 8, *MaMbMc* is the Antimedial triangle, and *Na* is the Nagel point.

References for Problem 9: Nagel point in [7], de Longchamps Point in [7], Cevian Corner triangles in [3], and point X(9943) Midpoint of the de Longchamps point of triangle *ABC* and the Orthocenter of the Intouch triangle.

Problem 9. *The Intouch triangle PaPbPc is homothetic with the Triangle of the de Longchamps Points of the Cevian Corner Triangles of the Nagel Point QaQbQc. The center of the homothety X is the midpoint of the de Longchamps point of triangle ABC and the Orthocenter of the Intouch triangle. The ratio of the homothety k is*

$$k = -1 < 0.$$

Figure 9 illustrates Problem 9. In figure 9,

- *PaPbPc* is the Intouch triangle,
- *Na* is the Nagel point,

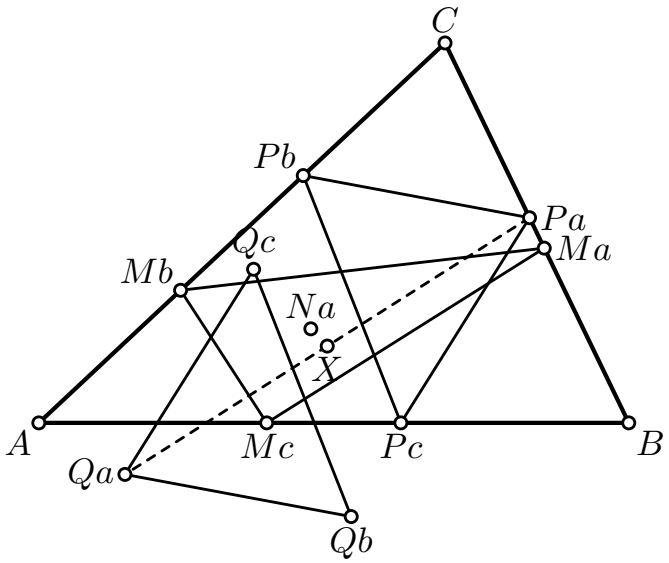

FIGURE 9.

- *MaMbMc* is the Extouch triangle, that is, the Cevian triangle of the Nagel point,
- *Qa* is the de Longchamps point of triangle *AMbMc*,
- *Qb* is the de Longchamps point of triangle *BMcMa*,
- *Qc* is the de Longchamps point of triangle *CMaMb*,
- *QaQbQc* is the Triangle of the de Longchamps Points of the Cevian Corner Triangles of the Nagel Point, and
- *X* is the center of the homothety.

References for Problem 10: Excentral triangle in [7], Anticomplement in [7], point X(3340) Intersection of the Line through the Incenter and Circumcenter and the Line through the Gergonne point and the Anticomplement of the Nagel point in [6].

Problem 10. *The Intouch triangle PaPbPc is homothetic with the Triangle of Reflections of the Vertices of the Excentral Triangle in the Incenter QaQbQc. The center of the homothety X is the Intersection of the Line through the Incenter and Circumcenter and the Line through the Gergonne point and the Anticomplement of the Nagel point.. The ratio of the homothety k is*

$$k = \frac{-4abc}{(a+b-c)(b+c-a)(c+a-b)} < 0.$$

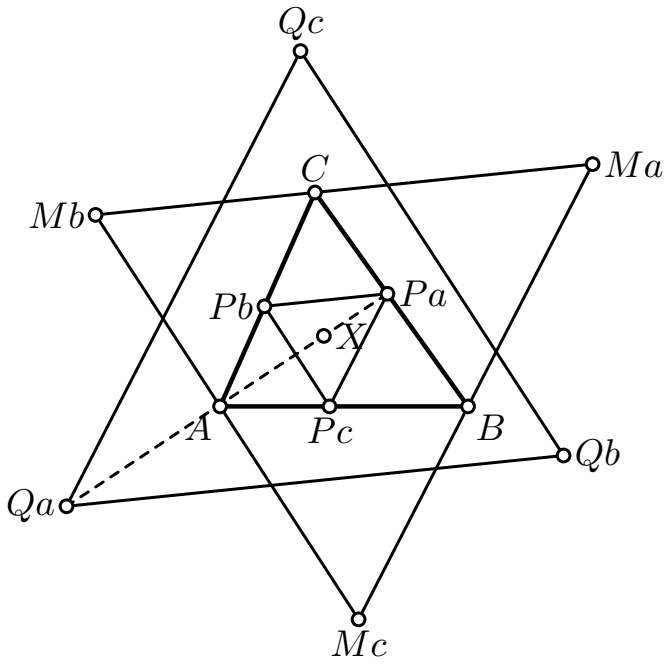

FIGURE 10.

Figure 10 illustrates Problem 10.

In figure 10,

- $PaPbPc$ is the Intouch triangle,
- $MaMbMc$ is the Excentral triangle,
- Qa is the reflection of point Ma about the Incenter,
- Qb is the reflection of point Mb about the Incenter,
- Qc is the reflection of point Mc about the Incenter,
- $QaQbQc$ is the Triangle of Reflections of the Vertices of the Excentral Triangle in the Incenter, and
- X is the center of the homothety.

Acknowledgement

The authors are grateful to Professor René Grothmann for his wonderful computer program *C.a.R.* http://car.rene-grothmann.de/doc_en/index.html. See also http://www.journal-1.eu/2016-1/Grothmann-CaR-pp.45-61.pdf. The authors are also grateful to Professor Troy Henderson http://www.tlhiv.org/ for his wonderful computer program *MetaPost Previewer* for creation of eps graphics http://www.tlhiv.org/mppreview/.

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