Educational Matters
Научно-методически статии

A SURVEY OF MATHEMATICS DISCOVERED BY COMPUTERS. PART 2.

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Abstract. This is the second part of the survey on mathematics discovered by computers. The first part is published in 2015 in the International Journal of Computer Discovered Mathematics. The second part of the survey contains articles published in 2015 – 2017.

Keywords: computer discovered mathematics; learning through inquiry; learning through discovery; Euclidean geometry; problems for students

Introduction

In 1958, in a seminal paper predicting future successes of artificial intelligence and operational research, Herbert Simon and Alan Newell (Simon & Newell 1958) suggested that:

"Within ten years a digital computer will discover an important mathematical theorem".

The prediction of Simon and Newell is accomplished in 2006 by the computer program "Discoverer". The leader of the team of the "Discoverer is Sava Grozdev (Bulgaria), with members Hiroshi Okumura (Japan) and Deko Dekov (Bulgaria). This is the second part of the survey on mathematics discovered by computers. The first part is published in (Grozdev & Dekov, 2015). Currently there are about 150 papers about the "Discoverer", containing more than 10 000 new theorems. A few of these papers are quoted in the first part of this survey. The recent papers, published in 2015 – 2017 are included in this paper.

In the references, IJCDM is an abbreviation for "International Journal of Computer Discovered Mathematics". The IJCDM is available at the web free of charge.

The computer program "Discoverer" is the first computer program, able easily to discover new theorems in mathematics, and possibly, the first computer program, able easily to discover new knowledge in science.

The ideas of the "Discoverer" are applicable to many areas of science. The authors of the "Discoverer" are interested is school geometry, so that the current realization of the "Discoverer" is devoted to Euclidean geometry.

We invite the interested persons to use the ideas of the "Discoverer" for effective discoveries in geometry, as well as in the genetics, biology, biotechnology, chemistry, physics, medicine, pharmacy.

The Discoverer

Discovery of theorems with the "Discoverer" is easy.

The "Discoverer" is a very simple computer program, based on simple, in some sense trivial ideas and algorithms.

The prototype of the "Discoverer" has been created by using the programming language JavaScript. Since 2012, a new version of the "Discoverer" is created by using the programming language PHP and data base MySQL.

The "Discoverer" is based on a few hundred independent modules. Each module could be independently extended, improved or deleted. This gives the opportunity the "Discoverer" to be created by a sequence of extensions. Also, we can add easily new modules.

The first group of modules creates the data base of the "Discoverer". The data base is created by using a simple production system (Nilsson, 1985).

The second group of modules creates the new theorems.

The reader may find description of the "Discoverer" in the paper (Grozdev & Dekov, 2014).

Note that the "Discoverer" has linear complexity, so that the process always terminates, relatively very fast. A simple desktop computer could be used effectively.

The discovered set of theorems is complete. If there is a theorem within a definite set of theorems, the "Discoverer" discovers it.

The "Discoverer" produces theorems by using a natural language. The reader could use the theorems without any changes.

The Paulson Criterion

We need a criterion for possible realization of the Simon-Newell prediction. There are a few criteria. Below we consider the Paulson criterion.

The Paulson criterion is as follows (Paulson, 2015): In order a theorem in mathematics to be accepted as a theorem discovered by a computer, the following data have to be published in a mathematical journal:

(1) The statement of the theorem.

(2) A statement that the result is discovered by a computer.

(3) Direct or indirect statement that the theorem is new, not published before.

(4) The name of the computer program which has discovered the result.

(5) The name of the authors who have created the computer program.

The Strong Paulson criterion is as follows: The Paulson criterion plus

(6) The basic ideas of the computer program have to be published.

(7) If a number of theorems are published by the creators of the computer program, it should be clear that the theorems are not discovered by the people.

The computer program "Discoverer" satisfies the Strong Paulson criterion. See the papers about the "Discoverer".

Simon and Newell speak about "important" theorem. The criterion about importance is not clear. We accept the following criterion: A new theorem is "important" if it generalizes published theorems, or it improves published theorems. If a theorem solves a hypothesis, it also should be considered as "important".

The computer program "Discoverer" satisfies the above criterion for "importance". See the papers about the "Discoverer".

Problems for students

The papers quoted in the two parts of this review contain also hundreds new theorems presented as problems. These problems are suitable for higher school students studying in the area of pedagogy of mathematics.

We strongly encourage the teachers and professors to use the problems for home works, essay for students and so on.

Scholarly Essays by Students

The aim of the "Discoverer" is to activate the interest of the students to mathematics and by this way to improve the high school and university education.

Note that an essay, based on the "Discoverer", is presented by M. Sirazitdinov at the European Student Conference in Mathematics at Thessaloniki, Greece, in 2016 (Sirazitdinov, 2016).

The use of the "Discoverer" in education process we call "learning through discovery". The "learning through discovery" is a new important direction within the "learning through inquiry".

In a number of papers Grozdev and Dekov have discussed the methodology for writing a scholarly essay by a student. An example from the paper (Grozdev & Dekov, 2015) is given below. In this special case the student investigates the reflections of remarkable points in the geometry of the triangle.

(1) The student chooses a set of remarkable points in the plane of (possibly from the database of the "Discoverer").

(2) The "Discoverer" produces a list of all reflections of these points. Note that the student could calculate these points by hand.

(3) "Discoverer" discovers which of the above produced points are not included in the Kimberling's ETC. The help of "Discoverer" at this stage is essential.

(4) "Discoverer" discovers new theorems about the points which are not available in Kimberling's ETC. The help of "Discoverer" at this stage is essential.

(5) The student uses the computer program for dynamic geometry, like C.a.R. or GeoGebra, in order to investigate the ruler-and-compass constructions of the new points. The student produces macros for the new points and animations for the ruler-and-compass constructions. The student produces also computer graphics for his or her essay.

(6) The student uses computer algebra system, like Maple, in order to prepare the proofs of the theorems of the essay. The student calculates also the barycentric coordinates of the new points.

(7) The student prepares the scholarly essay. The essay contains as supplementary material the HTML-files, produced by "Discoverer", the C.a.R. files and graphics, and the Maple files, produced by the student.

During the work on the scholarly essay, the student will improve his skills to use a discovery system, like "Discoverer", a system for dynamic geometry, like C.a.R. (or GeoGebra), and a computer algebra system, like Maple. These skills are between the basic skills which the student has to master and improve during his education.

The Future

Euclid is said to have said to the first Ptolemy who inquired if there was a shorter way to learn geometry than the Elements: there is no royal road to geometry.

Now we have a royal road to geometry. It is not necessary we to be inventive. The "Discoverer" will tell us what is necessary. All which we have to do is to write our problem and to go to drink coffee. We will drink coffee and the "Discoverer" will work for us. It is easy.

Conclusion

The era of new knowledge discovered only by human being is over. We are going into a new era – the era of the computers-discoverers. The computers-discoverers are important. They could extend effectively the current science and technology.

It is time we to open our eyes and to see the advantages of the computers-discoverers. They work fast, they work hard, they work day and night, they do not need salaries, they do not make errors, and most important, they can do what the people cannot do.

Today "Discoverer" is the only computer program, able to discover new theorems in mathematics. We hope that tomorrow a pleiad of computers-discoverers will help the people to move faster in science.

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ОБЗОР НА МАТЕМАТИКАТА, ОТКРИТА ОТ КОМПЮТРИ. ЧАСТ II.

Резюме. Това е втората част на обзор на математиката, открита от компютри. Първата част на обзора е публикувана през 2015 г. в списание International Journal of Computer Discovered Mathematics. Втората част на обзора включва статиите, публикувани през 2015 – 2017 г.

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